Syllabi of Semester-IV based on Under Graduate Curriculum Framework -2022

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Sem-IV

DISCIPLINE SPECIFIC CORE COURSE – 10: SEQUENCES AND SERIES OF FUNCTIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Sequences and Series of Functions	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The objective of the course is to introduce:

- The sequences and series of real-valued functions as a generalization to the sequences and series of real numbers.
- The situations under which the process of convergence of a sequence and series of real-valued functions may commute with the processes of calculus while taking differentiation, or integration.
- An important class of series functions (i.e., power series), and the definitions of the elementary functions- exponential, logarithmic and trigonometric.

Learning Outcomes: This course will enable the students to:

- Learn about Cauchy criterion for uniform convergence and Weierstrass M-test for uniform convergence of series of real-valued functions.
- Know about the constraints for the inter-changeability of differentiability and integrability with infinite sum.
- Approximate transcendental functions in terms of power series as well as, differentiation and integration of power series.

SYLLABUS OF DSC-10

UNIT – I: Sequences of Functions (18 hours)

Pointwise and uniform convergence of sequence of functions, The uniform norm, Cauchy criterion for uniform convergence, Continuity of the limit function of a sequence of functions, Interchange of the limit and derivative, and the interchange of the limit and integrability of a sequence of functions, Bounded convergence theorem.

UNIT – II: Series of Functions **(12 hours)**

Pointwise and uniform convergence of series of functions, Theorems on the continuity, derivability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass M-test for uniform convergence.

UNIT – III: Power Series **(15 hours)**

Definition of a power series, Radius of convergence, Absolute convergence (Cauchy-Hadamard theorem), Differentiation and integration of power series, Abel's theorem, Weierstrass approximation theorem; The exponential, logarithmic and trigonometric functions: definitions and their basic properties.

Essential Readings

1. Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). Wiley India Edition. Indian Reprint.
2. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

Suggestive Readings

- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
- Denlinger, Charles G. (2011). Elements of Real Analysis. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

DISCIPLINE SPECIFIC CORE COURSE – 11: MULTIVARIATE CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The primary objective of this course is to introduce:

- The extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- The geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- The techniques of integration to functions of two and three independent variables.
- The applications of multivariate calculus tools to physics, economics, optimization etc.

Learning Outcomes: This course will enable the students to:

- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double, and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems.

SYLLABUS OF DSC-11

UNIT – I: Calculus of Functions of Several Variables (18 hours)

Basic concepts, Limits and continuity, Partial derivatives, Tangent planes, Total differential, Differentiability, Chain rules, Directional derivatives and the gradient, Extrema of functions of two variables, Method of Lagrange multipliers with one constraint.

UNIT – II: Double and Triple Integrals (15 hours)

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integrals over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

UNIT – III: Green's, Stokes' and Gauss Divergence Theorem (12 hours)

Vector field, Divergence and curl, Line integrals and applications to mass and work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

Essential Reading

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. Pearson Education. Indian Reprint.

Suggestive Reading

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). Indian Reprint.

DISCIPLINE SPECIFIC CORE COURSE – 12: NUMERICAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Numerical Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The main objective of this course is to introduce:

- Various computational techniques to find approximate value for possible root(s) of algebraic and non-algebraic equations.
- Methods to solve system of linear equations and ordinary differential equations.
- The use of computer algebra system (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem-solving skills.

Learning Outcomes: The course will enable the students to:

- Learn some numerical methods to find the zeroes of nonlinear functions of a single variable, up to a certain given level of precision.
- Learn Gauss−Jacobi, Gauss−Seidel methods to solve system of linear equations.
- Compute the values for a tabulated function at points not in the table using interpolation techniques.
- Learn applications of numerical differentiation and integration to convert differential equations into difference equations for numerical solutions.

SYLLABUS OF DSC-12

UNIT – I: Methods for Solving Algebraic and Transcendental Equations (12 hours)

Rate and order of convergence; Bisection method, Method of false position, Fixed point iteration method, Newton's method, and Secant method, their order of convergence and convergence analysis.

UNIT – II: Techniques to Solve Linear Systems and Interpolation (15 hours)

LU decomposition and its applications; Iterative methods: Gauss−Jacobi, Gauss−Seidel methods; Lagrange and Newton interpolation, Piecewise linear interpolation.

UNIT – III: Numerical Differentiation and Integration (18 hours)

First and higher order approximation for the first derivative, Approximation for the second derivative; Numerical integration by closed Newton−Cotes formulae: Trapezoidal rule, Simpson's rule and its error analysis; Euler's method to solve ODE's, Modified Euler method, Runge−Kutta Method (fourth-order).

Essential Reading

1. Bradie, Brian. (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third impression 2011.

Suggestive Readings

- Gerald, Curtis F., & Wheatley, Patrick O. (2007). Applied Numerical Analysis (7th ed.). Pearson Education. India.
- Jain, M. K., Iyengar, S. R. K., & Jain, R. K. (2012). Numerical Methods for Scientific and Engineering Computation. (6th ed.). New Age International Publisher, India, 2016.

Note: Non programmable scientific calculator may be allowed in the University examination.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Use of computer algebra system (CAS) software: Python/SageMath/Mathematica/MATLAB/Maple/Maxima/ Scilab etc., for developing the following numerical programs:

1. Bisection method.
2. Newton-Raphson method.
3. Secant method.
4. LU decomposition method.
5. Gauss−Jacobi method.
6. Gauss−Seidel method.
7. Lagrange interpolation.
8. Newton interpolation.
9. Trapezoidal rule.
10. Simpson's rule.
11. Euler's method.
12. Runge−Kutta Method (fourth-order).

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

B.Sc. (Hons) Mathematics, Semester-IV, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): BIOMATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Biomathematics	4	3	1	0	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to:

- Develop and analyse the models of the biological phenomenon with emphasis on population growth and predator-prey models.
- Interpret first-order autonomous systems of nonlinear differential equations using the Poincaré phase plane.
- Apply the basic concepts of probability to understand molecular evolution and genetics.

Learning Outcomes: The course will enable the students to:

- Get a better comprehension of mathematical models, utilised in biology.
- To identify and explain the findings from models of population studies, species' communication, adaptation, and dynamics of disorder.
- Create a basic model of molecular evolution by making use of probability and matrices.

SYLLABUS OF DSE-2(i)

UNIT – I: Mathematical Modeling for Biological Processes (15 hours)

Formulation a model through data, A continuous population growth model, Long-term behavior and equilibrium states, The Verhulst model for discrete population growth, Administration of drugs, Differential equation of chemical process and predator-prey model (Function response: Types I, II and III).

UNIT – II: Epidemic Model: Formulation and Analysis (15 hours)

Introduction to infectious disease, The SIS, SIR and SEIR models of the spread of an epidemic, Analyzing equilibrium states, Phase plane analysis, Stability of equilibrium points, Classifying the equilibrium state; Local stability, Limit cycles, Poincaré-Bendixson theorem.

UNIT – III: Bifurcation, Chaos and Modeling Molecular Evolution (15 hours)

Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation and period-doubling, Chaos, Stability of limit cycles, Introduction of the Poincaré plane; Modeling molecular evolution: Matrix models of base substitutions for DNA sequences, Jukes-Cantor and Kimura models, Phylogenetic distances.

Essential Readings

1. Robeva, Raina S., et al. (2008). An Invitation to Biomathematics. Academic press.
2. Jones, D. S., Plank, M. J., & Sleeman, B. D. (2009). Differential Equations and Mathematical Biology (2nd ed.). CRC Press, Taylor & Francis Group.
3. Allman, Elizabeth S., & Rhodes, John A. (2004). Mathematical Models in Biology: An Introduction. Cambridge University Press.

Suggestive Readings

- Linda J. S. Allen (2007). An Introduction to Mathematical Biology. Pearson Education.
- Murray, J. D. (2002). Mathematical Biology: An Introduction (3rd ed.). Springer.
- Shonkwiler, Ronald W., & Herod, James. (2009). Mathematical Biology: An Introduction with Maple and MATLAB (2nd ed.). Springer.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: Primary objective of this course is to introduce:

- Mathematical modelling as the representation of a system by a set of mathematical relations or equations.

- Mathematical epidemiological models susceptible-infectious-recovered (SIR) and its variant SEIR (S-Exposed-IR) for the spread of diseases.
- Monte Carlo simulation techniques, and simplex method for solving linear programming problems.

Learning Outcomes: The course will enable the students to:

- Understand the methodology of solving SIR models for disease spread.
- Learn significance of dieting model that provides important insights and guides to a biomedical issue that is of interest to the general public.
- Understand nonlinear systems and phenomena with stability analysis ranges from phase plane analysis to ecological and mechanical systems.
- Use Monte Carlo simulation technique to approximate area under a given curve, and volume under a given surface.

SYLLABUS OF DSE-2(ii)

UNIT – I: Mathematical Epidemiological and Dieting Models (15 hours)

Modeling concepts and examples, Scaling of variables, and approximations of functions; SIR and SEIR models for disease spread: Methodology, Standard and solvable SIR models, Basic reproduction number; Dieting model with analysis and approximate solutions.

UNIT – II: Modeling with Nonlinear Systems and Phenomena (15 hours)

Stability and the phase plane, Almost linear systems; Ecological models: Predators and competitors, Critical points, Oscillating populations, Survival of single species, Peaceful Coexistence of two species, Interaction of logistic populations, Wildlife conservation preserve; Nonlinear mechanical systems: Hard and soft spring oscillations, Damped nonlinear vibrations.

UNIT – III: Simulation and Optimization Modeling (15 hours)

Monte Carlo simulating deterministic, and probabilistic behavior, Generating random numbers; Linear programming model: Geometric and algebraic solutions, Simplex method and its tableau format, Sensitivity analysis.

Essential Readings

1. Mickens, Ronald E. (2022). Mathematical Modelling with Differential Equations. CRC Press, Taylor & Francis Group.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2023). Differential Equations and Boundary Value Problems: Computing and Modeling (6th ed.). Pearson.
3. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). Brooks/Cole, Cengage Learning India Pvt. Ltd.

Suggested Readings

- Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modeling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press. Taylor & Francis Group.
- Ross, Shepley L. (2014). Differential Equations (3rd ed.). Wiley India Pvt. Ltd.
- Simmons, George F. (2017). Differential Equations with Applications and Historical Notes (3rd ed.). CRC Press. Taylor & Francis Group.

Practical (30 hours)- Practical work to be performed in Computer Lab: Modeling of the following problems using: R/Python/SageMath/Mathematica/MATLAB/Maxima/Scilab etc.

1. a) Simulation of SIR model and its variants using some initial parameter values, and finding basic reproduction number for analysis.
   b) Analysis of the dieting process, which includes both body-mass loss and gain.
2. Nonlinear Systems and Phenomena.
   a) Plot phase plane portraits and solutions of first-order equations.
   b) Obtain interesting and complicated phase portraits for almost linear systems.
   c) Discuss large wildlife conservation preserve model and obtain (i) The period of oscillation of the rabbit and fox populations, (ii) The maximum and minimum numbers of rabbits and foxes.
   d) Discuss the Rayleigh and van der Pol models.
3. (i) Random number generation and then use it for the following:
   a) Simulate area under a given curve.
   b) Simulate volume under a given surface.

   (ii) [2] Chapter 7 (Projects 7.4 and 7.5).

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(iii): MECHANICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mechanics	4	3	1	0	Class XII pass with Mathematics	DSC-5: Calculus DSC-6: Ordinary Differential Equations

Learning Objectives: The core purpose of the course is to:

- Give the know-how of the concept of mechanics to the students.
- Make the students familiar with the notion of statics and a level up with the dynamics, hydrostatics, and related concepts.
- Finally winds up the journey with meaningful usage of the knowledge attained.

Learning Outcomes: This course will strengthen the concept of:

- Notion of statics, dynamics, and hydrostatics with applications.
- Methods discussed in this course and maneuver the practical aspect of couples, projectiles, and fluid pressure.
- Applicability of the knowledge of the friction, harmonic oscillators as well as thrust on plane surfaces.

SYLLABUS OF DSE-2(iii)

UNIT – I: Statics **(15 hours)**

Fundamental laws of Newtonian mechanics, Law of parallelogram of forces, Equilibrium of a particle, Lamy's theorem, Equilibrium of a system of particles, External and internal forces, Couples, Reduction of a plane force system, Work, Principle of virtual work, Potential energy and conservative field, Mass centers, Centers of gravity, Friction.

UNIT – II: Dynamics **(18 hours)**

Kinemetics of a particle, Motion of a particle, Motion of a system, Principle of linear momentum, Motion of mass center, Principle of angular momentum, Motion relative to mass center, Principle of energy, D'Alembert's principle; Moving frames of reference, Frames of reference with uniform translational velocity, Frames of reference with constant angular velocity; Applications in plane dynamics- Motion of a projectile, Harmonic oscillators, General motion under central forces, Planetary orbits.

UNIT – III: Hydrostatics **(12 hours)**

Shearing stress, Pressure, Perfect fluid, Pressure at a point in a fluid, Transmissibility of liquid pressure, Compression, Specific gravity, Pressure of heavy fluid- Pressure at all points in a horizontal plane, Surface of equal density; Thrust on plane surfaces.

Essential Readings

1. Synge, J. L., & Griffith, B. A. (1987). Principles of Mechanics (3rd ed.). McGraw-Hill. Indian Reprint.
2. Ramsey, A. S. (1961). Hydrostatics. Cambridge University Press. Indian Reprint.

Suggested Readings

- Roberts, A. P. (2003). Statics and Dynamics with Background Mathematics. Cambridge University Press.
- Ramsey, A. S. (1985). Statics (2nd ed.). Cambridge University Press.

B.A. (Prog.) Sem-IV with Mathematics as Major
Category-II

DISCIPLINE SPECIFIC CORE COURSE (DSC-4): INTRODUCTION TO GRAPH THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Introduction to Graph Theory	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

Learning Outcomes: This course will enable the students to:

- Learn modeling of real-world problems by graphs.
- Know characteristics of different classes of graphs.
- Learn representation of graphs in terms of matrices.
- Understand some properties of graphs and their applications in practical situations.

SYLLABUS OF DSC-4

UNIT-I: Graphs, Types of Graphs and Basic Properties (12 hours)

Graphs and their representation, Pseudographs, Subgraphs, Degree sequence, Euler's theorem, Isomorphism of graphs, Paths and circuits, Connected graphs, Euler trails and circuits, Hamiltonian paths and cycles, Adjacency matrix, Weighted graphs, Travelling salesman problem, Dijkstra's algorithm.

UNIT-II: Directed Graphs and Applications, Trees (18 hours)

The Chinese postman problem; Digraphs, Bellman-Ford algorithm, Tournaments, Directed network, Scheduling problem; Trees and their properties, Spanning trees, Kruskal's algorithm, Prim's algorithm, Acyclic digraphs and Bellman's algorithm.

UNIT-III: Planar Graphs, Graph Coloring and Network Flows (15 hours)

Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring, Applications of graph coloring, Circuit testing and facilities design, Flows and cuts, Max flow-min cut theorem, Matchings, Hall's theorem.

Essential Reading

1. Goodaire, Edgar G., & Parmenter, Michael M. (2011). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.

Suggestive Readings

- Bondy, J. A. & Murty, U.S.R. (2008), Graph Theory with Applications. Springer.
- Chartrand, Gary, & Zhang, P. (2012). A First Course in Graph Theory. Dover Publications.
- Diestel, R. (1997). Graph Theory (Graduate Texts in Mathematics). Springer Verlag.
- West, Douglas B. (2001). Introduction to graph theory (2nd ed.). Pearson India.

DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

Learning Outcomes: This course will enable the students to:

- Understand groups, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals and factor rings.

SYLLABUS OF DISCIPLINE A-4

UNIT-I: Introduction to Groups (12 hours)

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

UNIT-III: Rings, Integral Domains and Fields (15 hours)

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

Suggested Reading

- Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.

B.Sc. (Prog.)/ BA (Prog.) Sem-IV with Mathematics as non-Major

Category-III

DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

Learning Outcomes: This course will enable the students to:

- Understand groups, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals and factor rings.

SYLLABUS OF DISCIPLINE A-4

UNIT-I: Introduction to Groups (12 hours)

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

UNIT-III: Rings, Integral Domains and Fields **(15 hours)**

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

Suggested Reading

- Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.

B.Sc. (Physical Sciences/Mathematical Sciences) Sem-IV with Mathematics as one of the Core Discipline

Category-III

DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

Learning Outcomes: This course will enable the students to:

- Understand groups, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals and factor rings.

SYLLABUS OF DISCIPLINE A-4

UNIT-I: Introduction to Groups **(12 hours)**

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

UNIT-III: Rings, Integral Domains and Fields (15 hours)

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

Suggested Reading

- Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.

DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Sem-IV

Category-III

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): ELEMENTS OF DISCRETE MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Discrete Mathematics	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: Students are introducing to:

- Order (or partial order) and related properties.
- Notion of a lattice which is also a step towards abstract algebra.
- Concept of Boolean algebra and its applications to minimizing a Boolean polynomial and switching circuits, which has further applications in computer science.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of sets, relations, functions, and induction.
- Understand mathematical logic and logical operations to various fields.
- Understand the notion of order and maps between partially ordered sets.
- Minimize a Boolean polynomial and apply Boolean algebra techniques to decode switching circuits.

SYLLABUS OF DSE-2(i)

UNIT-I: Sets, Relations and Functions (18 hours)

Sets, Propositions and logical operations, Conditional statements, Mathematical induction, Relations and equivalence relation, Equivalence classes, Partial order relation, Partially ordered set, Hasse diagrams, Chain, Maximal and minimal elements, least and greatest elements, Least upper bound, Greatest lower bound, Zorn's lemma, Functions and bijective functions, Functions between POSETS, Order isomorphism.

UNIT-II: Lattices (12 hours)

Lattice as a POSET, Lattice as an algebra and their equivalence, Bounded lattices, Sublattices, Interval in a lattice, Products and homomorphism of lattices, Isomorphism of lattices; Distributive, Complemented, Partition and pentagonal lattices.

UNIT-III: Boolean Algebra and Switching Circuits (15 hours)

Boolean algebra, De Morgan's laws, Boolean expressions, Truth tables, Logic diagrams, Boolean functions, Disjunctive normal forms (as join of meets), Minimal forms of Boolean polynomials, Quine Mc-Cluskey method, Karnaugh maps, Switching circuits, Applications of switching circuits.

Essential Readings

1. Rudolf Lidl, & Gunter Pilz (2004). Applied Abstract Algebra (2nd ed.). Undergraduate text in Mathematics, Springer (SIE), Indian Reprint.
2. Bernard Kolman, Robert C. Busby, & Sharon Cutler Ross (2009). Discrete Mathematical Structures (6th ed.). Pearson education Inc., Indian reprint.

Suggested Reading

- Rosen, Kenneth H. (2017). Discrete Mathematics and its applications with combinatorics and Graph Theory (7th ed.). McGraw Hill Education.

DISCIPLINE SPECIFIC ELECTIVE COURSE-2(ii): INTRODUCTION TO GRAPH THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Introduction to Graph Theory	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

Learning Outcomes: This course will enable the students to:

- Learn modeling of real-world problems by graphs.
- Know characteristics of different classes of graphs.
- Learn representation of graphs in terms of matrices.
- Understand some properties of graphs and their applications in practical situations.

SYLLABUS OF DSE-2(ii)

UNIT-I: Graphs, Types of Graphs and Basic Properties (12 hours)

Graphs and their representation, Pseudographs, Subgraphs, Degree sequence, Euler's theorem, Isomorphism of graphs, Paths and circuits, Connected graphs, Euler trails and circuits, Hamiltonian paths and cycles, Adjacency matrix, Weighted graphs, Travelling salesman problem, Dijkstra's algorithm.

UNIT-II: Directed Graphs and Applications, Trees (18 hours)

The Chinese postman problem; Digraphs, Bellman-Ford algorithm, Tournaments, Directed network, Scheduling problem; Trees and their properties, Spanning trees, Kruskal's algorithm, Prim's algorithm, Acyclic digraphs and Bellman's algorithm.

UNIT-III: Planar Graphs, Graph Coloring and Network Flows (15 hours)

Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring, Applications of graph coloring, Circuit testing and facilities design, Flows and cuts, Max flow-min cut theorem, Matchings, Hall's theorem.

Essential Reading

1. Goodaire, Edgar G., & Parmenter, Michael M. (2011). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.

Suggestive Readings

- Bondy, J. A. & Murty, U.S.R. (2008), Graph Theory with Applications. Springer.
- Chartrand, Gary, & Zhang, P. (2012). A First Course in Graph Theory. Dover Publications.
- Diestel, R. (1997). Graph Theory (Graduate Texts in Mathematics). Springer Verlag.
- West, Douglas B. (2001). Introduction to graph theory (2nd ed.). Pearson India.

DISCIPLINE SPECIFIC ELECTIVE COURSE-2(iii): LINEAR PROGRAMMING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The solution of linear programming problem using simplex method.
- The solution of transportation and assignment problems.
- Game theory which makes possible the analysis of the decision-making process of two interdependent subjects.

Learning Outcomes: This course will enable the students to:

- Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- Write the dual of a linear programming problem.
- Solve the transportation and assignment problems.
- Learn about solution of rectangular games using graphical method and dominance.
- Formulate game to a pair of associated prima-dual linear programming problems.

SYLLABUS OF DSE-2(iii)

UNIT-I: Linear Programming Problem, Simplex Method, and Duality (18 hours)

Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity; Introduction to the simplex method**:** Optimality criterion and unboundedness, Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples.

UNIT-II: Transportation and Assignment Problems (15 hours)

Definition of transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problem; Hungarian method of solving assignment problem.

UNIT-III: Two-Person Zero-Sum Games (12 hours)

Introduction to game theory, rectangular games, Mixed strategies, Dominance principle; Formulation of game to primal and dual linear programming problems.

Essential Readings

1. Thie, Paul R., & Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Hillier, F. S., & Lieberman, G. J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.

COMMON POOL OF GENERIC ELECTIVES (GE) Sem-IV COURSES OFFERED BY DEPARTMENT OF MATHEMATICS

Category-IV

GENERIC ELECTIVES (GE-4(i)): ELEMENTS OF REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

Learning Outcomes: This course will enable the students to:

- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Apply limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF GE-4(i)

UNIT-I: Basic Properties of the Set of Real Numbers (12 hours)

Field and order properties of $\mathbb{R}$, basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of $\mathbb{R}$.

UNIT-II: Real Sequences (18 hours)

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

UNIT-III: Infinite Series of Real Numbers (15 hours)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series, Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's nth root test, Raabe's test; Alternating series, Leibniz alternating series test, Absolute and conditional convergence.

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Additional Readings:

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

GENERIC ELECTIVES (GE-4(ii)): LINEAR PROGRAMMING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The solution of linear programming problem using simplex method.
- The solution of transportation and assignment problems.
- Game theory which makes possible the analysis of the decision-making process of two interdependent subjects.

Learning Outcomes: This course will enable the students to:

- Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- Write the dual of a linear programming problem.
- Solve the transportation and assignment problems.
- Learn about solution of rectangular games using graphical method and dominance.
- Formulate game to a pair of associated prima-dual linear programming problems.

SYLLABUS OF GE-4(ii)

UNIT-I: Linear Programming Problem, Simplex Method, and Duality (18 hours)

Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity; Introduction to the simplex method**:** Optimality criterion and unboundedness, Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples.

UNIT-II: Transportation and Assignment Problems (15 hours)

Definition of transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problem; Hungarian method of solving assignment problem.

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