UNIVERSITY OF DELHI

DEPARTMENT OF MATHEMATICS

B.Sc.(Prog.) Physical Sciences/Mathematical Sciences

Learning Outcomes based Curriculum Framework (LOCF)

2019

Introduction

The modern citizen is routinely confronted by a maze of numbers and data of various forms in today's information-overload world. An increased knowledge of mathematics is essential to be able to make sense out of this, Mathematics is at the heart of many of today's advancements in science and technology. Studying mathematics along with physics and chemistry can provide a firm foundation for further study in a variety of other disciplines. Students who have learned to logically question assertions, recognize patterns, and distinguish the essential and irrelevant aspects of problems can think deeply and precisely, nurture the products of their imagination to fruition in reality, and share their ideas and insights while seeking and benefiting from the knowledge and insights of others.

Programme Learning Outcomes in course:

A well-structured Mathematical component in B.Sc.(Programme) Physical/Mathematical Sciences empowers the students to:

• Solve problems using a broad range of significant mathematical techniques, including calculus, algebra, geometry, analysis, numerical methods, differential equations, probability and statistics along with hands-on learning through CAS and LaTeX.

• Analyze quantitative data using statistical analysis techniques.

• Combine the principles of physics and chemistry, as supported by mathematics to describe the foundational concepts of the physical world and apply these concepts to new situations.

• Apply the techniques of mathematics to understand experimental observations and predict outcomes.

• Collaborate with others, including multidisciplinary groups, to solve scientific problems, and to recognize ethical issues in each respective profession.

CBCS Course Structure for Under -Graduate B.Sc. Programme

Courses	***Credits** Theory+ Practical	**Theory+ Tutorial**
I. Core Courses	$12\times 4 = 48$	$12\times 5 = 60$
(12 Papers)		
Core Course Practical / Tutorial*		
Two papers – English		
Two papers – MIL		
Four papers – Discipline 1.		
Four papers – Discipline 2.		
Core Course Practical / Tutorial*	$12\times 2 = 24$	$12\times 1 = 12$
(12 Practicals/Tutorials*)		
II. Elective Courses		
(6 Papers)	$6\times 4 = 24$	$6\times 5 = 30$
Two papers- Discipline 1 specific		
Two papers- Discipline 2 specific		
Two papers- Inter disciplinary		
Two papers from each discipline of choice and two papers of interdisciplinary nature.		
Elective Course Practical / Tutorials*	$6\times 2 = 12$	$6\times 1 = 6$
(6 Practical/ Tutorials*)		
Two papers- Discipline 1 specific		
Two papers- Discipline 2 specific		
Two papers- Generic (Inter disciplinary)		
Two papers from each discipline of choice including papers of interdisciplinary nature.		
III. Ability Enhancement Courses		
Ability Enhancement Compulsory Courses (AECC)	$2\times 4 = 8$	$2\times 4 = 3$
(2 Papers of 2 credits each)		
Environmental Science		
English /MIL Communication		
Skill Enhancement Courses (SEC)	$4\times 4 =16$	$4\times 4 = 16$
(4 Papers of 4 credits each)		
Total credits:	**132**	**132**

*** Wherever there is a practical there will be no tutorial and vice-versa**
Note: One-hour lecture per week equals 1 Credit, 2 Hours practical class per week equals 1 credit. Practical in a group of 15-20 students in Computer Lab and Tutorial in a group of 12-15 students.

SEMESTER WISE PLACEMENT OF THE COURSES

Sem-ester	**Core Course (12)**	**Ability Enhancement Compulsory Course (AECC) (2)**	**Skill Enhancement Course (SEC) (4)**	**Discipline Specific Elective (DSE) (6)**
I	Calculus and Matrices			
II	Calculus and Geometry			
III	Abstract Algebra		**SEC-1** Computer Algebra Systems	
IV	Real Analysis		**SEC-2** Mathematical Typesetting System: LaTeX	
V			**SEC-3** Transportation and Network Flow Problems	**DSE-1** (i) Differential Equations (with Practicals) OR (ii) Mechanics and Discrete Mathematics
VI			**SEC-4** Statistical Software: **R**	**DSE-2** (i) Numerical Methods (with Practicals) OR (ii) Probability and Statistics

Mathematics Courses Details for the B.Sc. Programme:

Semester-I

Paper I: Calculus and Matrices

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The primary objective of this course is to gain proficiency in differential calculus, and introduce the basic tools of matrices and complex numbers which are used to solve application problems in a variety of settings ranging from chemistry and physics to business and economics. Differential calculus develops the concepts of limit, continuity and derivative, and is fundamental for many fields of mathematics.
Course Learning Outcomes: This course will enable the students to:

i) Define and use fundamental concepts of calculus including limits, continuity and differentiability.
ii) Solve systems of linear equations and find eigenvalues and corresponding eigenvectors for a square matrix, and check for its diagonalizability.
iii) Perform operations with various forms of complex numbers to solve equations.

Unit 1: Calculus **(Lectures: 30)**
Graphs of simple concrete functions such as polynomial, Trigonometric, Inverse trigonometric, Exponential and logarithmic functions; Limits and continuity of a function including $\varepsilon-\delta$ approach, Properties of continuous functions including Intermediate value theorem; Differentiability, Successive differentiation, Leibnitz theorem, Recursion formulae for higher derivatives; Rolle's theorem, Lagrange's mean value theorem with geometrical interpretations and simple applications, Taylor's theorem, Taylor's series and Maclaurin's series, Maclaurin's series expansion of functions such as e^x, $\sin x$, $\cos x$, $\log(1+x)$ and $(1+x)^m$; their use in polynomial approximation and error estimation; Functions of two or more variables, Graphs and level curves of functions of two variables, Partial differentiation up to second order.

Unit 2: Matrices **(Lectures: 25)**
Elementary row operations, Row reduction and echelon forms, Solution of systems of linear equations in matrix form, Linear independence and dependence, The rank of a matrix and applications; Elementary linear transformations like shear, translation, dilation, rotation, refection, and their matrix form, The matrix of a general linear transformation; Eigenvectors & eigenvalues of square matrices up to order 3 and diagonalization.

Unit 3: Complex Numbers **(Lectures: 15)**
Geometrical representation of addition, subtraction, multiplication and division of complex numbers; Lines, circles, and discs in terms of complex variables; Statement of the Fundamental Theorem of Algebra and its consequences; De Moivre's theorem and its application to solve simple equations in complex variables.

References:

1. Andreescu, Titu & Andrica Dorin. (2014). *Complex Numbers from A to...Z.* (2nd ed.). Birkhäuser.
2. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
3. Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.
4. Lay, David C., Lay, Steven, R., & McDonald Judi, J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson.
5. Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

Additional Reading:

i. Prasad, Gorakh (2016). *Differential Calculus* (19th ed.). Pothishala Pvt. Ltd. Allahabad.

Teaching Plan (Paper I: Calculus and Matrices):

Week 1: Graphs of simple concrete functions such as polynomial, Trigonometric, Inverse trigonometric, Exponential and logarithmic functions. [5] Sections 1.1, 1.2, 1.3, 7.2, 7.3, and 7.6.
Weeks 2 and 3: Limits and continuity of a function including $\varepsilon-\delta$ approach, Properties of continuous functions including Intermediate value theorem. [2] Chapter 1.
Week 4: Differentiability, Successive differentiation, Leibnitz theorem, Recursion formulae for higher derivatives. [5] Chapter 3 (Sections 3.2, 3.3, and 3.6), and Exercise 26, Page 184.
Week 5: Rolle's theorem, Lagrange's mean value theorem with geometrical interpretations and simple applications, Taylor's theorem, Taylor's series and Maclaurin's series, Maclaurin's expansion of functions such as $e^x, \sin x, \cos x, \log(1+x)$ and $(1+x)^m$; their use in polynomial approximation and error estimation. [5] Chapter 4 (Sections 4.2, and 4.3). [2] Chapter 9 (Sections 9.8, and 9.9).
Week 6: Functions of two or more variables, Graphs and Level curves of functions of two variables, Partial differentiation up to second order. [2] Chapter 13 (Sections 13.1, and 13.3).
Weeks 7 and 8: Elementary row operations, Row reduction and echelon forms, Solution of systems of linear equations in matrix form, Linear independence and dependence, The rank of a matrix and applications. [4] Chapter 1 (Sections 1.1, 1.2, 1.4, 1.6, and 1.7). [3] Section 6.6 (Pages 287 to 291)].
Weeks 9 and 10: Elementary linear transformations like shear, translation, dilation, rotation, refection, and their matrix form, The matrix of a general linear transformation. [4] Chapter 1 (Sections 1.8, and 1.9).
Week 11: Eigenvectors & eigenvalues of square matrices up to order 3 and diagonalization.
[4] Chapter 5 (Sections 5.1 to 5.3).
Weeks 12 to 14: Geometrical representation of addition, subtraction, multiplication and division of complex numbers; Lines, Circles, Discs in terms of complex variables; Statement of the Fundamental Theorem of Algebra and its consequences; De Moivre's theorem and its application to solve simple equations in complex variables. [5] Appendix A.7.
[1] Sections 1.2, 2.1.2, 2.1.3, 2.1.4, 2.2.3, 3.5.1, 3.5.2, and 3.6.1.

Semester-II

Paper II: Calculus and Geometry

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)`
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objectives of this course are to consider applications of derivatives for sketching of curves and conics and application of definite integrals for calculating volumes of solids of revolution, length of plane curves and surface areas of revolution which are helpful in understanding their applications in plenary motion, design of telescope and to many real-world problems.

Course Learning Outcomes: This course will enable the students to:

i) Sketch curves in a plane using its mathematical properties in the different coordinate systems of reference.
ii) Compute area of surfaces of revolution and the volume of solids by integrating over cross-sectional areas.
iii) Be well versed with conics and quadric surfaces so that they should able to relate the shape of real life objects with the curves/conics.

Unit 1: Derivatives for Graphing and Applications (Lectures:25)
The first derivative test for relative extrema, Concavity and inflection points, Second derivative test for relative extrema, Curve sketching using first and second derivative tests, Limits to infinity and infinite limits, Graphs with asymptotes, L'Hôpital's rule; Parametric representation of curves and tracing of parametric curves (except lines in $\mathbb{R}^3$), Polar coordinates and tracing of curves in polar coordinates.

Unit 2: Volume and Area of Surfaces (Lectures: 20)
Volumes by slicing disks and method of washers, Volumes by cylindrical shells, Arc length, Arc length of parametric curves, Area of surface of revolution; Reduction formulae.

Unit 3: Geometry and Vector Calculus (Lectures: 25)
Techniques of sketching conics, Reflection properties of conics, Rotation of axes and second degree equations, Classification into conics using the discriminant; Introduction to vector functions and their graphs, Operations with vector-valued functions, Limits and continuity of vector functions, Differentiation of vector-valued functions, gradient, divergence, curl and their geometrical interpretation; Spheres, Cylindrical surfaces; Illustrations of graphing standard quadric surfaces like cone, ellipsoid.

References:

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Strauss, M. J., Bradley, G. L., & Smith, K. J. (2007). *Calculus* (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Sixth impression 2011.

Additional Reading:

i. Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

Teaching Plan (Paper II: Calculus and Geometry):

Weeks 1 and 2: The first derivative test for relative extrema, Concavity and inflection points, Second derivative test for relative extrema, Curve sketching using first and second derivative tests.
[2] Chapter 4 (Section 4.3)
Weeks 3 and 4: Limits to infinity and infinite limits, Graphs with asymptotes, Vertical tangents and cusps, L'Hôpital's rule. [2] Chapter 4 (Sections 4.4, and 4.5). [1] Sections 3.3, and 6.5.
Week 5: Parametric representation of curves and tracing of parametric curves (except lines in $\mathbb{R}^3$), Polar coordinates and the relationship between Cartesian and polar coordinates. Tracing of curves in polar coordinates. [2] Chapter 9 [Section 9.4 (pages 471 to 475)]. [1] Chapter 10 (Section 10.2).
Weeks 6 and 7: Volumes by slicing disks and method of washers. Volumes by cylindrical shells, Arc length, Arc length of parametric curves. [1] Chapter 5 (Sections 5.2, 5.3 and 5.4).
Week 8: Area of surface of revolution. [1] Chapter 5 (Section 5.5).
Week 9: Reduction formulae, and to obtain the iterative formulae for the integrals of the form**:**
$\int \sin^n x\,dx,\ \int \cos^n x\,dx,\ \int \tan^n x\,dx, \int \sec^n x\,dx,$ and $\int \sin^m x \cos^n x\,dx.$
[1] Chapter 7 [Sections 7.2 and 7.3 (Pages 497 to 503)]
Weeks 10 and 11: Techniques of sketching conics**:** Parabola, Ellipse and Hyperbola. [1] Section 10.4.
Week 12: Reflection properties of conics, Rotation of axes, second degree equations and their classification into conics using the discriminant. [1] Chapter 10 (Sections 10.4 and 10.5).
Week 13: Vector-valued functions, Differentiation of vector-valued functions, Gradients, Divergence, Curl and their geometrical interpretation. [1] Sections 12.1, 12.2, and 15.1).
Week 14: Spheres, Cylindrical surfaces. Illustrations of graphing standard quadric surfaces like cone, ellipsoid. [1] Chapter 11 (Sections 11.1, and 11.7).

Semester-III
Paper III: Abstract Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of this course is to introduce the fundamental theory of groups, rings and vector spaces, a major part of abstract algebra, which is an essential tool in number theory, geometry, topology and has applications in cryptography, coding theory, quantum chemistry and physics.
Course Learning Outcomes: The course will enable the students to:

i) Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups etc;
ii) Explain the significance of the notion of cosets, normal subgroups, and of factor groups;
iii) Understand the fundamental concepts of Rings, Fields, Subrings, Integral domains, Vector spaces over a field, and linear transformations.

Unit 1: Groups **(Lectures: 35)**
Definition and examples of groups, Abelian and non-Abelian groups, The group $\mathbb{Z}_n$ of integers under addition modulo n and the group $U(n)$ of units under multiplication modulo n; Cyclic groups from sets of numbers, Group of n^{th} roots of unity, The general linear group; Elementary properties of groups; Groups of symmetries of (i) an isosceles triangle, (ii) an equilateral triangle, (iii) a rectangle, and (iv) a square; The permutation group Sym (n), and properties of permutations; Order

of an element, Subgroups and its examples, Subgroup tests, Cyclic subgroup, Center of a group, Properties of cyclic groups; Cosets and its properties, Lagrange's theorem, Index of a subgroup; Definition and examples of normal subgroups.

Unit 2: Rings, Integral Domains and Fields **(Lectures: 15)**
Definition and examples of rings, Commutative and noncommutative rings, Properties of rings, Subrings and ideals; Integral domains and fields, Examples of fields: $\mathbb{Z}_p, \mathbb{Q}, \mathbb{R},$ and $\mathbb{C}$.

Unit 3: Vector Spaces and Linear Transformations **(Lectures: 20)**
Definition and examples of vector spaces, Subspaces, Linear independence, Basis and dimension of a vector space; Linear transformations, Null spaces, Ranges and illustrations of the rank-nullity theorem.

References:

1. Gallian, Joseph. A. (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.
2. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2003). *Linear Algebra* (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.

Additional Readings:

i. Beachy, John A., & Blair, William D. (2006). *Abstract Algebra* (3rd ed.). Waveland Press.
ii. Lay, David C., Lay, Steven, R., & McDonald Judi, J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson.

Teaching Plan (Paper III: Abstract Algebra):

Weeks 1 and 2: Groups: Definition and examples of Abelian and non-Abelian groups, The group $\mathbb{Z}_n$ of integers under addition modulo n and the group $U(n)$ of units under multiplication modulo n; Cyclic groups from sets of numbers, Group of n^{th} roots of unity, The general linear group; Elementary properties of groups. [1] Chapter 2.
Week 3: Groups of symmetries of (i) an isosceles triangle, (ii) an equilateral triangle, (iii) a rectangle, and (iv) a square; The permutation group Sym (n), and properties of permutations.
[1] Chapter 1, Chapter 5 (Examples 1 to 7 and illustrations of Theorems 5.1 to 5.7 without proofs).
Weeks 4 and 5: Order of an element, Subgroups and its examples, Subgroup tests, Cyclic Subgroup, Center of a group, Properties of cyclic groups. [1] Chapters 3, and 4.
Week 6: Cosets and its properties, Lagrange's Theorem, Index of a subgroup.
[1] Chapter 7 up to Corollary 4, Page 149.
Week 7: Normal subgroups: Definition, examples and characterizations, Factor groups.
[1] Chapter 9 (Theorem 9.1, and Theorem 9.2 (Statement only) up to Examples 11, Page 189.
Weeks 8 and 9: Definition and examples of rings, commutative and noncommutative rings, Properties of rings, Subrings and ideals. [1] Chapter 12, and Chapter 14 up to Example 4, Page 268.
Week 10: Integral domains and fields, Examples of fields: $\mathbb{Z}_p, \mathbb{Q}, \mathbb{R},$ and $\mathbb{C}$.
[1] Chapter 13 up to Example 10, Page 258.
Weeks 11 and 12: Definition and examples of vector spaces, Subspaces, Linear independence, Basis and dimension of a vector space. [1] Chapter 19.
Weeks 13 and 14: Linear transformations, Null spaces, Ranges and illustrations of the rank-nullity theorem. [2] Chapter 2 (Section 2.1).

Skill Enhancement Paper

SEC-1: Computer Algebra Systems

Total Marks: 100 (Theory: 38, Internal Assessment: 12, and Practical: 50)
Workload: 2 Lectures, 4 Practicals (per week) **Credits:** 4 (2+2)
Duration: 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

Course Objectives: This course aims at providing basic knowledge to Computer Algebra Systems (CAS) and their programming language in order to apply them for plotting functions, finding roots to polynomials, computing limits and other mathematical tools.

Course Learning Outcomes: This course will enable the students to use CAS:

i) as a calculator;
ii) for plotting functions;
iii) for various applications of algebra, calculus and matrices.

Unit 1: Introduction to CAS and Graphics **(Lectures: 10)**
Computer Algebra Systems (CAS), Use of a CAS as a calculator, Simple programming in a CAS; Computing and plotting functions in 2D, Customizing Plots, Animating Plots; Producing table of values, Working with piecewise defined functions, Combining graphics.

Unit 2: Applications in Algebra **(Lectures: 6)**
Factoring, Expanding and finding roots of polynomials, Working with rational and trigonometric functions, Solving general equations.

Unit 3: Applications of Calculus **(Lectures: 6)**
Computing limits, First and higher order derivatives, Maxima and minima, Integration, Computing definite and indefinite integrals.

Unit 4: Working with Matrices **(Lectures: 6)**
Performing Gaussian elimination, Operations (transpose, determinant, and inverse), Minors and cofactors, Solving systems of linear equations, Rank and nullity of a matrix, Eigenvalue, eigenvector and diagonalization.

References:

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Torrence, Bruce F., & Torrence, Eve A. (2009). *The Student's Introduction to Mathematica®: A Handbook for Precalculus, Calculus, and Linear Algebra* (2nd ed.). Cambridge University Press.

Note: Theoretical and Practical demonstration should be carried out ***only in one*** of the CAS**:** Mathematica/MATLAB/Maple/Maxima/Scilab or any other.

Practicals to be done in the Computer Lab using CAS Software:

[1] Chapter 12 (Exercises 1 to 4 and 8 to 12).

[2] Chapter 3 [Exercises 3.2 (1), 3.3 (1, 2 and 4), 3.4 (1 and 2), 3.5 (1 to 4), 3.6 (2 and 3)].

[2] Chapter 4 (Exercises 4.1, 4.2, 4.5, 4.7 and 4.9).

[2] Chapter 5 [Exercises 5.1 (1), 5.3, 5.5, 5.6 (1, 2 and 4), 5.10 (1 and 3), 5.11 (1 and 2)].

[2] Chapter 7 [Exercises 7.1 (1), 7.2, 7.3 (2), 7.4 (1) and 7.6].

Teaching Plan (Theory of SEC-1: Computer Algebra Systems):

Weeks 1 and 2: Computer Algebra Systems (CAS), Use of a CAS as a calculator, Simple programming in a CAS. [1] Chapter 12 (Sections 12.1 to 12.5).
Weeks 3 to 5: Computing and plotting functions in 2D, Customizing Plots, Animating Plots, Producing table of values, Working with piecewise defined functions, Combining graphics.
[2] Chapter 1, Chapter 3 (Sections 3.1 to 3.6, and 3.8)
Weeks 6 to 8: Factoring, Expanding and finding roots of polynomials, Working with rational and trigonometric functions, Solving general equations. [2] Chapter 4 (Sections 4.1 to 4.3, 4.5 to 4.7, and 4.9).
Weeks 9 to 11: Computing limits, First and higher order derivatives, Maxima and minima, Integration, computing definite and indefinite integrals. [2] Chapter 5 (Sections 5.1, 5.3, 5.5, 5.6, 5.10, and 5.11).
Weeks 12 to 14: Performing Gaussian elimination, Operations (transpose, determinant, and inverse), Minors and cofactors, Solving systems of linear equations, Rank and nullity of a matrix, Eigenvalue, Eigenvector and diagonalization. [2] Chapter 7 (Sections 7.1 to 7.4, and 7.6 to 7.8).

Semester-IV

Paper IV: Real Analysis

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The course will develop a deeper and more rigorous understanding of defining terms and proving results about convergence of sequences and series of real numbers, having vide applications in real-world problems.
Course Learning Outcomes: This course will enable the students to:

i) Familiar with the concept of sequences, series and recognize convergent, divergent, bounded, Cauchy and monotone sequences.
ii) Test the convergence and divergence of series using the ratio test, Leibnitz test.
iii) Understand and apply the basics of Riemann integration.

Unit 1: Real Line and Real Sequences **(Lectures: 30)**
Finite and infinite sets, Examples of countable and uncountable sets; Absolute value and the Real line; Bounded sets, Suprema and infima, The Completeness property of $\mathbb{R}$, Archimedean property of $\mathbb{R}$; Real sequences, Convergence, Sum and product of convergent sequences, Order preservation and squeeze theorem; Monotone sequences and their convergence; Proof of convergence of some simple sequences such as $\frac{(-1)^n}{n}, \frac{1}{n^2}, \left(1+\frac{1}{n}\right)^n, x^n$ with $|x| < 1$, a_n/n, where a_n is a bounded sequence; Subsequences and the Bolzano–Weierstrass theorem; Limit superior and limit inferior of a bounded sequence; Cauchy sequences, Cauchy convergence criterion for sequences.

Unit 2: Infinite Series of Real Numbers **(Lectures: 20)**
Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series; Positive term series, The integral test, Convergence of p-series, Comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's root test; Alternating series, Leibniz test; Definition and examples of absolute and conditional convergence.

Unit 3: Uniform Convergence and Riemann Integration (Lectures: 20)
Sequences and series of functions, Pointwise and uniform convergence, Uniform norm, Cauchy general principle for uniform convergence of series of functions, Weierstrass M-test; Definition of power series, Radius and interval of convergence, Power series expansions for $\exp(x)$, $\sin x$ and $\cos x$ and their properties. Riemann Integration and examples, Integrability of continuous and monotone functions.

References:

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition.
2. Denlinger, Charles G. (2015). *Elements of Analysis*. Jones & Bartlett India Pvt. Ltd.
3. Ross, Kenneth A. (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

Additional Reading:

i. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Teaching Plan (Paper IV: Real Analysis):

Weeks 1 and 2: Finite and infinite sets, Examples of countable and uncountable sets; Absolute value of the real line; Bounded sets, Suprema and infima, Statement of order completeness property of $\mathbb{R}$, Archimedean property of $\mathbb{R}$. [1] Chapter 1 (Section 1.3), and Chapter 2 (Sections 2.2 to 2.5).
Week 3: Real sequences, Convergence, Sum and product of convergent sequences, Order preservation and squeeze theorem. [1] Chapter 3 (Sections 3.1, and 3.2).
Week 4: Monotone sequences and their convergence; Proof of convergence of some simple sequences such as $\frac{(-1)^n}{n}, \frac{1}{n^2}, \left(1+\frac{1}{n}\right)^n, x^n$ with $|x| < 1$, a_n/n, where a_n is a bounded sequence. [1] Section 3.3.
Weeks 5 and 6: Subsequences and the Bolzano–Weierstrass theorem (statement and examples); Limit superior and limit inferior of a bounded sequence (definition and examples); Statement and illustrations of Cauchy convergence criterion for sequences. [1] Chapter 3 (Sections 3.4, and 3.5).
Weeks 7 and 8: Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series; Positive term series, State the integral test and prove the convergence of p-series, Comparison test, Limit comparison test and examples.
[2] Chapter 8 (Section 8.1). [1] Chapter 3 (Section 3.7).
Weeks 9 and 10: D'Alembert's ratio test, Cauchy's root test; Alternating series, Leibnitz's test; Absolute and conditional convergence. [2] Chapter 8 (Sections 8.2 and 8.3).
Weeks 11 and 12: Sequences and series of functions, Pointwise and uniform convergence, Uniform norm, Cauchy general principle for uniform convergence of series of functions, Weierstrass M- test.
[1] Chapter 8 (Section 8.1 up to 8.1.9 except 8.1.3 and 8.1.5), Chapter 9 (Section 9.4: 9.4.1, 9.4.5 (Statement only), and 9.4.6).
Week 13: Definition of power series, Radius and interval of convergence, Power series expansions for $\exp(x)$, $\sin x$ and $\cos x$ and their properties. [3] Chapter 4 [Article 23, 23.1 (without proof)].
[1] Chapter 9 (9.4.7 to 9.4.9 (without proof), Example 9.4.14).
Week 14: Riemann Integration and examples, Integrability of Continuous and Monotone Functions
[3] Chapter 6 (Article 32 (only statements of the results), Article 33 up to 33.2).

Skill Enhancement Paper

SEC-2: Mathematical Typesetting System: LaTeX

Total Marks: 100 (Theory: 38, Internal Assessment: 12, and Practical: 50)
Workload: 2 Lectures, 4 Practicals (per week) **Credits:** 4 (2+2)
Duration: 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination**: 2 Hrs.

Course Objectives: The purpose of this course is to help you begin using LaTeX, a mathematical typesetting system designed for the creation of beautiful books—and especially for books that contain a lot of mathematics, complicated symbols and formatting.

Course Learning Outcomes: This course will enable the students to:
i) Create and typeset a LaTeX document;
ii) Typeset a mathematical document;
iii) Draw pictures in LaTeX, and create beamer presentations.

Unit 1: Getting Started with LaTeX **(Lectures: 6)**
Introduction to TeX and LaTeX, Creating and typesetting a simple LaTeX document, Adding basic information to documents, Environments, Footnotes, Sectioning, Displayed material.

Unit 2: Mathematical Typesetting **(Lectures: 8)**
Accents and symbols; Mathematical typesetting (elementary and advanced): Subscript/ Superscript, Fractions, Roots, Ellipsis, Mathematical symbols, Arrays, Delimiters, Multiline formulas, Putting one thing above another, Spacing and changing style in math mode.

Unit 3: Graphics and PSTricks **(Lectures: 8)**
Pictures and graphics in LaTeX, Simple pictures using PSTricks, Plotting of functions.

Unit 4: Getting Started with Beamer **(Lectures: 6)**
Beamer, Frames, Setting up beamer document, Enhancing beamer presentation.

References:

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Lamport, Leslie (1994). *LaTeX*: *A Document Preparation System*, User's Guide and Reference Manual (2nd ed.). Pearson Education. Indian Reprint.

Additional Reading:

i. Dongen, M. R. C. van (2012). *LaTeX and Friends*. Springer-Verlag.

Practicals to be done in the Computer Lab using a suitable LaTeX Editor:

[1] Chapter 9 (Exercises 4 to 10), Chapter 10 (Exercises 1, 3, 4, and 6 to 9), and

Chapter 11 (Exercises 1, 3, 4, 5).

Teaching Plan (Theory of SEC-2: Mathematical Typesetting System: LaTeX):

Weeks 1 to 3: Introduction to TeX and LaTeX, Creating and typesetting a simple LaTeX document, adding basic information to documents, Environments, Footnotes, Sectioning, Displayed material.
[1] Chapter 9 (Sections 9.1 to 9.5). [2] Chapter 2 (Sections 2.1 to 2.5).
Weeks 4 to 7: Accents and symbols; Mathematical typesetting (elementary and advanced)**:** Subscript/Superscript, Fractions, Roots, Ellipsis, Mathematical symbols, Arrays, Delimiters, Multiline formulas, Putting one thing above another, Spacing and changing style in math mode.
[1] Chapter 9 (Sections 9.6, and 9.7). [2] Chapter 3 (Sections 3.1 to 3.3).
Weeks 8 to 11: Pictures and Graphics in LaTeX, Simple pictures using PS Tricks, Plotting of functions.
[1] Chapter 9 (Section 9.8), and Chapter 10 (Sections 10.1 to 10.3). [2] Chapter 7 (Sections 7.1, and 7.2).
Weeks 12 to 14: Beamer, Frames, Setting up beamer document, Enhancing beamer presentation.
[1] Chapter 11 (Sections 11.1 to 11.4).

Semester-V
Skill Enhancement Paper

SEC-3: Transportation and Network Flow Problems

Total Marks: 100 (Theory: 55, Internal Assessment: 20 and Practical: 25)
Workload: 3 Lectures, 2 Practicals (per week) **Credits:** 4 (3+1)
Duration: 14 Weeks (42 Hrs. Theory + 28 Hrs. Practical) **Examination**: **3** Hrs.

Course Objectives: This course aims at providing applications of linear programming to solve real-life problems such as transportation problem, assignment problem, shortest-path problem, minimum spanning tree problem, maximum flow problem and minimum cost flow problem.

Course Learning Outcomes: This course will enable the students to solve:

i) Transportation, Assignment and Traveling salesperson problems.
ii) Network models and various network flow problems.

Unit 1: Transportation Problems **(Lectures: 12)**
Transportation problem and its mathematical formulation, Northwest-corner method, Least cost method and Vogel approximation method for determination of starting basic feasible solution, Algorithm for solving transportation problem.

Unit 2: Assignment and Traveling Salesperson Problems **(Lectures: 9)**
Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem, Traveling salesperson problem.

Unit 3: Network Models **(Lectures: 12)**
Network models, Minimum spanning tree algorithm, Shortest-route problem, Maximum flow model.

Unit 4: Project Management with CPM/PERT **(Lectures: 9)**
Project network representation, CPM and PERT.

References:

1. Hillier, Frederick S., & Lieberman, Gerald J. (2017). *Introduction to Operations Research*, (10th ed.). McGraw Hill Education (India) Pvt. Ltd. New Delhi.
2. Taha, Hamdy A. (2007). *Operations Research*: *An Introduction* (8th ed.). Pearson Education India. New Delhi.

Additional Reading:

i. Bazaraa, Mokhtar S., Jarvis, John J., & Sherali, Hanif D. (2010). *Linear Programming and Network Flows* (4th ed.). John Wiley & Sons.

Practicals to be done in the Computer Lab using a suitable Software:

Use TORA/Excel spreadsheet to solve transportation problem, Assignment problem, Traveling salesperson problem, Shortest-route problem, Minimum spanning tree algorithm, Maximum flow model, CPM and PERT calculations of exercises from the chapters 5 and 6 of [2].

[1] Case 9.1: Shipping Wood to Market, and Case 9.3: Project Pickings.

Teaching Plan (Theory of SEC-3: Transportation and Network Flow Problems):

Weeks 1 to 4: Transportation problem and its mathematical formulation, northwest-corner method, least cost method and Vogel approximation method for determination of starting basic feasible solution. Algorithm for solving transportation problem. [2] Chapter 5 (Sections 5.1, and 5.3).
Weeks 5 to 7: Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem, traveling salesperson problem.[2] Chapter 5 (Section 5.4), and Chapter 9 (Section 9.3)
Weeks 8 to 11: Network models, minimum spanning tree algorithm, shortest-route problem, maximum flow model. [2] Chapter 6 (Sections 6.1 to 6.4).
Weeks 12 to 14: Project network, CPM and PERT. [2] Chapter 6 (Section 6.5).

Discipline Specific Elective (DSE) Course -1

DSE-1 (i): Differential Equations (with Practicals)
OR
DSE-1 (ii): Mechanics and Discrete Mathematics

DSE-1 (i): Differential Equations (with Practicals)

Total Marks: 150 (Theory: 75, Internal Assessment: 25, and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) **Credits:** 6 (4+2)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: This course helps the students to develop skills and knowledge of standard concepts in ordinary and partial differential equations and also provide the standard methods for solving differential equations.

Course Learning Outcomes: The student will be able to:

i) Solve the exact, linear and Bernoulli equations and find orthogonal trajectories.
ii) Apply the method of variation of parameters to solve linear differential equations.
iii) Formulate and solve various types of partial differential equations of first and second order.

Unit 1: First Order Ordinary Differential Equations (Lectures: 16)
First order exact differential equations, Integrating factors, Rules to find an integrating factor; Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories; Basic theory of higher order linear differential equations, Wronskian, and its properties; Solving differential equation by reducing its order.

Unit 2: Second Order Linear Differential Equations (Lectures: 16)
Linear homogenous equations with constant coefficients, Linear non-homogenous equations, The method of variation of parameters, The Cauchy-Euler equation; Simultaneous differential equations.

Unit 3: Partial Differential Equations (Lectures: 24)
Partial differential equations**:** Basic concepts and definitions with mathematical problems; First order partial differential equations**:** Classification, Construction, Geometrical interpretation, Method of characteristics and general solutions, Canonical forms and method of separation of variables; Second order partial differential equations**:** Classification, Reduction to canonical forms; Linear second order partial differential equations with constant coefficients**:** Reduction to canonical forms with general solutions.

References:

1. Myint-U, Tyn & Debnath, Lokenath. (2007). *Linear Partial Differential Equation for Scientists and Engineers* (4th ed.). Springer, Third Indian Reprint, 2013.
2. Kreyszig, Erwin (2011). *Advanced Engineering Mathematics* (10th ed.). John Wiley & Sons, Inc. Wiley India Edition 2015.
3. Ross, Shepley L. (1984). *Differential Equations* (3rd ed.). John Wiley & Sons, Inc.

Additional Readings:

i. Ross, Clay C. (2004). *Differential Equations*: *An Introduction with Mathematica®* (2nd ed.). Springer.
ii. Sneddon, I. N. (2006). *Elements of Partial Differential Equations*, Dover Publications. Indian Reprint.

Practical /Lab work to be performed in a Computer Lab:

Use of computer algebra systems (CAS), for example Mathematica/MATLAB/ Maple/Maxima/Scilab, etc., for developing the following programs:

1) Solution of first order differential equation.
2) Plotting of second order solution family of differential equation.
3) Plotting of third order solution family of differential equation.
4) Solution of differential equation by variation of parameter method.
5) Solution of systems of ordinary differential equations.
6) Solution of Cauchy problem for first order PDE.
7) Plotting the characteristics for the first order PDE.
8) Plot the integral surfaces of a given first order PDE with initial data.

Teaching Plan (Theory Paper: DSE-1 (i): Differential Equations):

Week 1: First order ordinary differential equations: Basic concepts and ideas. [2] Chapter 1 (Section 1.1). [3] Chapter 1 (Sections 1.1, and 1.2).
Week 2: First order exact differential equations. Integrating factors and rules to find integrating factors. [2] Chapter 1 (Section 1.4). [3] Chapter 2 (Sections 2.1, and 2.2).
Weeks 3 and 4: Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories; Basic theory of higher order linear differential equations, Wronskian, and its properties; Solving a differential equation by reducing its order.
[3] Chapter 2 (Sections 2.3, and 2.4), Chapter 3 (Section 3.1), and Chapter 4 (Section 4.1)
Weeks 5 and 6: Linear homogenous equations with constant coefficients. Linear non-homogenous equations. [2] Chapter 2 (Section 2.2). [3] Chapter 4 (Sections 4.2, 4.3, and 4.6).
Week 7: The method of variation of parameters, The Cauchy-Euler equation.
[3] Chapter 4 (Sections 4.4, and 4.5)
Week 8: Simultaneous differential equations. [3] Chapter 7 (Sections 7.1, and 7.3).
Week 9: Partial differential equations: Basic concepts and definitions with mathematical problems. Classification of first order partial differential equations. [1] Chapter 2 (Sections 2.1, and 2.2).
Week 10: Construction and Geometrical interpretation of first order partial differential equations.
[1] Chapter 2 (Sections 2.3, and 2.4).
Week 11: Method of characteristics, General solutions of first order partial differential equations.
[1] Chapter 2 (Section 2.5)
Week 12: Canonical forms and method of separation of variables for first order partial differential equations. [1] Chapter 2 (Sections 2.6, and 2.7).
Week 13: Classification of second order partial differential equations, reduction to canonical forms.
[1] Chapter 4 (Sections 4.1, and 4.2).
Week 14: Second order partial differential equations with constant coefficients, General solutions.
[1] Chapter 4 (Sections 4.3, and 4.4).

DSE-1 (ii): Mechanics and Discrete Mathematics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: This course helps the students to develop skills and knowledge of standard concepts in mechanics and discrete mathematics. Also to demonstrate the students that how differential mechanics and discrete mathematics can be useful in solving daily life problems.
Course Learning outcomes: The student will be able to:
i) Explain the concept of mechanics and discrete mathematics.
ii) Know the knowledge they have gained to solve real problems.
iii) Understand graphs, their types and its applications in study of shortest path algorithms.

Unit 1: Statics and Dynamics **(Lectures: 35)**
Conditions of equilibrium of a particle and of coplanar forces acting on a rigid body, Laws of friction, Problems of equilibrium under forces including friction, Centre of gravity, Work and potential energy; Velocity and acceleration of a particle along a curve: Radial and transverse components (plane curve), Tangential and normal components (space curve); Newton's Laws of motion, Simple harmonic motion, Simple Pendulum, Projectile motion.

Unit 2: Graphs **(Lectures: 20)**
Types of graphs**:** Simple graph, Directed graph, Multi graph, and Pseudo graph; Graph modelling, Terminology and basics; Special Graphs**:** Complete Graph, Cycles, *n*-dimensional cubes, Bipartite graph, Complete Bipartite graph; Subgraph and basic algebraic operations on graphs, Connectivity, Path, Cycles, Tree to be introduced as a connected graph with no cycles.

Unit 3: Shortest-Path Problems, Euler and Hamiltonian Cycles **(Lectures: 15)**
Introduction to shortest path (least number of edges) problem, Solution of shortest path problem for simple graphs using complete enumeration; Euler and Hamiltonian graphs (for undirected graphs only)**:** Königsberg bridge problem, Statements and interpretations of (i) Necessary and sufficient conditions for Euler cycles and paths (ii) Suficient condition for Hamiltonian cycles; Finding Euler cycles and Hamiltonian cycles in a given graph.

References:

1. Ramsay, A. S. (1998). *Statics*, CBS Publishers and Distributors, Delhi (Indian Reprint).
2. Roberts, A. P. (2003). *Statics and Dynamics with Background Mathematics*, Cambridge University Press.
3. Rosen, Kenneth H. (2012). *Discrete Mathematics and its Applications* (7th ed.). McGraw-Hill Education (India) Pvt. Ltd.

Teaching Plan (DSE-1 (ii): Mechanics and Discrete Mathematics):

Week 1: Conditions of equilibrium of a particle and of coplanar forces acting on a rigid body.
[1] Chapter 5 (Section 5.2).
Week 2: Laws of friction, Problems of equilibrium under forces including friction.
[1] Chapter 9 (Sections 9.1, and 9.2).
Week 3: Centre of gravity, Work and potential energy.
[1] Chapter 10 (Section 10.1), and Chapter 11 (Sections 11.1, and 11.6).
Week 4: Velocity and acceleration of a particle along a curve**:** Radial and transverse components (plane curve). [2] Chapter 10 (Sections 10.4 to 10.6).
Week 5: Tangential and normal components (space curve), Newton's Laws of motion
[2] Chapter 10 (Section 10.1)
Week 6: Simple harmonic motion. [2] Chapter 11 (Section 11.4).
Week 7: Simple Pendulum, Projectile Motion. [2] Chapter 11 (Sections 11.5 to 11.7).
Week 8: Types of graphs**:** Simple graph, Directed graph, Multi graph, and Pseudo graph, Graph modeling.
[3] Chapter 10 (Section 10.1)
Weeks 9 and 10: Terminology and basics; Special Graphs**:** Complete Graph, Cycles, *n*-dimensional cubes, Bipartite graph, Complete Bipartite graph; Subgraph and basic algebraic operations on graphs.
[3] Chapter 10 (Section 10.2)
Week 11: Connectivity, Path, Cycles, Tree to be introduced as a connected graph with no cycles.
[3] Chapter 10 (Section 10.4)
Week 12: Introduction to shortest path (least number of edges) problem, Solution of shortest path problem for simple graphs using complete enumeration. [3] Chapter 10 (Section 10.6).
Weeks 13 and 14: Euler and Hamiltonian graphs (for undirected graphs only)**:** Königsberg bridge problem, Statements and interpretations of (i) Necessary and sufficient conditions for Euler cycles and paths (ii) Suficient condition for Hamiltonian cycles; Finding Euler cycles and Hamiltonian cycles in a given graph.
[3] Chapter 10 (Section 10.5).

Semester-VI

Skill Enhancement Paper

SEC-4: Statistical Software: R

Total Marks: 100 (Theory: 38, Internal Assessment: 12, and Practical: 50)
Workload: 2 Lectures, 4 Practicals (per week) **Credits:** 4 (2+2)
Duration: 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

Course Objectives: The purpose of this course is to help you begin using **R**, a powerful free software program for doing statistical computing and graphics. It can be used for exploring and plotting data, as well as performing statistical tests.

Course Learning Outcomes: This course will enable the students to:

i) Use **R** as a calculator;
ii) Read and import data in **R**.
iii) Explore and describe data in **R** and plot various graphs in **R**.

Unit 1: Getting Started with R - The Statistical Programming Language (Lectures: 10)
Introducing **R**, using **R** as a calculator; Explore data and relationships in **R**; Reading and getting data into **R:** combine and scan commands, viewing named objects and removing objects from **R**, Types and structures of data items with their properties, Working with history commands, Saving work in **R**; Manipulating vectors, Data frames, Matrices and lists; Viewing objects within objects, Constructing data objects and their conversions.

Unit 2: Descriptive Statistics and Tabulation (Lectures: 6)
Summary commands**:** Summary statistics for vectors, Data frames, Matrices and lists; Summary tables.

Unit 3: Distribution of Data (Lectures: 6)
Stem and leaf plot, Histograms, Density function and its plotting, The Shapiro-Wilk test for normality, The Kolmogorov-Smirnov test.

Unit 4: Graphical Analysis with R (Lectures: 6)
Plotting in **R:** Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts, Bar charts; Copy and save graphics to other applications.

References:

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Gardener, M. (2012). *Beginning* **R:** *The Statistical Programming Language*, Wiley Publications.

Additional Reading:

i. Verzani, John (2014). *Using R for Introductory Statistics* (2nd ed.). CRC Press, Taylor & Francis Group.

Practicals to be done in the Computer Lab using Statistical Software R:

[1] Chapter 14 (Exercises 1 to 3)

[2] Relevant exercises of Chapters 2 to 5, and 7

Note: The practical may be done on the database to be downloaded from https://data.gov.in/

Teaching Plan (Theory of SEC-4: Statistical Software: R):

Weeks 1 to 3: Introducing **R**, using **R** as a calculator; Explore data and relationships in **R**, Reading and getting data into **R:** Combine and scan commands, viewing named objects and removing objects from **R**, Types and structures of data items with their properties, Working with history commands, Saving work in **R**. [1] Chapter 14 (Sections 14.1 to 14.4). [2] Chapter 2.
Weeks 4 and 5: Manipulating vectors, Data frames, Matrices and lists; Viewing objects within objects, Constructing data objects and their conversions. [2] Chapter 3.
Weeks 6 to 8: Summary commands**:** Summary statistics for vectors, Data frames, Matrices and lists; Summary tables. [2] Chapter 4.
Weeks 9 to 11: Stem and leaf plot, Histograms, Density function and its plotting, The Shapiro-Wilk test for normality, The Kolmogorov-Smirnov test. [2] Chapter 5.
Weeks 12 to 14: Plotting in **R:** Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts, Bar charts; Copy and save graphics to other applications.
[1] Chapter 14 (Section 14.7). [2] Chapter 7.

Discipline Specific Elective (DSE) Course -2

DSE-2 (i): Numerical Methods (with Practicals)
OR
DSE-2 (ii): Probability Theory and Statistics

DSE-2 (i): Numerical Methods (with Practicals)

Total Marks: 150 (Theory: 75, Internal Assessment: 25, and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) **Credits:** 6 (4+2)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students for the study of certain algorithms that uses numerical approximation for the problems of mathematical analysis. Also, the use of Computer Algebra Systems (CAS) by which the intractable problems can be solved both numerically and analytically.

Course Learning Outcomes: After completion of this course, students will be able to:
i) Find the consequences of finite precision and the inherent limits of numerical methods.
ii) Appropriate numerical methods to solve algebraic and transcendental equations.
iii) How to solve first order initial value problems of ODE’s numerically using Euler methods.

Unit 1: Errors and Roots of Transcendental and Polynomial Equations (Lectures: 16)
Floating point representation and computer arithmetic, Significant digits; Errors**:** Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

Unit 2: Algebraic Linear Systems and Interpolation (Lectures: 20)
Gaussian elimination method (with row pivoting), Gauss-Jordan method; Iterative methods**:** Jacobi method, Gauss-Seidel method; Interpolation**:** Lagrange form, Newton form, Finite difference operators, Gregory-Newton forward and backward difference interpolations, Piecewise polynomial interpolation (Linear and Quadratic).

Unit 3: Numerical Differentiation, Integration and ODE (Lectures: 20)
Numerical differentiation**:** First and second order derivatives; Numerical integration**:** Trapezoid rule, Simpson's rule; Extrapolation methods**:** Richardson extrapolation, Romberg integration; Ordinary differential equation**:** Euler's method, Modified Euler's methods **(**Heun and Mid-point).

References:

1. Chapra, Steven C. (2018). *Applied Numerical Methods with* MATLAB *for Engineers and* Scientists (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). *Applied Numerical Analysis Using* MATLAB. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). *Numerical Methods for Scientific and Engineering Computation* (6th ed.). New Age International Publishers. Delhi.

Additional Reading:

i. Bradie, Brian (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

Practical /Lab work to be performed in the Computer Lab:

Use of Computer Algebra System (CAS), for example MATLAB/Mathematica/ Maple/Maxima/Scilab etc., for developing the following Numerical Programs:

1) Bisection Method
2) Secant Method and Regula-Falsi Method
3) Newton-Raphson Method
4) Gaussian elimination method and Gauss-Jordan method
5) Jacobi Method and Gauss-Seidel Method
6) Lagrange Interpolation and Newton Interpolation
7) Trapezoid and Simpson's rule.
8) Romberg integration
9) Euler methods for solving first order initial value problems of ODE's.

Teaching Plan (Theory of DSE-2(i): Numerical Methods):
Weeks 1 and 2: Floating point representation and computer arithmetic, Significant digits; Errors**:** Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions. [2] Chapter 1 (Sections 1.2.3, 1.3.1, and 1.3.2). [3] Chapter 1 (Sections 1.2, and 1.3).
Week 3 and 4: Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.
[2] Chapter 2 (Sections 2.1 to 2.3). [3] Chapter 2 (Sections 2.2 and 2.3
Week 5: Gaussian elimination method (with row pivoting), Gauss-Jordan method;
Iterative methods**:** Jacobi method, Gauss-Seidel method. [2] Chapter 3 (Sections 3.1, and 3.2), Chapter 6 (Sections 6.1, and 6.2). [3] Chapter 3 (Sections 3.2, and 3.4).

Week 6: Interpolation**:** Lagrange form, and Newton form. [2] Chapter 8 (Section 8.1).
[3] Chapter 4 (Section 4.2)
Weeks 7 and 8: Finite difference operators, Gregory-Newton forward and backward difference interpolations. [3] Chapter 4 (Sections 4.3, and 4.4).
Week 9: Piecewise polynomial interpolation**:** Linear, and Quadratic.
[2] Chapter 8 [Section 8.3 (8.3.1, and 8.3.2)]. [1] Chapter 18 (Sections 18.1 to 18.3).
Weeks 10 and 11: Numerical differentiation**:** First and second order derivatives;
Numerical integration**:** Trapezoid rule, Simpson's rule.
[2] Chapter 11 [Sections 11.1 (11.1.1, and 11.1.2), and 11.2 (11.2.1, and 11.2.2)]
Weeks 12 and 13: Extrapolation methods**:** Richardson extrapolation, Romberg integration;
Ordinary differential equations**:** Euler's method. [2] Chapter 11 [Section 11.1 (11.1.4), and 11.2 (11.2.4)].
[1] Chapter 22 (Sections 22.1, and 22.2).
Weeks 14: Modified Euler's methods**:** Heun's method, The Midpoint method. [1] Section 22.3.

DSE-2 (ii): Probability Theory and Statistics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: To provide a foundation in probability theory and statistics in order to solve applied problems and to prepare for providing the solutions that take account of their everyday experiences with their scientific interests.
Course Learning Outcomes: This course will enable the students to learn:
i) Basic probability axioms and familiar with discrete and continuous random variables.
ii) To measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.
iii) Central limit theorem, which helps to understand the remarkable fact that**:** the empirical frequencies of so many natural populations, exhibit a bell shaped curve.

Unit 1: Univariate Discrete and Continuous Distributions (Lectures: 35)
Sample space, Probability set function, Real random variables - Discrete and continuous, cumulative distribution function, Probability mass/density functions, Mathematical expectation, Moments, Moment generating function, Characteristic function; Discrete distributions**:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson; Continuous distributions**:** Uniform, Gamma, Exponential and Normal; Normal approximation to the binomial distribution.

Unit 2: Bivariate Distribution (Lectures: 15)
Joint cumulative distribution function and its properties, Joint probability density function, Marginal distributions, Expectation of function of two random variables, Joint moment generating function, Conditional distributions and expectations.

Unit 3: Correlation, Regression and Central Limit Theorem (Lectures: 20)
Independent random variables, Covariance, Correlation coefficient; Linear regression for two variables and the method of least squares; Chebyshev's theorem, Statement and interpretation of (weak) law of large numbers and strong law of large numbers; Central Limit Theorem for independent and identically distributed random variables with finite variance.

References:

1. Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2013). *Introduction to Mathematical Statistics* (7th ed.), Pearson Education, Inc.
2. Miller, Irwin, & Miller, Marylees (2014). John E. Freund's: *Mathematical Statistics with Applications* (8th ed.). Pearson Education Ltd. Indian Reprint. Dorling Kindersley.
3. Ross, Sheldon M. (2014). *Introduction to Probability Models* (11th ed.). Elsevier Inc.

Additional Reading:

i. Mood, Alexander M., Graybill, Franklin A. & Boes, Duane C. (1974). *Introduction to The Theory of Statistics* (3rd ed.). McGraw-Hill Education, Indian Edition (2017).

Teaching Plan (DSE-2 (ii): Probability Theory and Statistics):

Week 1: Sample space, Probability set function and examples. [1] Chapter 1 (Sections 1.1, and 1.3).
Week 2: Random variable, Probability mass /density function, Cumulative distribution function and its properties. [1] Chapter 1 (Section 1.5).
Week 3: Discrete and continuous random variables. [1] Sections 1.6, and 1.7, except Transformations.
Week 4: Expectation of random variables, and some special expectations**:** Mean, Variance, Standard deviation, Moments and moment generating function, Characteristic function. [1] Sections 1.8, and 1.9.
Week 5: The discrete distributions**:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric, and Poisson. [2] Chapter 5 (Sections 5.2 to 5.5, and 5.7).
Week 6: The continuous distributions**:** Uniform, Gamma, and Exponential. [2] Sections 6.2, and 6.3.
Week 7: Normal distribution, and normal approximation to the binomial distribution. [2] Sections 6.5, 6.6.
Week 8 and 9: Random vector**:** Discrete and continuous, Joint cumulative distribution function and its properties, Joint probability mass/density function, Marginal probability mass function, and Expectation of two random variables, Joint moment generating function. [1] Chapter 2 (Section 2.1).
Week 10: Conditional distribution and expectations. [1] Chapter 2 (Section 2.3).
Week 11: Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Independent random variables. [1] Chapter 2 (Sections 2.4, and 2.5).
Weeks 12 and 13: Linear regression for two variables, and the method of least squares; Chebyshev's theorem. [2] Chapter 14 (Sections 14.1 to 14.3), Chapter 4 (Section 4.4).
Week 14: Statement and interpretation of the strong law of large numbers; Central limit theorem and the weak law of large numbers. [3] Chapter 2 (Section 2.8, and Exercise 76, page 89).