## M.Phil./ Ph.D. Coursework Syllabus

The course MATH17-01 Research Methodology is compulsory and the research scholar should study any three other courses. Each course has 4 credit. Final examination will be of 3 hours duration and for 70 marks. Internal assessment will be for 30 marks and will be based on tests, assignments, presentation. Qualifying marks: 55% in each paper (internal + final examination put together).

### MATH17-R01 : Research Methodology

Scientific research and literature survey. History of mathematics, finding and solving research problems, role of a supervisor, survey of a research topic, publishing a paper, reviewing a paper, research grant proposal writing, copyright issues, ethics and plagiarism.

Research tools. Searching google (query modifiers), MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, h-index, Google Scholar, ORCID, JStor, Online and open access journals, Virtual library of various countries

Scientific writing and presentation. Writing a research paper, survey article, thesis writing; LaTeX, PSTricks, Beamer, HTML and MathJaX

Software for Mathematics. Mathematica/Matlab/Scilab/GAP

Reference:


[1] J. Stillwell, Mathematics and its History, Springer International Edition, 4th Indian Reprint, 2005
[2] L. Lamport, LaTeX, a Document Preparation System, 2nd ed, Addison-Wesley, 1994.
[3] Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, Jean A. Dieudonne, How to Write Mathematics, American Mathematical Society, 1973.
[4] Nicholas J. Higham, Handbook of Writing for the Mathematical Sciences, 2ed, SIAM, 1998.
[5] Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts, Mathematical Writing, Mathematical Association of America, Washington, D.C., 1989.
[6] Frank Mittelbach, Michel Goossens, Johannes Braams, David Carlisle, Chris Rowley, The LaTeX Companion, 2nd edition (TTCT series), Addison-Wesley, 2004.
[7] Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel, Herbert Voss, The LaTeX Graphics Companion, 2nd edition (TTCT series), Addison-Wesley, 2004
[8] Mathtools documentation (http://mirrors.ctan.org/macros/latex/contrib/mathtools/mathtools.pdf)
[9] Pstricks documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)
[10] MathJax documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)


### MATH17-R02 Advanced Commutative Algebra

Localization of rings and its properties, Integral extensions, Discrete valuation rings, Dedekind domains, Graded rings and modules, Associated graded rings, I-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, Hom functor, Tensor functor, I-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor.

*References:*


[1] H. Matsumura, Commutative ring theory, Cambridge, 1989.
[2] Balwant Singh, Basic commutative algebra, World Scientific, 2011.
[3] D. Eisenbud, Commutative algebra with a view towards algebraic geometry, Springer-Verlag, 1995.
[4] M.F. Atiyah & I.G. Macdonald, Introduction to commutative algebra, Addison-Wesley, 1969.

## MATH17-R03 Topics in Analysis

Uniform convergence and differentiation, The Stone-Weierstrass theorem, Contraction principle, Non-expansive maps and Browder fixed point theorem. Integration of vector functions—Bochner integrability.

Differential calculus in normed linear spaces, Gateaux and Frechet derivative of functions, mean value theorems, chain rule, higher order derivatives, Taylor's formula, local and global inverse function theorems, implicit function theorem, extremum problems and Lagrange multipliers.

Spherical distance in the extended complex plane, uniform convergence and local uniform convergence with respect to this metric for sequence of meromorphic functions, normality of families of meromorphic functions and various characterizations.

Criterions for normality of families of holomorphic functions and their applications to Montel's theorem, Miranda's theorem and Bloch's theorem. Criterions for normality of families of meromorphic functions and their applications to Montel's theorem, Zalcman's theorem and Gu's theorem.

*References:*


[1] W. Cheney, Analysis for applied Mathematics, Springer-Verlag, 2001
[2] Chi-Tai Chuang: Normal families of meromorphic functions, World Scientific, Singapore, 1993
[3] J. B. Conway, Functions of one complex variable, Narosa, New Delhi, 2002
[4] P. Drabek and J. Milota, Methods of Nonlinear analysis, applications to Differential equations, Birkhauser, Berlin 2007.
[5] W. Rudin, Principles of Mathematical Analysis, McGraw Hill, 1976.


## MATH17-R04 Advanced Functional Analysis

*Topological Vector Spaces.* Types of topological vector spaces, separation properties, linear mappings, boundedness and continuity, quotient spaces , examples , Banach –Steinhaus theorem , open mapping theorem , closed graph theorem, Hahn Banach Theorem on topological vector spaces, Weal topologies, weak*-topology of a dual space, compact convex sets, extreme points , Milman's theorem, vector –valued integration, vector valued holomorphic functions.

*Banach Algebras*. Definition and examples of Banach algebras and *-Banach algebras, Complex homomorphisms, spectrum, Symbolic calculus, , group of invertible elements, Ideals and quotient algebras, Gelfand transform, Applications to non-commutative Banach algebras, spectral theorem, symbolic calculus for normal operators, characterization of C*-algebras, unbounded operators

*References*


[1] E. Kaniuth, A course in Commutative Banach Algebras, Springer , 2009.
[2] W.Rudin, Functional Analysis, Mcgraw Hill Education, 2015
[3] H.H. Schaefer, M.P.Wolf, Topological Vector Spaces. Springer, 2012


## MATH17-R05 Topology and Mixing

*Topological transitivity:* Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for continuous interval maps , Characterizing topological mixing in terms of topological transitivity for

continuous interval maps , Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on intervals **,** Various other definitions of Chaos and their interrelationships.

*Topological Entropy:* Definition using open covers, Examples and properties, Bowen's definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov-Sinai Theorem, Topological entropy of an expansive homeomorphism, of the two sided shift, of the topological Markov chain, of any homeomorphism of the unit circle, of any homeomorphism of closed unit interval, an upper bound for the topological entropy of a diffeomorphism of a finite dimensional Riemannian manifold.

*References*


[1] L. Alseda, J. Llibre, M. Misiurewicz, Combinatorial Dynamics and Entropy in Dimension One, Advanced Series in Nonlinear Dynamics, 2000.
[2] L. S. Block and W. A. Coppel, Dynamics in One dimension, Springer, 1992.
[3] M. Brin and G.Stuck , Introduction to Dynamical Systems, Cambridge University Press, 2002
[4] R. L. Devaney, A First Course in Chaotic Dynamical Systems, Westview Press, 1992.
[5] C. Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC press, 1999.
[6] S. Ruette, Chaos for continuous interval maps: A survey of relationship between various kinds of chaos, 2003.
[7] P.Walters, An Introduction to Ergodic Theory, Springer-Verlag New York, 1982.


**MATH17-R06 Convex Analysis and Semidefinite Optimization**

Convex sets, Convexity-preserving operations for a set, Relative interior, Asymptotic cone, Extreme points, Face, Projection operator, Separation theorems, Bouligand tangent and normal cones.

Convex functions, Closedness, Affinity, Epigraphical hull and Lower-bound function of a set, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties.

Sublinear functions, Support function, Calculus of support functions, Norms and their duals, Polarity.

Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem.

Convex conjugate of a function, Biconjugate of a function, Coercivity, Subdifferentials of extended-valued functions, Calculus rule of conjugate functions.

Positive semidefinite cone, Primal and dual semidefinite program, Fenchel and Lagrangian duality in semidefinite optimization

*References*


[1] Jean-Baptiste Hiriart-Urruty and Claude Lemarechal, Fundamentals of Convex Analysis, Springer-Verlag, Berlin, 1996.
[2] Jonathan M. Borwein Adrian and S. Lewis, Convex Analysis and Nonlinear Optimization: Theory and Examples, CMS Books in Mathematics, Springer Verlag, New York, 2006.
[3] R. Tyrrell Rockafellar, Convex Analysis, Princeton University Press, Princeton, New Jersey, 1997.
[4] Y. Kanno, Nonsmooth Mechanics and Convex Optimization, CRC Press, 2011.

## MATH17-R07 Hyperbolic System of Conservation Laws and Boundary Layer Theory

*Hyperbolic system of conservation laws:* Fundamental concepts and examples, Scalar and system of conservation laws, Riemann Problem, Entropy condition, Classical and non-classical shocks, Similarity method.

*Boundary layer theory:* Laminar boundary layer, Turbulent flow, Turbulent boundary layer; Heat and Mass transfer, conduction, convection and radiation; Thermal boundary layer; Modeling and method of solution of the problems.

*References*


[1] G. B. Witham- Linear and Non-linear Waves, Wiley, 1999.
[2] V. D. Sharma-Quasilinear hyperbolic systems, Compressible flows and Waves, CRC, 2010.
[3] P. G. LeFloch-Hyperbolic Systems of Conservation Laws: The Theory of Classical and Non-classical Shock Waves, Birkhauser, Berlin 2002.
[4] H. Schlichting and K. Gersten- Boundary Layer Theory, Springer, 2000.
[5] Tuncer Cebeci-Analysis of turbulent flows, Elsevier, 2004.
[6] J.P. Holman, Heat Transfer, tenth edition, McGraw-Hill, New York, 2010
[7] G. W. Bluman and S. Kumei- Symmetries and differential equations, Springer, New York, 1989.
[8] E.F. Toro, Riemann Solvers and numerical methods for Fluid dynamics, Springer, 2009


## MATH17-R08 Partial Differential Equations: Theory and Numerics

Maximum Principles for second order linear parabolic, elliptic and hyperbolic partial differential equations. Weak solutions for second order linear parabolic, elliptic and hyperbolic partial differential equations. Lax-Milgram Theorem. Local existence, uniqueness and regularity results for second order linear parabolic, elliptic and hyperbolic partial differential equations.

Dispersion and Dissipation analysis of PDEs and its finite difference schemes, Discontinuous solutions. Finite difference schemes for systems of parabolic and hyperbolic PDEs.Analysis of well-posed initial value problem of parabolic and hyperbolic systems, Convergence estimates for parabolic and hyperbolic PDES. Finite difference schemes for curved boundaries of elliptic PDEs.

*References*


[1] L C. Evans, Partial Differential Equation, American Mathematical Society, 1998.
[2] R. C. McOwen, Partial Differential Equations: Methods and Applications, 2nd Ed., Pearson Education, Inc., 2003.
[3] G. B. Folland, Introduction to Partial Differential Equations, 2nd Ed., Prentice-Hall of India, 1995.
[4] M. Renardy and R. C. Rogers, An Introduction to Partial Differential Equations, Springer-Verlag, 2004.
[5] John C. Strikwerda, Finite Dofference Schemes and Partial Differential Equations, SIAM, Philadelphia, 2004.
[6] J.W.Thomas, Numerical Partial Differential Equations: Finite Difference Methods, Springer-Verlag, New York, 1995.