Mathematical Model for Fire Spread

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What is Fire?

- **Heat is supplied from the fire to the potential fuel, the surface is dehydrated and further heating raises the surface temperature until the fuel begins to pyrolyze and release combustible gases.**

When the gas evolution rate from the potential fuel is sufficient to support combustion, the gas is ignited by the flame and the fire advances to a new position.

- **Gradually, a constant rate of spread is attained which is called “quasi-steady state” wherein the fire advances at a rate that is the average of all the elemental rates.**

Objective

A mathematical model to determine rate of fire spread and its intensity is formulated. The model is developed by R.C. Rothermal (1972) and it is considered as the basis in the National Fire Danger Rating System.

The model considers physical and chemical properties of the fuel and the environmental conditions in which it is expected to tingle.

- The physical properties incorporated are

1. fuel loading
2. fuel depth
3. fuel particle surface area to volume ratio
4. fuel particle moisture, and
5. the moisture content at which extinction is expected.

- **Environmental inputs are**

1. **mean wind velocity**
2. **slope of terrain.**

Mathematical Model

R	**quasi-steady rate of spread, ft/min**
I_{Xig}	Horizontal heat flux absorbed by a unit volume of the fuel at the time of ignition B.t.u./ft^2 min
ρ_{be}	Effective bulk density (the amount of fuel per unit volume of the fuel bed raised to ignition ahead of the advancing fire), lb/ft^3
Q_{ig}	Heat of pre-ignition (the heat required to bring a unit weight of fuel to ignition), B.t.u./lb
$\left(\frac{\partial I_z}{\partial z}\right)_{z_c}$	The gradient of the vertical intensity evaluated at a plane at a constant depth, z_c, of the fuel bed, B.t.u./ft^3 min
x	Horizontal coordinate
z	Vertical coordinate

- **Using conservation of energy principle to a unit volume of fuel ahead of an advancing fire in a homogeneous fuel bed (by Fransen (1971))**

$$R = \frac{I_{Xig} + \int_{-\infty}^{0} \left(\frac{\partial I_z}{\partial z} \right)_{z_c} dx}{\rho_{be} Q_{ig}} = \frac{\text{heat flux received from the source}}{\text{heat required for ignition by the potential fuel}} \quad (1)$$

Analysis

- **The fuel-reaction zone interface is fixed and the unit volume is moving at a constant depth, z_c, from x = - infinity towards the interface at x = 0. The unit volume ignites at the interface.**

- **The heat required for ignition depends upon**

(a) Ignition temperature

(b) moisture content of the fuel, and

(c) Amount of fuel involved in the ignition process.

- **The energy per unit mass required for ignition is the heat of pre-ignition, Q_{ig}.**

$$Q_{ig} = f(M_f, T_{ig}), B.t.u./lb \qquad (2)$$

where

M_f : ratio of fuel moisture to ovendry weight

T_{ig} : ignition temp.

- **The amount of fuel involved in the ignition process is the effective bulk density, ρ_{be} .**
- **An effective heating number ε is ratio of the effective bulk density to the actual bulk density.**

$$\varepsilon = \frac{\rho_{be}}{\rho_b} \qquad (3)$$

- **The effective heating number is dimensionless which will be nearly unity for fine fuels and decrease to zero as fuel size increases. Therefore,**

$$\rho_{be} = f\left(\text{bulk density, fuel size}\right) \qquad (4)$$

- **Propagating Flux**

It is given by

$$I_p = I_{Xig} + \int_{-\infty}^{0} \left(\frac{\partial I_z}{\partial z} \right)_{z_c} dx \qquad (5)$$

$\Downarrow$ $\Downarrow$

horizontal flux — gradient of the vertical flux integrated from -infinity to the fire front

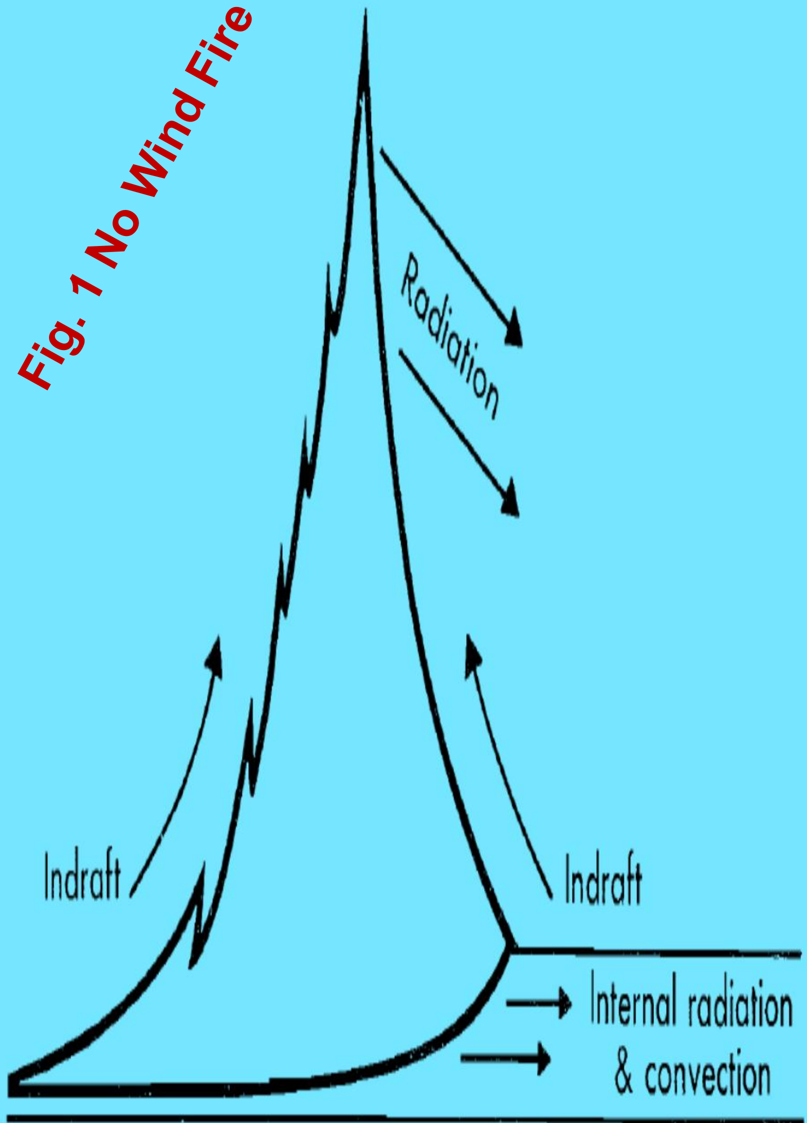


Fig. 1 No Wind Fire

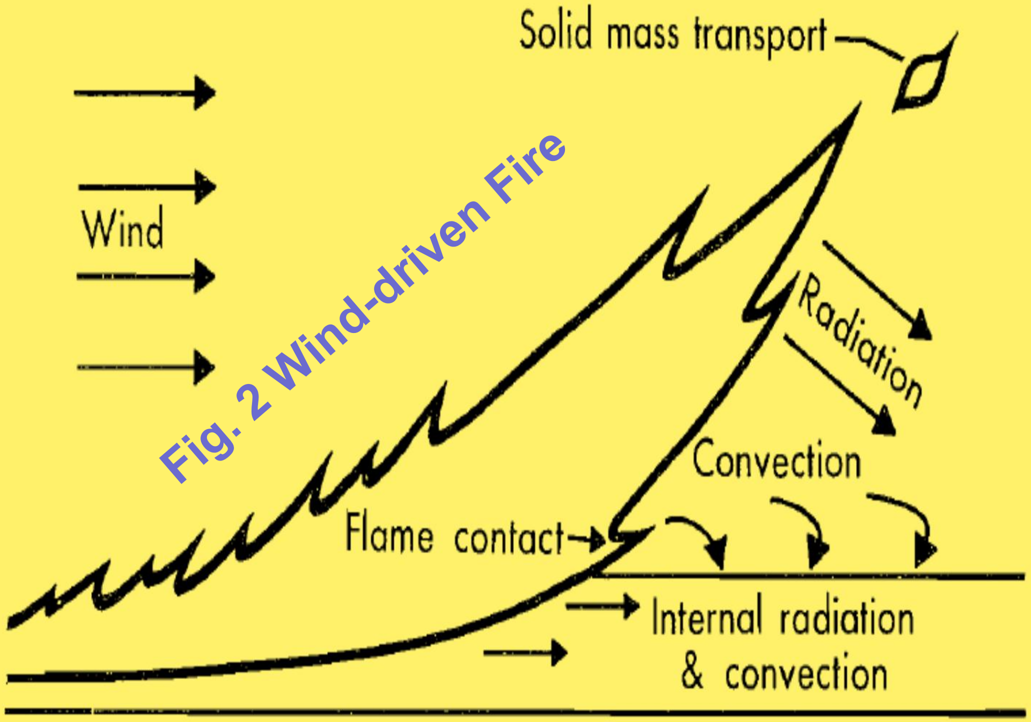


Fig. 2 Wind-driven Fire

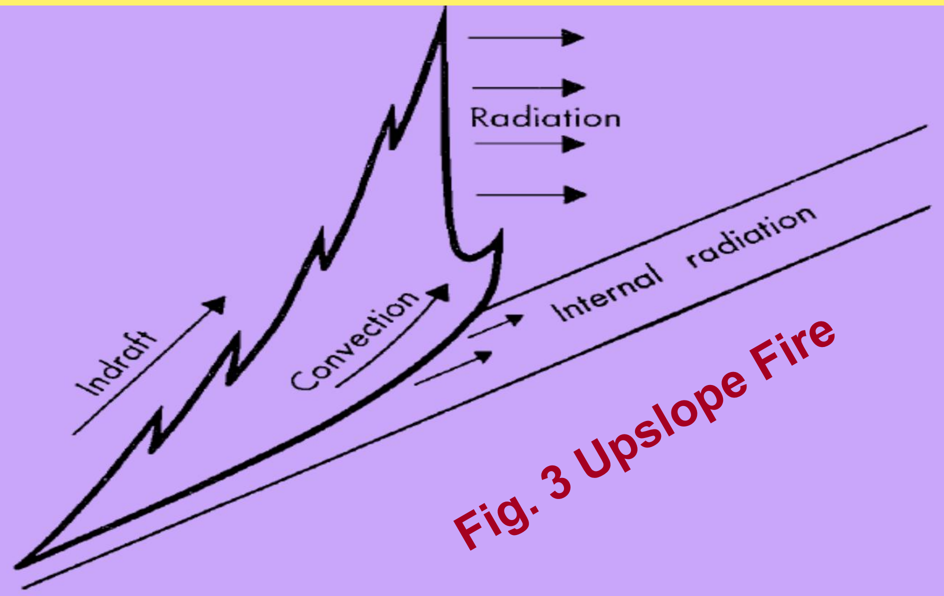


Fig. 3 Upslope Fire

- **The figures indicate that the vertical flux is more significant during wind-driven and upslope fires because the flame tilts over the potential fuel, thereby, increasing radiation, causing direct flame contact and convective heat transfer to the potential fuel.**

- **Assume that the vertical flux is small for no-wind fires and let $I_p = (I_p)_0$. This is the basic heat flux component to which all additional effects of wind and slope are related.**
- **With (3) – (5) in (1) and $I_p = (I_p)_0$ and *R*=*R_0* for the no-wind case, we get**

$$(I_p)_0 = R_0 \rho_b \varepsilon Q_{ig} \qquad (6)$$

Reaction Intensity

- **The energy release rate of the fire front is produced by burning gases released from the organic matters in the fuels.**

- **The rate of change of this organic matter from a solid to a gas is a good approximation of the subsequent heat release rate of the fire.**

- **The heat release rate per unit area of the front is called the reaction intensity and is defined as**

$$I_R = -\frac{dw}{dt} h \qquad (7)$$

mass loss rate per unit area in the fire front   Heat content of fuel

- **The reaction intensity is a function of fuel parameters such as particle size, bulk density, moisture, and chemical composition.**
- **The reaction intensity is the source of the no-wind propagating flux**

$$\left(I_p\right)_0 = f\left(I_R\right) \qquad (8)$$

Effect of Wind & Slope

- **Wind and slope change the propagating heat flux by exposing the potential fuel to additional convective and radiant heat.**

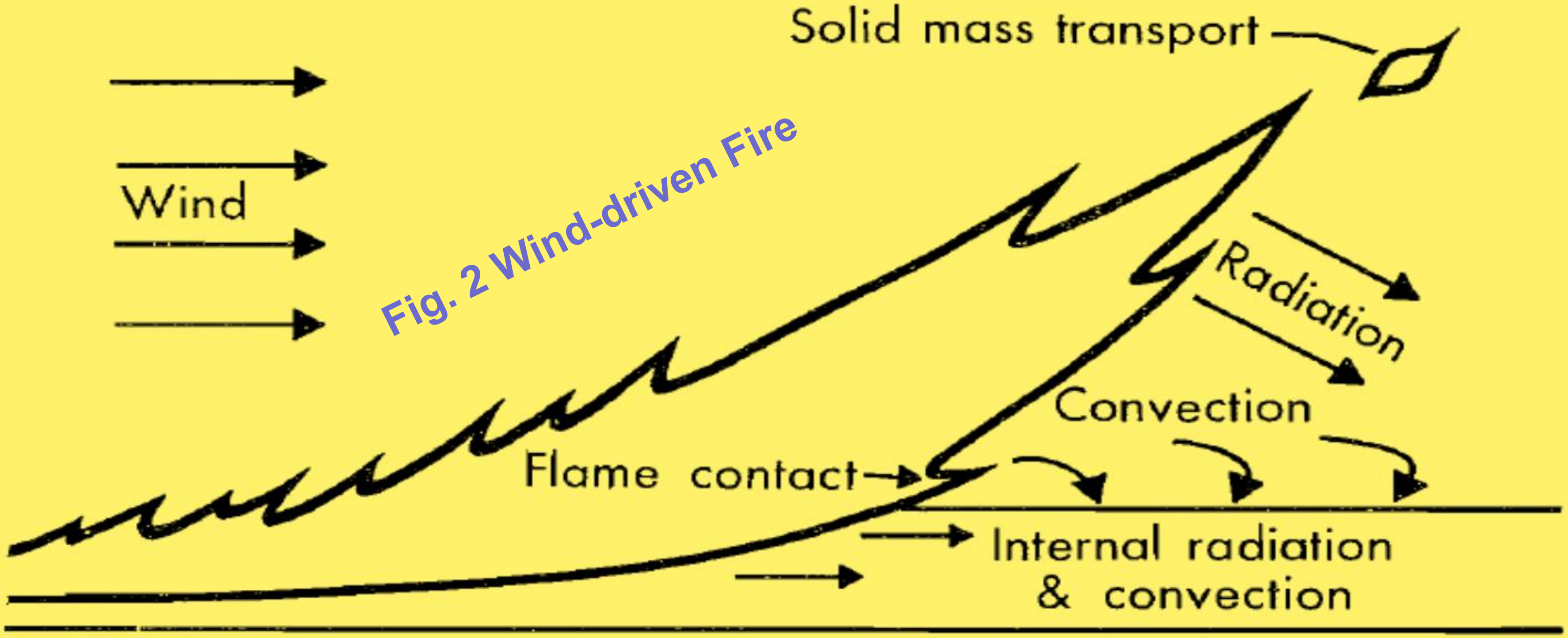


Fig. 2 Wind-driven Fire

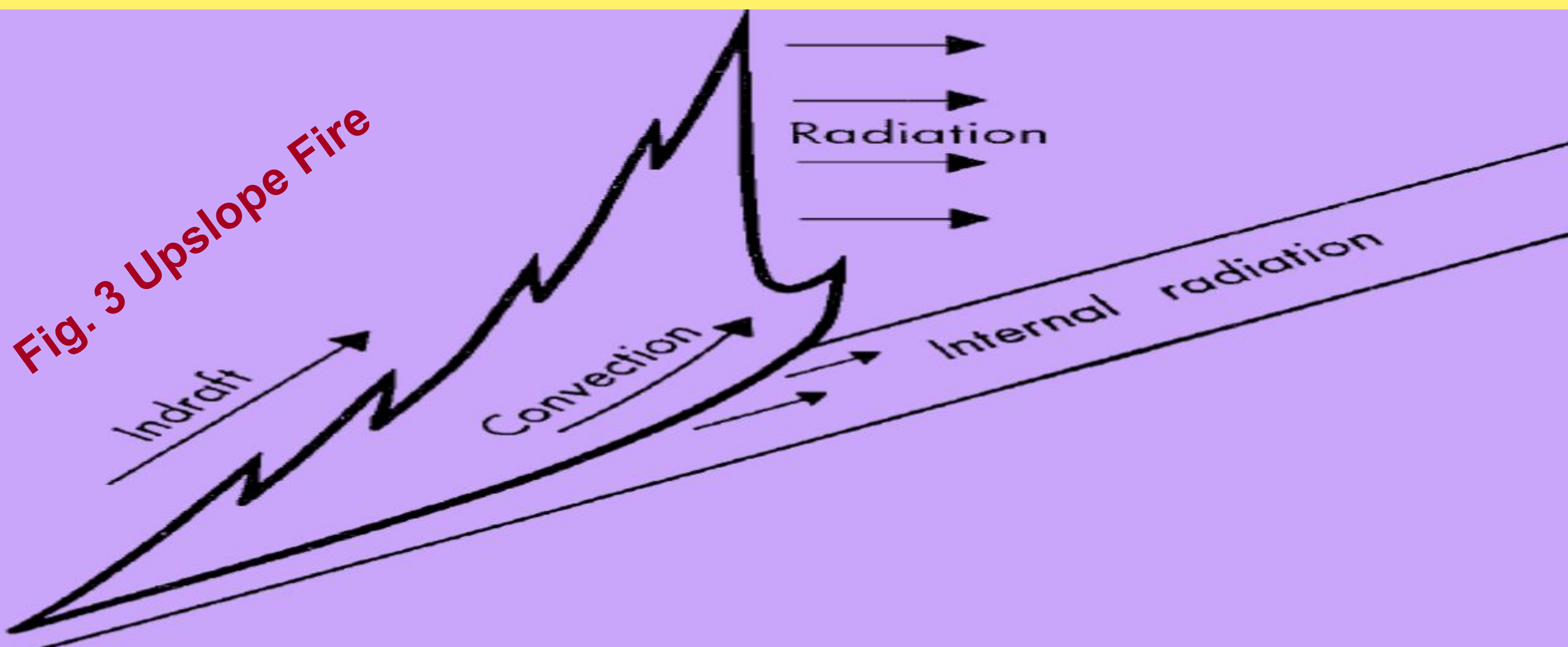


Fig. 3 Upslope Fire

- **Let ϕ_w and ϕ_s represent the additional propagating flux produced by wind and slope. The total propagating flux is**

$$I_p = \left(I_p\right)_0 \left(1 + \phi_w + \phi_s\right) \qquad (9)$$

- **Approximate rate of spread (eq.(1)) becomes**

$$R = \frac{\left(I_p\right)_0 \left(1 + \phi_w + \phi_s\right)}{\rho_b \varepsilon Q_{ig}} \qquad (10)$$

Heat of Pre-ignition

C_{pd}	**Specific heat of dry wood**
ΔT_{ig}	Temperature range to ignition
M_f	Fuel moisture lb. water / lb. dry wood
C_{pw}	Specific heat of water
ΔT_b	Temperature range to boiling
V	Latent heat of vaporization

- Q_{ig} **for cellulosic fuels is determined by considering the change in specific heat from ambient to ignition temperature and the latent heat of vaporization of the moisture as**

$$Q_{ig} = C_{pd} \Delta T_{ig} + M_f \left(C_{pw} \Delta T_b + V \right) \qquad (11)$$

- **Taking temperature to ignition in the range of 20^{0}c to 320^{0}c and boiling temperature to be @ 100^{0}c, eq. (11) becomes**

$$Q_{ig} = 250 + 1116 M_f, B.t.u./lb \qquad (12)$$

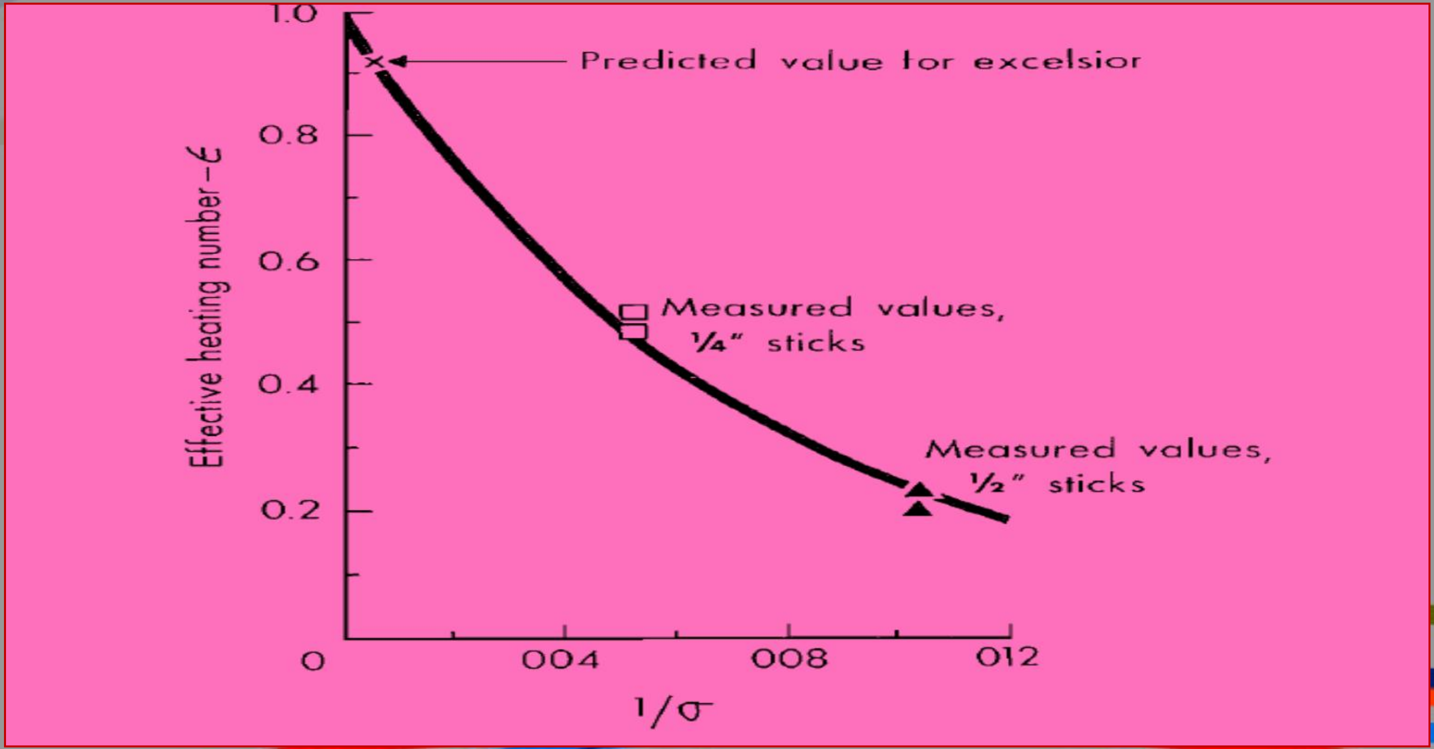
Effective Bulk Density

- **To determine the effective bulk density, we need to compute the efficiency of heating as a function of particle size.**

- **An exponential fit is given by**

$$\varepsilon = exp\left(-\frac{138}{\sigma}\right) \qquad (13)$$

where σ is particle surface area to volume ratio, ft^{-1}



- **Rearrange eq.(7) as**

$$I_R = -\left(\frac{dw}{dx}\right)\left(\frac{dx}{dt}\right)h \qquad (15)$$

where $\dfrac{dx}{dt} = R$, the quassy-steady rate of spread

$$\therefore I_R dx = -Rhdw \qquad (16)$$

- **To solve (16), integrate x over the reaction zone depth D, and w over the limits of loading in the reaction zone.**

$$I_R \int_0^D dx = -Rh \int_{w_n}^{w_r} dw \Rightarrow I_R D = Rh(w_n - w_r) \quad (18)$$

where

D	Reaction zone depth (front to rear)
w_n	Net initial fuel loading, lb/ft^2
w_r	Residue loading immediately after passage of the reaction zone, lb/ft^2

- **The time taken for the fire front to travel a distance equivalent to the depth of one reaction zone is the reaction time,**

$$\tau_R = \frac{D}{R} \qquad (19)$$

Then eq. (18) becomes

$$I_R = \frac{h(w_n - w_r)}{\tau_R} \qquad (20)$$

- **Next, we define a maximum reaction intensity where there is no loading residue left after the reaction zone is passed and where the reaction time remains unchanged.**

$$I_{R\max} = \frac{h w_n}{\tau_R} \qquad (21)$$

- **The reaction zone efficiency is**

$$\eta_\delta = \frac{I_R}{I_{R\max}} = \frac{w_n - w_r}{w_n} \qquad (22)$$

$$\therefore I_R = \frac{w_n h \eta_\delta}{\tau_R} \qquad (23)$$

In (23), the net fuel loading is given by

$$w_n = \frac{w_0}{1 + S_T} \qquad (24)$$

where

$w_0 =$ ovendry fuel loading, lb/ft^2

$S_T =$ fuel mineral content, lb minerals/lb dry fuel

Reaction Velocity

- **The reaction velocity denotes the completeness and rate of fuel consumption. It is defined as the ratio of the reaction zone efficiency to the reaction time.**

$$\Gamma = \frac{\eta_\delta}{\tau_R} * \text{reaction velocity/min.} \qquad (25)$$

Effective Fuel Parameters for Reaction Velocity

1. **Moisture content**
2. **Mineral content**
3. **Particle size, and**
4. **Fuel bed bulk density**

Γ'	**Potential reaction velocity/min**
η_M	Moisture damping coefficient having values ranging from 1 to 0
η_s	Mineral damping coefficient having value ranging from 1 to 0

Then

$$\Gamma = \Gamma' \eta_M \eta_s \qquad (26)$$

then the reaction intensity is

$$I_R = w_n h \Gamma' \eta_M \eta_s \qquad (27)$$

One can evaluate reaction velocity and the moisture and mineral damping coeffiecients by experiments.

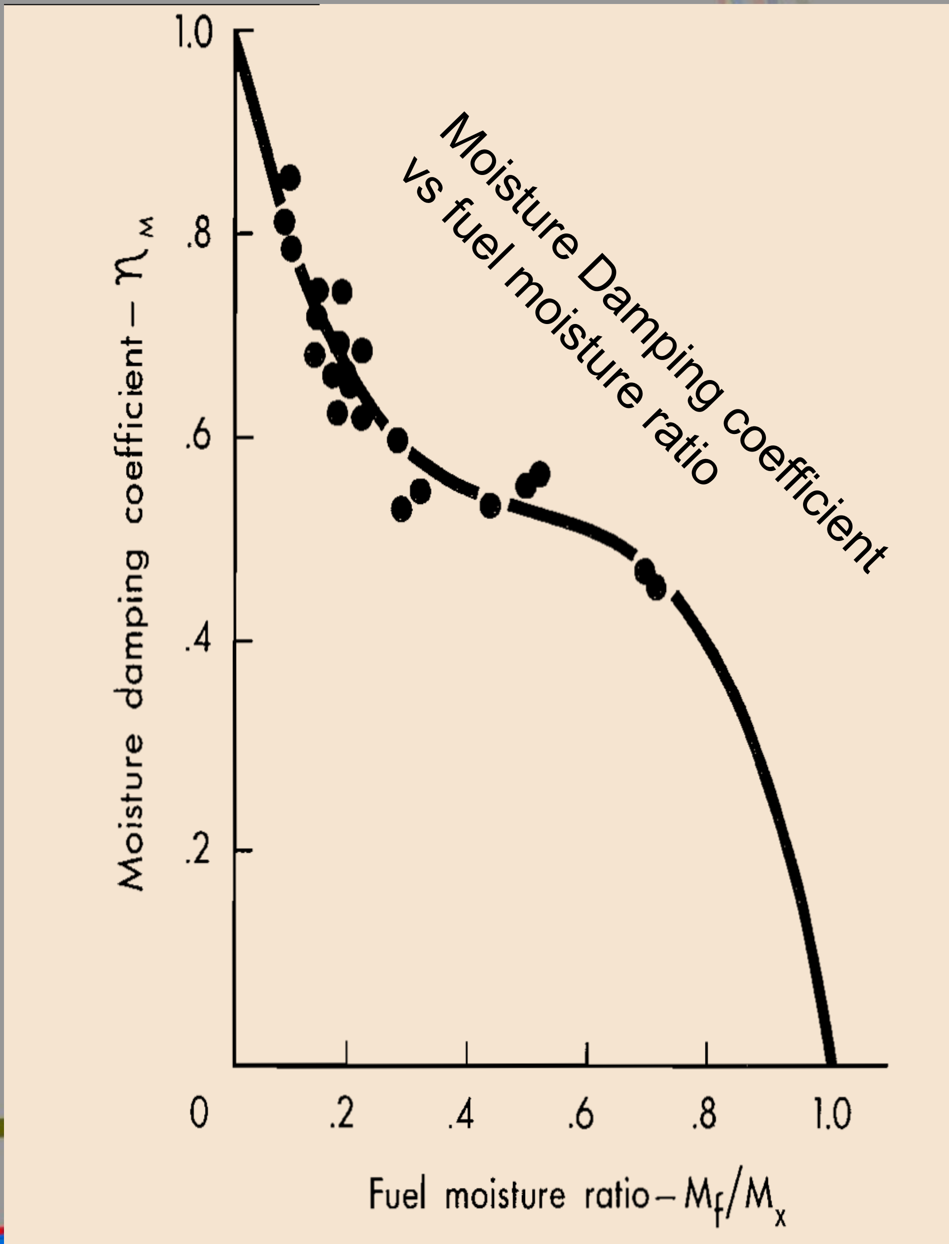
Moisture Damping coefficient

- It is defined as

$$\eta_M = \left.\frac{I_R}{I_{R\max}}\right|_{M_f=0} \qquad (28)$$

In adjacent fig., M_f is fuel moisture and M_x is the moisture content of the fuel at which the fire will not spread.

Moisture damping coefficient accounts for the decrease in intensity caused by the combination of fuel that initially contained moisture.

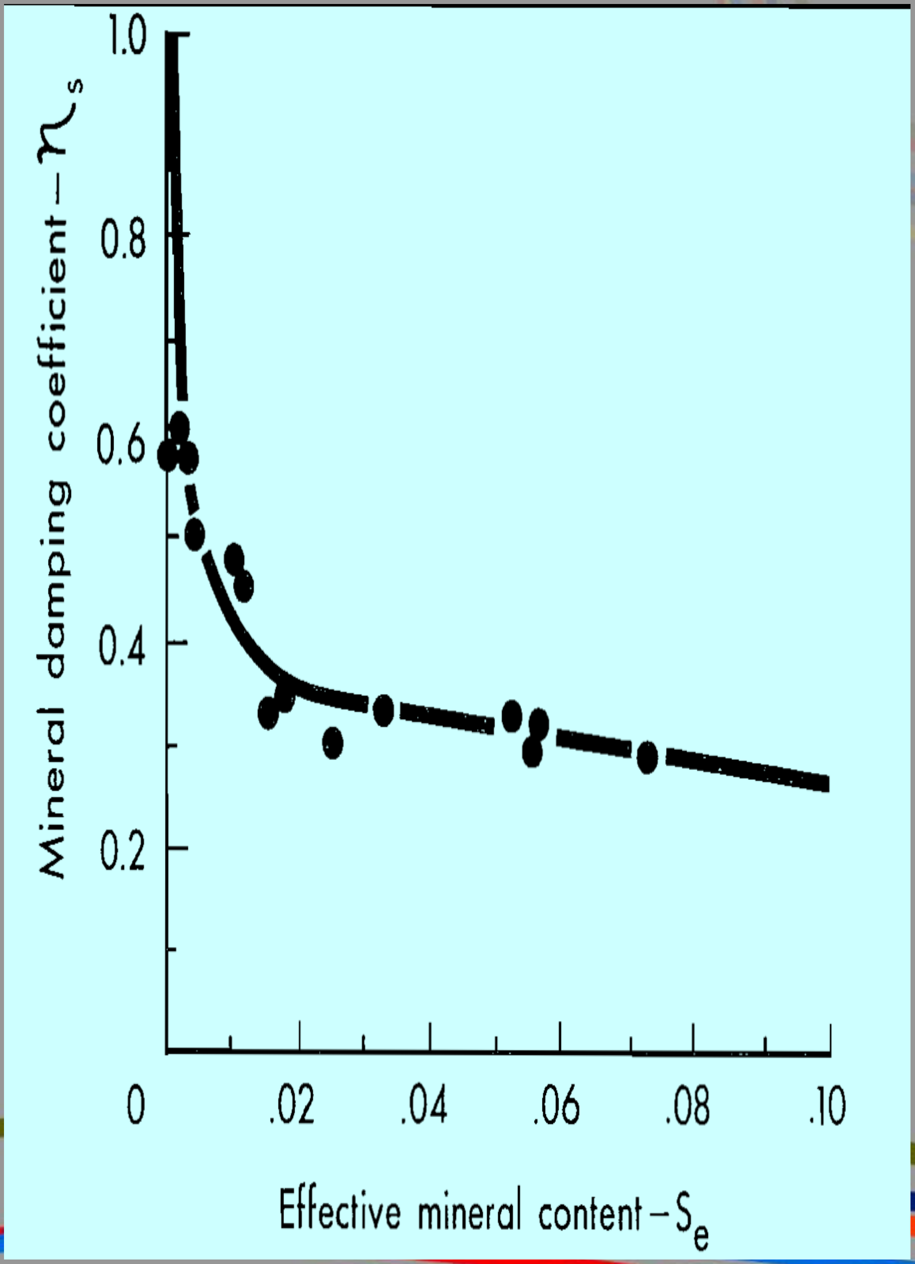


Mineral Damping coefficient

- **It was evaluated from thermo gravimetric analysis (TGA) data of natural fuels by Philot (1968). It is assumed that the ration of the normalized decomposition rate will be same as the normalized reaction intensity.**

$$\eta_s = 0.174 S_e^{-0.19} \qquad (30)$$

where S_e is effective mineral content.



Physical Fuel Parameters

- **Next, we need to consider two parameters:**

1. **The reaction intensity – fuel bed compactness, and**
2. **Fuel particle size**

Assume that low values of fire intensity and rate of spread occur at the two extremes of compactness (loose and dense).

In dense beds, this can be attributed to low air-to-fuel ratio and to poor penetration of the heat beyond the upper surface of the fuel array.

- **In loose beds, low intensity and poor spread are attributed to heat transfer looses between particles and to lack of fuels.**

- **Between these two extremes, there must be an optimum best equilibrium of air, fuel and heat transfer for both maximum fire intensity and reaction velocity.**

- **The compactness of the fuel bed is quantified by the packing ration, which is the ratio of the fuel array to fuel particle density.**

$$\text{packing ratio}, \beta = \frac{\rho_b}{\rho_p} \qquad (31)$$

$$\text{where } \rho_b : \text{fuel array bulk density, lb/ft}^3$$

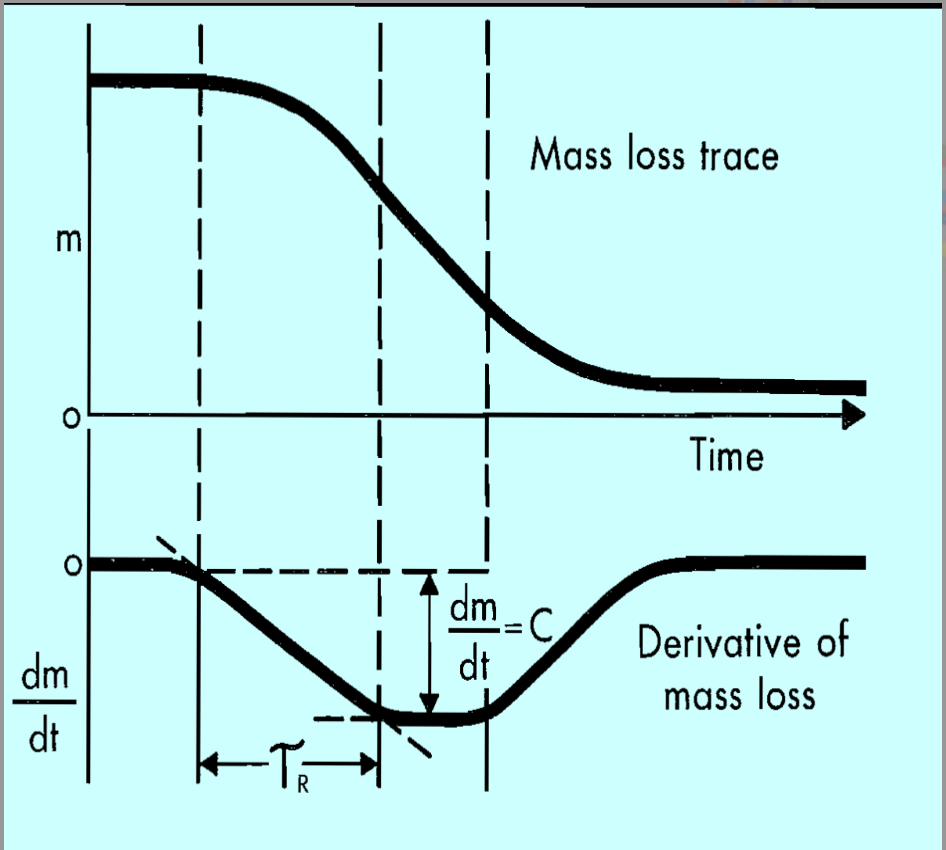
$$\rho_p : \text{fuel particle density, lb/ft}^3$$

- **The surface area-to-volume ratio, σ is used to quantify the particle size.**
- **For fuels that are long w.r.t. the thickness**

$$\sigma = \frac{4}{d} \qquad (32)$$

here d denotes diameter of circular particles or edge length of square particles.

The reaction time, τ_R is defined on the derivative curve as the time from initial mass loss until the loss stabilizes at a steady rate.



During the reaction time mass loss rate is linear. Also the duration of the constant mass rate is dependent on the length of platform.

The reaction time could be thought of as the fire burned off.

The mass loss rate is

$$\frac{dm}{dt} = \left(w_n - w_r\right) RW \qquad (33)$$

W : width of the platform.

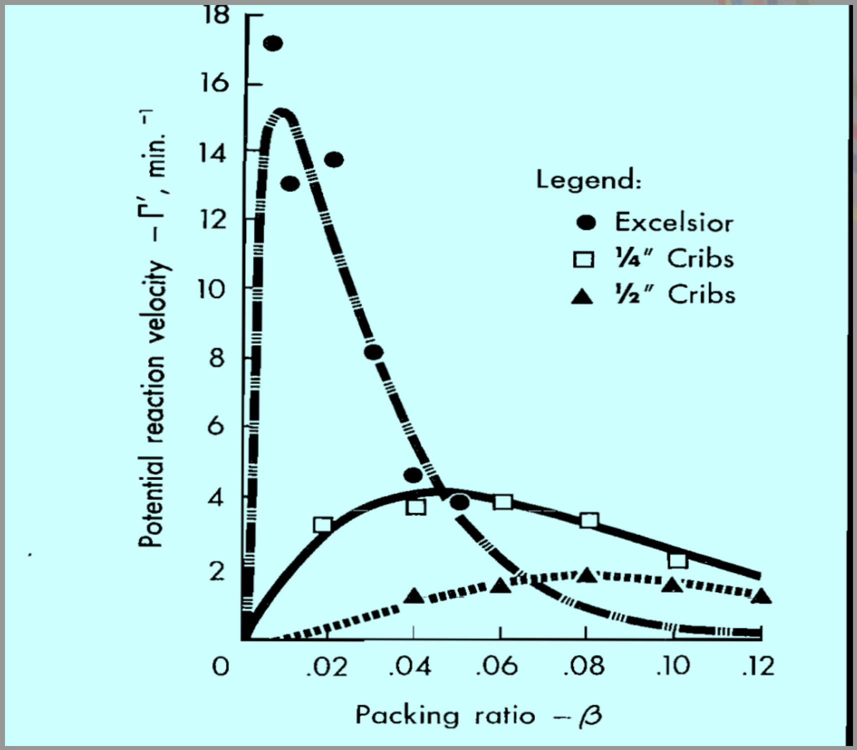
The efficiency of fire is

$$\eta_\delta = \frac{1}{w_n RW} \frac{dm}{dt} \qquad (34)$$

- **With efficiency and the reaction time, the reaction velocity is**

$$\Gamma = \frac{1}{w_n R W \tau_R} \frac{dm}{dt} \tag{35}$$

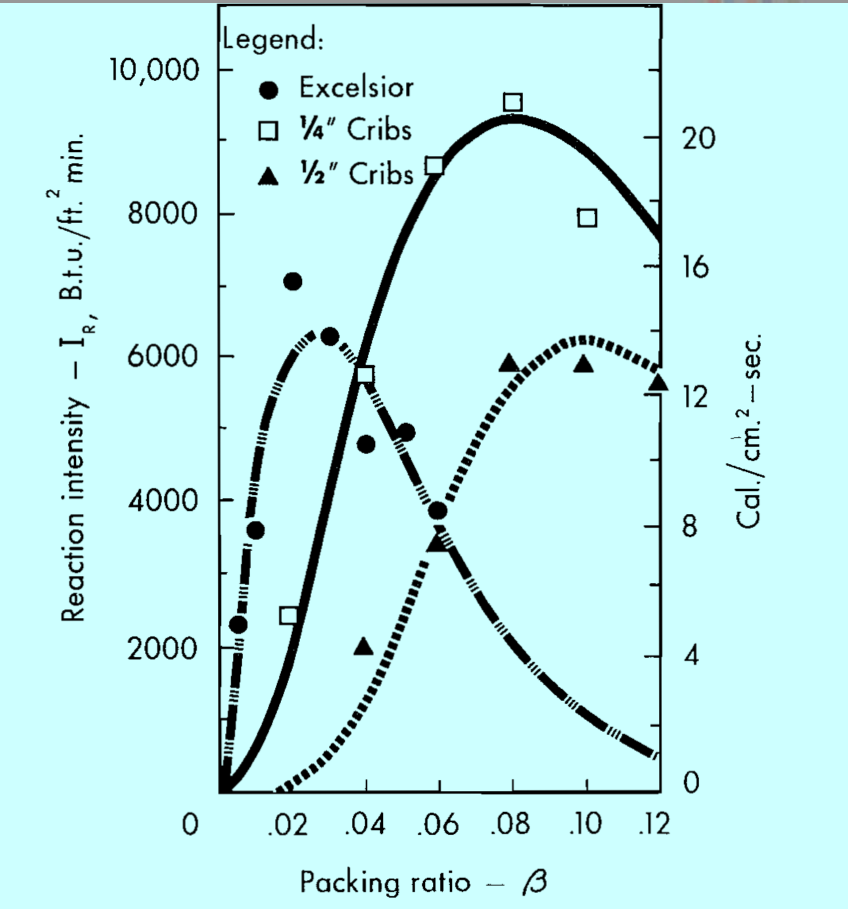
The potential reaction velocity to disassociate the reaction velocity from the effects of the moisture and minerals of the fuel is $\Gamma' = \dfrac{\Gamma}{\eta_M \eta_s}$.



The reaction velocity must drop to zero if there is no fuel to support combustion.

Tightly packed fine fuels have lower reaction velocity than do longer fuels at the same packing ratio.

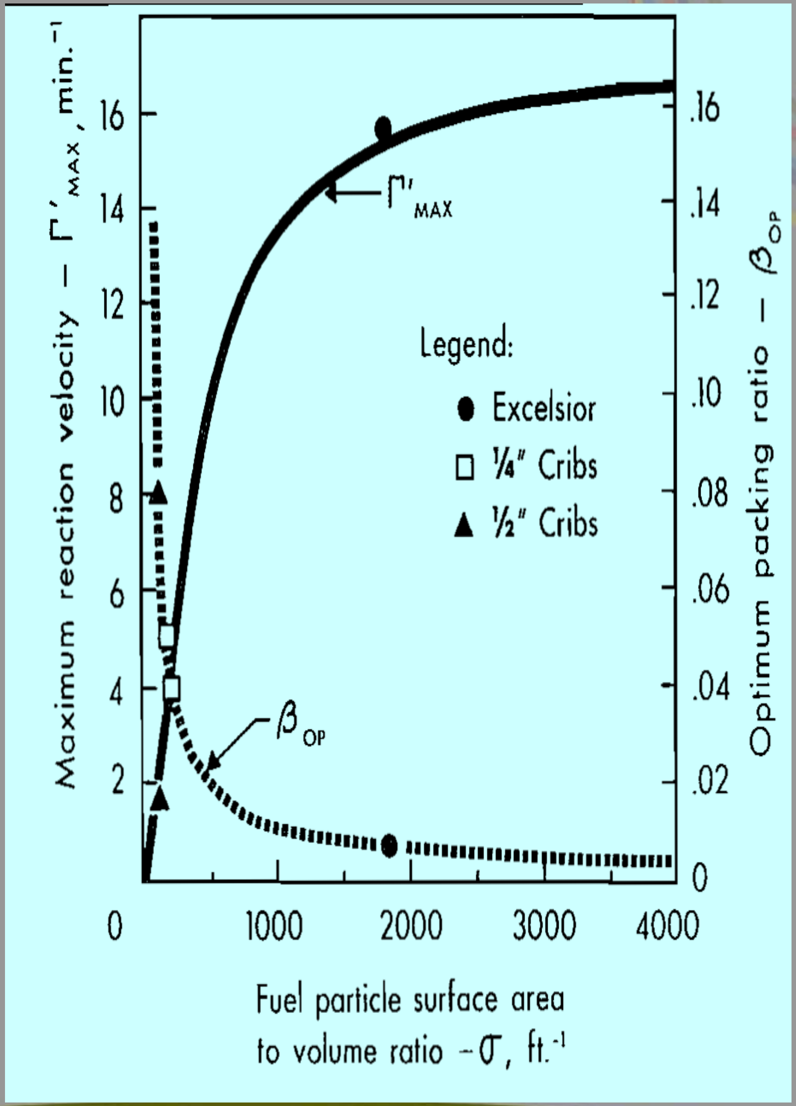
The loss of reaction intensity of the fine fuel can be seen as the difference in flaming vigor.



To determine the general equation as a function of β and σ is given by

$$\Gamma' = \frac{\sigma^{1.5}}{495 + 0.0594\sigma^{1.5}} \qquad (36)$$

$$\beta_{op} = 3.348\sigma^{-0.8189} \qquad (37)$$



Combining these two eqs. gives

$$\Gamma' = \Gamma'_{\max}\left(\frac{\beta}{\beta_{op}}\right)^{A}\exp\left[A\left(1-\frac{\beta}{\beta_{op}}\right)\right] \qquad (38)$$

$$\text{with } A = \frac{1}{4.77\sigma^{0.1}-7.27} \qquad (39)$$

These will predict reaction velocity for any combination of fuel particle size and any packing ratio. The eqs. will predict reasonable values when input parameters are extrapolated. This will help us to predict reaction intensity and, subsequently, rate of spread over a wide range of fuel and environmental combinations.

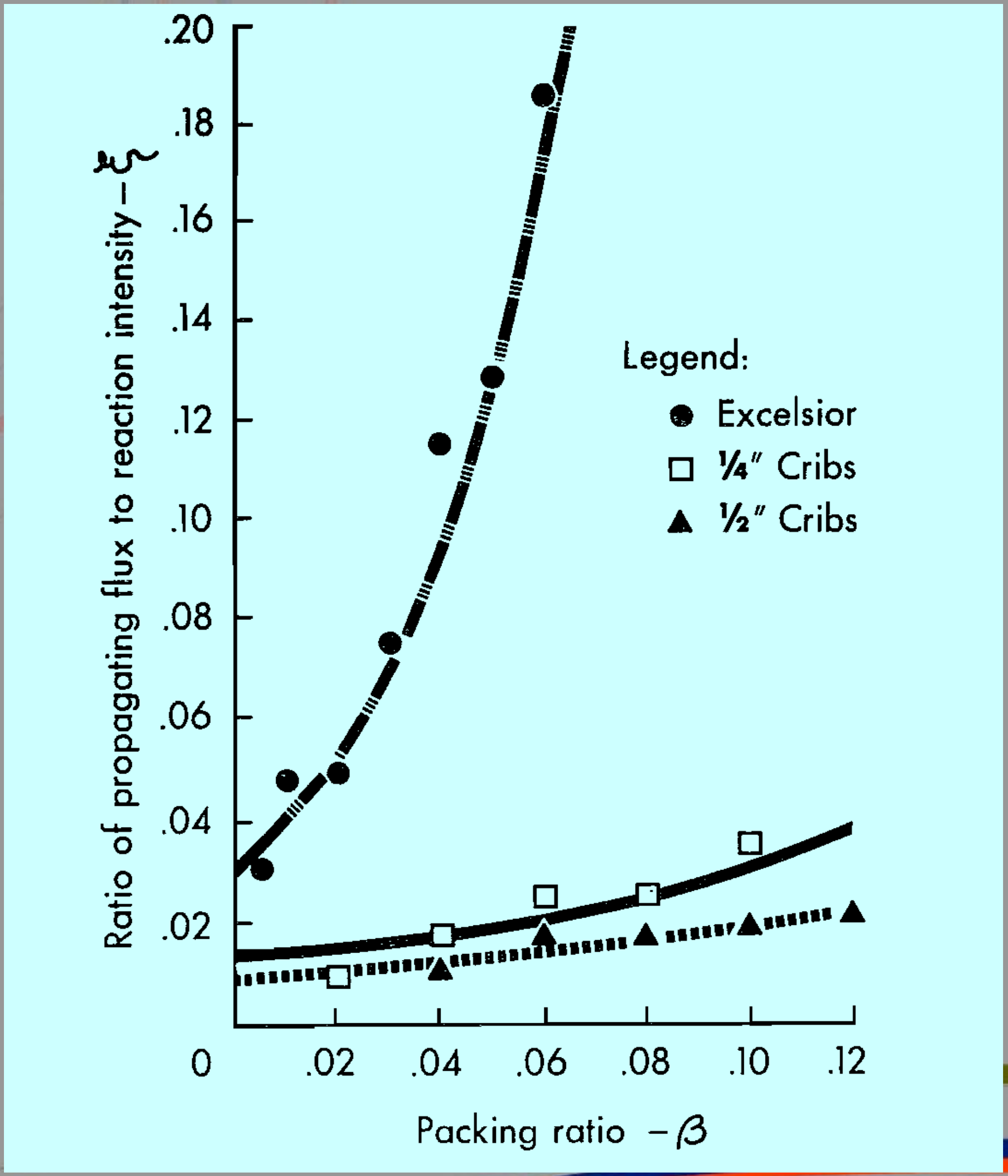
Propagating Flux

The no-wind propagating flux is

$$\left(I_p\right)_0 = R_0 \rho_b \varepsilon Q_{ig} \text{(we know from (6))}$$

A ratio ξ relating the propagating flux to the reaction intensity is

$$\xi = \frac{\left(I_p\right)_o}{I_R} \qquad (41)$$


Ratio of propagating flux to reaction intensity – ξ
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
.02
.04
.06
.08
.10
.12
Packing ratio – β
Legend:
Excelsior
¼″ Cribs
½″ Cribs

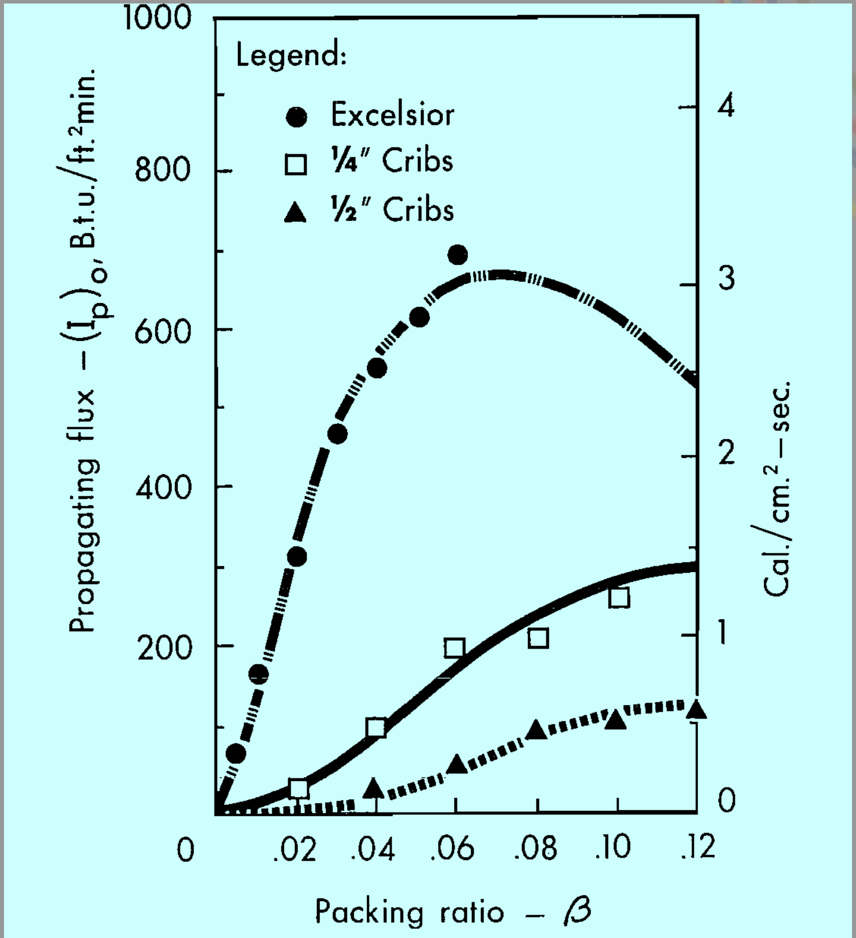
A respective fit is

$$\xi = \frac{1}{192 + 0.259\sigma} \exp\left[(0.792 + 0.681\sqrt{\sigma})(\beta + 0.1) \right] \qquad (42)$$

It is seen that $(I_p)_0$ increases with increase in β, but at a decreasing rate.

$(I_p)_0$ **will attain a maximum and then decreases.**

This is reasonable, considering the fact that the fuel array is becoming so compact that the intensity has decreased.



Rate of Spread

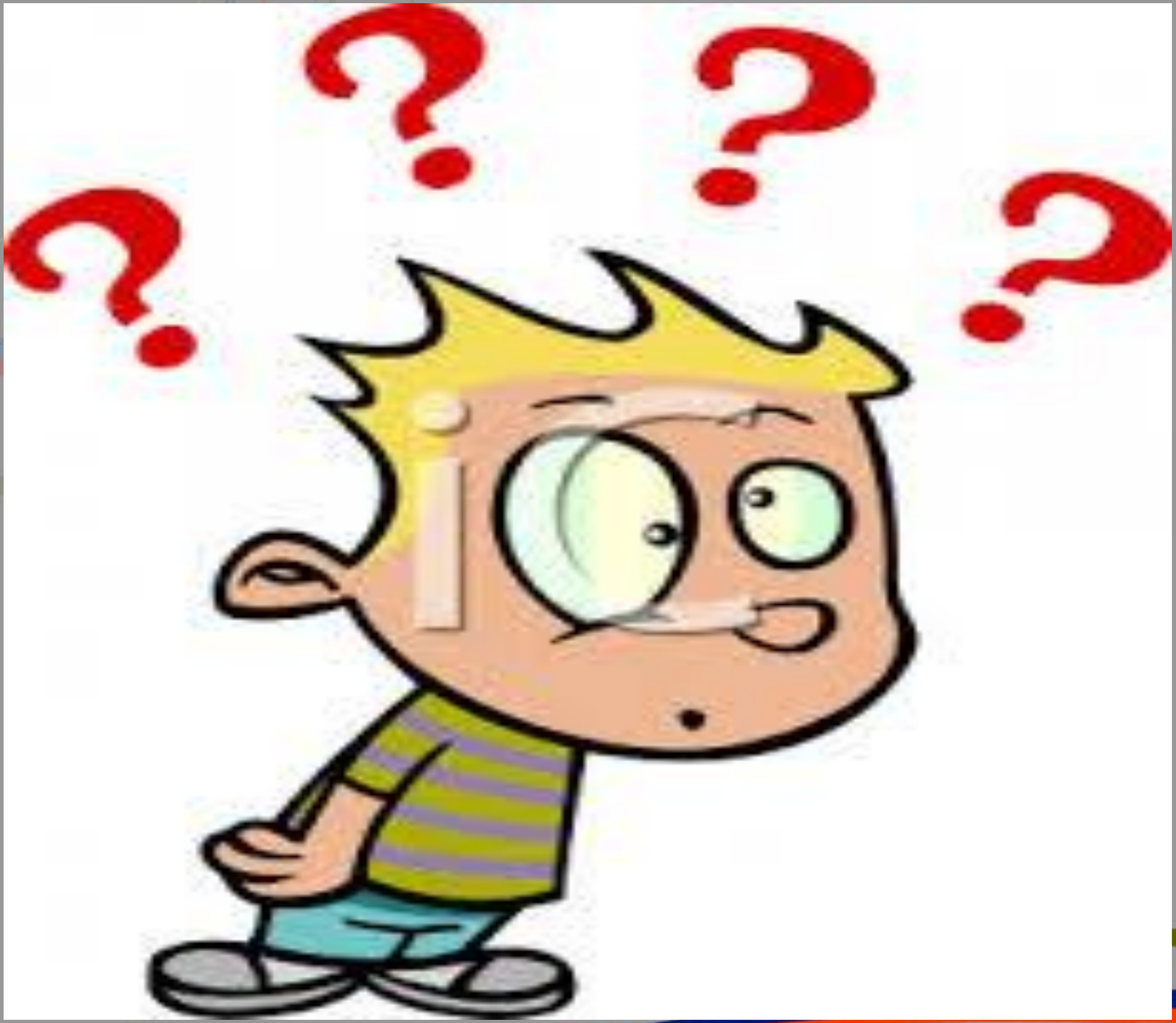
Combining the heat source and heat sink terms results into the no-wind rate of spread equation as

$$R_0 = \frac{I_R \xi}{\rho_b \varepsilon Q_{ig}} \qquad (43)$$


Fire Flames

Reference

1. Richard C. Rothermal (1972): Intermountain Forest and Range experiment station Forest service, Res. Paper 115.



Thank you very much for your attention