

FUNCTION PROJECTIVE SYNCHRONIZATION OF A NEW HYPER CHAOTIC SYSTEM

Priyambada Tripathi

Research Scholar
Department of Mathematics
University of Delhi, Delhi 110007
India
dupriyam@gmail.com

1. Introduction

- In this article a function projective synchronization(FPS) of two identical new hyper chaotic systems is defined and scheme of FPS is developed by using Open-Plus-Closed-Looping(OPCL) coupling method. A new hyper chaotic system has been constructed and then response system with parameters perturbation has been defined. Numerical simulations verify the effectiveness of this scheme, which has been performed by mathematica and MATLAB.
- Here, we have constructed a new hyper chaotic system and verified the chaotic behavior of this system by time series analysis and drawing chaotic attractors via mathematica and MATLAB. Hyperchaotic behavior of this system is discovered within some system parameters range, which has not yet been reported previously.

2. Preliminaries

In this section we mention some definitions and scheme of the main task.

- Chaos is a dynamical regime in which a system becomes extremely sensitive to initial conditions and reveals an unpredictable and random-like behavior. Small differences in initial conditions yield widely diverging outcomes for chaotic systems, rendering long term prediction impossible in general.
- A hyperchaotic system is defined as chaotic behavior with at least two positive Lyapunov exponents.

2.1. Function Projective Synchronization. Function Projective synchronization is defined in the following manner:

Let $\dot{x} = F(x, t)$ be the drive chaotic system, and $\dot{y} = F(y, t) + U$ be the response system, where $x = (x_1(t), x_2(t), \dots, x_m(t))^T$,

$y = (y_1(t), y_2(t), \dots, y_m(t))^T$, $U = (u_1(x, y), u_2(x, y), \dots, u_m(x, y))$ is a controller to be determined later.

Denote $e_i = x_i - f_i(x)y_i$, ($i = 1, 2, \dots, m$), $f_i(x)$, ($i = 1, 2, \dots, m$) are functions of x . If

$$\lim_{t \rightarrow \infty} \|e\| = 0,$$

$e = (e_1, e_2, \dots, e_m)$, there exists function projective synchronization (FPS) between these two identical chaotic (hyperchaotic) systems, and we call f a scaling function matrix.

2.2. Methodology for FPS via OPCL. Here, we will construct corresponding response system through the OPCL coupling method. Consider the following n -dimensional chaotic system as drive system

$$\frac{dx}{dt} = f(x) + \Delta f(x), \quad (1)$$

where $x \in \mathbb{R}^n$ and $\Delta f(x)$ is the perturbation part of the parameters. Now, consider the following n -dimensional chaotic system as response system according to coupling method :

$$\frac{dy}{dt} = f(y) + D(y, g), \quad (2)$$

where $y \in \mathbb{R}^n$. The coupling function is:

$$D(y, g) = \dot{g} - f(g) + (H - \frac{\partial f(g)}{\partial g})(y - g) \quad (3)$$

where $\frac{\partial f(g)}{\partial g}$ is the jacobian matrix of the dynamics system. H is an $n \times n$ Hurwitz constant matrix, whose eigen values are negative and $g = \beta(t)x$ with $\beta(t)$ is a scaling function which is continuously differentiable. When $\beta(t) = \pm 1$, system is complete synchronization or antisynchronization accordingly.

Our goal in this paper is to find out $D(y, g)$ and hence find error dynamics of the system such that

$$\lim_{t \rightarrow \infty} \|e\| = \lim_{t \rightarrow \infty} \|y - g\| = 0,$$

where $\|\cdot\|$ is the Euclidean norm, then the systems (1) and (2) are said to be Function Projective synchronized.

3. System Description

3.1. Hyper Chaotic Rabinovich-Fabrikant system. The Rabinovich-Fabrikant chaotic system is a set of three coupled ordinary differential equations exhibiting chaotic behavior for certain values of parameters. They are named after Mikhail Rabinovich and Anatoly

Fabrikant, who described them in 1979 [18]. The equations of system are :

$$\left. \begin{aligned} \dot{x}_1 &= x_2(x_3 - 1 + x_1^2) + \gamma x_1, \\ \dot{x}_2 &= x_1(3x_3 + 1 - x_1^2) + \gamma x_2, \\ \dot{x}_3 &= -2x_3(x_1x_2 + \alpha) \end{aligned} \right\}$$

where α and γ are constant parameters that control the evolution of the system. For some values of α and γ , the system is chaotic but for other it tends to a stable periodic orbit. Now, we construct a new hyper chaotic system by introducing one more differential equation with a new parameter δ in the above system as follow:

$$\left. \begin{aligned} \dot{x}_1 &= x_2(x_3 - 1 + x_1^2) + \gamma x_1, \\ \dot{x}_2 &= x_1(3x_3 + 1 - x_1^2) + \gamma x_2, \\ \dot{x}_3 &= -2x_3(x_1x_2 + \alpha), \\ \dot{x}_4 &= -3x_3(x_2x_4 + \delta) + x_4^2 \end{aligned} \right\} \quad (4)$$

This new system shows hyper chaotic behavior with some values of parameters and tend to stable periodic orbits with other values of parameters. We have investigated system's behavior for different values of δ . Figures are given below:

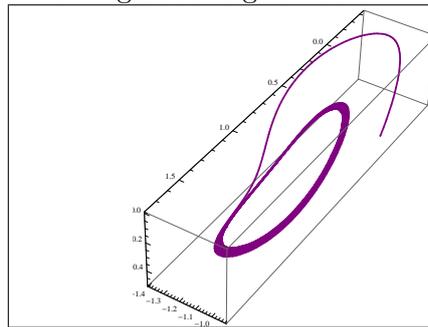


Fig.1 Chaotic behavior of the system with $\alpha = 0.14, \gamma = 1.1$ and $-0.01 \leq \delta \leq 7650$ tending to stable periodic orbits

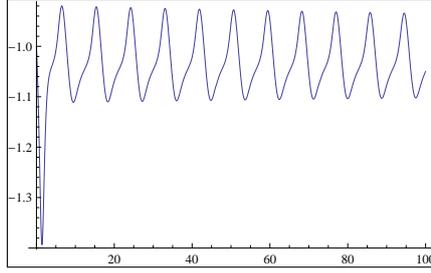


Fig.2 Time series analysis of $y1[t]$
with $\alpha = 0.14, \gamma = 1.1$ and
 $-0.01 \leq \delta \leq 7650$

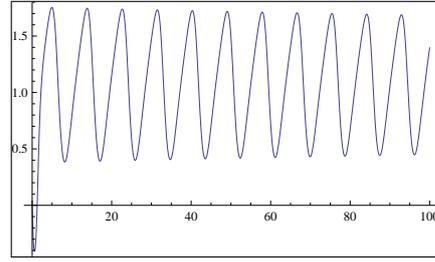


Fig.3 Time series analysis of $y2[t]$
with $\alpha = 0.14, \gamma = 1.1$ and
 $-0.01 \leq \delta \leq 7650$

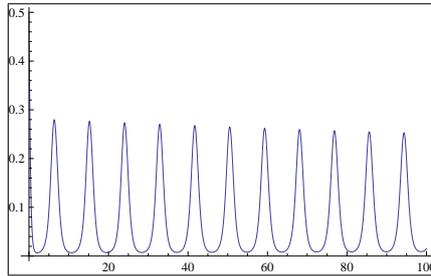


Fig.4 Time series analysis of $y3[t]$
with $\alpha = 0.14, \gamma = 1.1$ and
 $-0.01 \leq \delta \leq 7650$

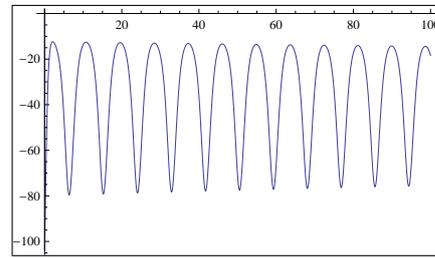


Fig.5 Time series analysis of $y4[t]$
with $\alpha = 0.14, \gamma = 1.1$ and
 $-0.01 \leq \delta \leq 7650$

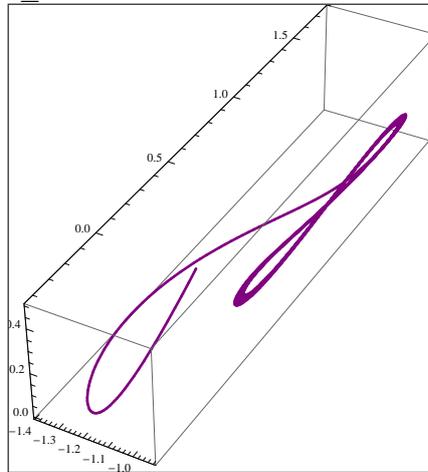


Fig.6 Chaotic Behavior of the
system with $\alpha = 0.87, \gamma = 1.1$
and $\delta = 1890$

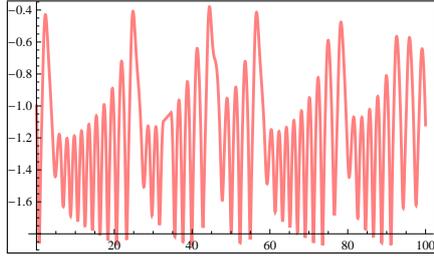


Fig.7 Time series analysis of $y_1[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = 1890$

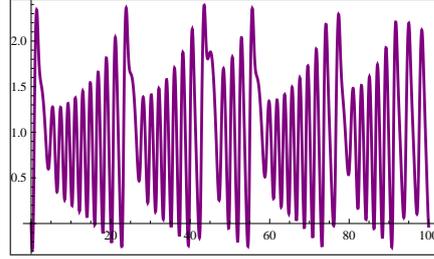


Fig.8 Time series analysis of $y_2[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = 1890$

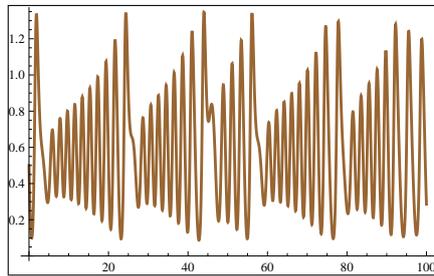


Fig.9 Time series analysis of $y_3[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = 1890$

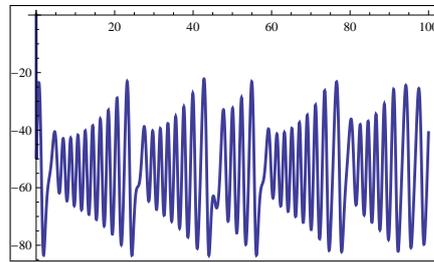


Fig.10 Time series analysis of $y_4[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = 1890$

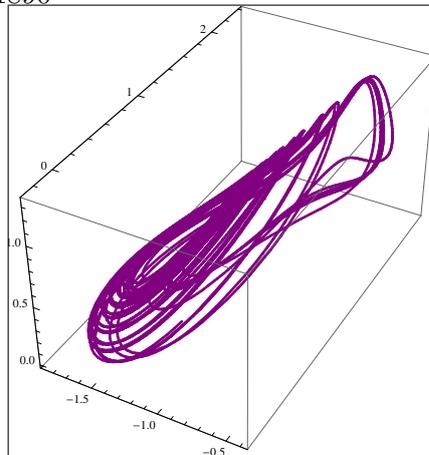


Fig.11 Chaotic Behavior of the system with $\alpha = 0.87, \gamma = 1.1$ and $\delta = -0.2$

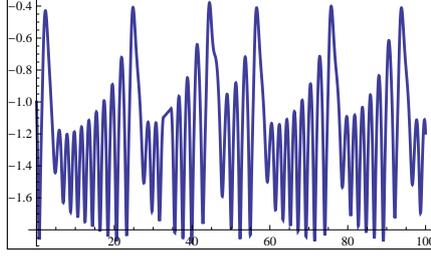


Fig.12 Time series analysis of $y1[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = -0.2$

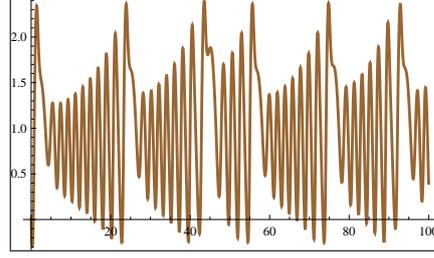


Fig.13 Time series analysis of $y2[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = -0.2$

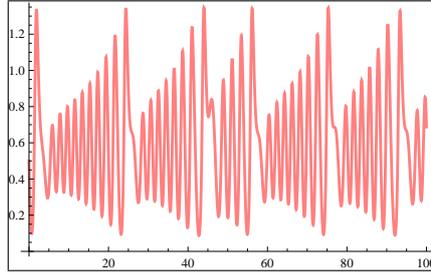


Fig.14 Time series analysis of $y3[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = -0.2$

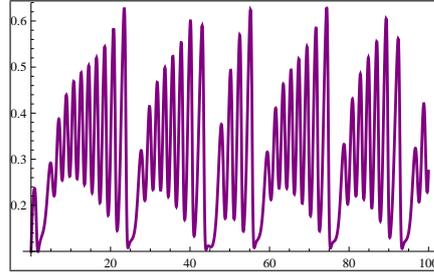


Fig.15 Time series analysis of $y4[t]$ with $\alpha = 0.87, \gamma = 1.1$ and $\delta = -0.2$

3.2. Illustration. In this section, we perform function projective synchronization of above described system via OPCL coupling method. Define following system as a drive system with parameters perturbation as

$$\left. \begin{aligned} \dot{x}_1 &= x_2(x_3 - 1 + x_1^2) + (\gamma + \Delta\gamma)x_1, \\ \dot{x}_2 &= x_1(3x_3 - x_1^2 + 1) + (\gamma + \Delta\gamma)x_2, \\ \dot{x}_3 &= -2x_3(x_1x_2 + \alpha + \Delta\alpha), \\ \dot{x}_4 &= -3x_3(x_2x_4 + \delta + \Delta\delta) + x_4^2 \end{aligned} \right\} \quad (5)$$

where $\Delta\gamma$, $\Delta\alpha$ and $\Delta\delta$ are the perturbation parts in the parameters. Now construct the corresponding response system via OPCL coupling method.

The Jacobian matrix of the system (5) is

$$\frac{\partial f(x)}{\partial x} = \begin{pmatrix} \gamma + \Delta\gamma + 2x_1x_2 & x_1^2 + x_3 - 1 & x_2 & 0 \\ 3x_3 - 3x_1^2 + 1 & \gamma + \Delta\gamma & 3x_1 & 0 \\ -2x_2x_3 & -2x_1x_3 & -2(\alpha + \Delta\alpha + x_1x_2) & 0 \\ 0 & -3x_3x_4 & -3x_2x_4 - 3\delta - 3\Delta\delta & 2x_4 - 3x_2x_3 \end{pmatrix}$$

Define Hurwitz matrix H as the unit negative matrix $-I$ (as $g = \beta x$),

Then $H - \frac{\partial f(g)}{\partial g}$

$$= \begin{pmatrix} -\gamma - \Delta\gamma - 2\beta^2 x_1 x_2 - 1 & -\beta^2 x_1^2 - \beta x_3 + 1 & -\beta x_2 & 0 \\ -3\beta x_3 + 3\beta^2 x_1^2 - 1 & -\gamma - \Delta\gamma - 1 & -3\beta x_1 & 0 \\ 2\beta^2 x_2 x_3 & 2\beta^2 x_1 x_3 & 2(\alpha + \Delta\alpha + \beta^2 x_1 x_2) - 1 & 0 \\ 0 & 3\beta^2 x_3 x_4 & 3\beta^2 x_2 x_4 + 3\delta + 3\Delta\delta & -2\beta x_4 + 3\beta^2 x_2 x_3 - 1 \end{pmatrix}$$

Therefore, response system after coupling is as follows

$$\left. \begin{aligned} \dot{y}_1 &= f(y_1) - f(g_1) + \dot{g}_1 + \left(H - \frac{\partial f(g_1)}{\partial g_1}\right)(y_1 - g_1) \\ \dot{y}_2 &= f(y_2) - f(g_2) + \dot{g}_2 + \left(H - \frac{\partial f(g_2)}{\partial g_2}\right)(y_2 - g_2) \\ \dot{y}_3 &= f(y_3) - f(g_3) + \dot{g}_3 + \left(H - \frac{\partial f(g_3)}{\partial g_3}\right)(y_3 - g_3) \\ \dot{y}_4 &= f(y_4) - f(g_4) + \dot{g}_4 + \left(H - \frac{\partial f(g_4)}{\partial g_4}\right)(y_4 - g_4) \end{aligned} \right\} \quad (6)$$

As error dynamics is defined as $\dot{e} = \dot{y} - \dot{g}$, so we have final equation of error dynamics after coupling and putting values of $f(y), f(g)$ and

$\left(H - \frac{\partial f(g)}{\partial g}\right)(y - g)$ in above equation as follows

$$\left. \begin{aligned} \dot{e}_1 &= \Delta\gamma e_1 + e_2 e_3 + e_2 e_1^2 + 2\beta x_1 e_1 e_2 + \beta x_2 e_1^2 - e_1 \\ \dot{e}_2 &= \Delta\gamma e_2 + 3e_1 e_3 + e_1^3 + 3\beta x_1 e_1^2 - e_2 \\ \dot{e}_3 &= -2\Delta\alpha e_3 - 2e_1 e_2 e_3 + e_2 e_1 e_2 \beta x_3 - 2\beta x_2 e_1 e_3 - 2\beta x_1 e_2 e_3 - e_3 \\ \dot{e}_4 &= -3\Delta\delta e_3 - 2e_2 e_3 e_4 - 3e_2 e_4 \beta x_3 - 3\beta x_2 e_4 e_3 - 3\beta x_4 e_2 - e_4 \end{aligned} \right\} \quad (7)$$

So, from the above error dynamics we can conclude that FPS between two identical hyper chaotic system can be achieved.

4. NUMERICAL SIMULATIONS

If Perturbation of Parameters of the response system of hyper chaotic Rabinovich-Fabrikant system are zero and $\beta = 0.5$ with the initial conditions of drive system $[x_1(0), x_2(0), x_3(0), x_4(0)] = [0, 2, 0.5, -0.2]$ and response systems $[y_1(0), y_2(0), y_3(0), y_4(0)] = [0.5, 1, -0.1, -0.15]$ respectively. So, the initial conditions for $[e_1(0), e_2(0), e_3(0), e_4(0)] = [0.5, 0, -0.35, -0.05]$ diagrams of convergence of errors given below are the witness of achieving function projective synchronization between master and slave system.

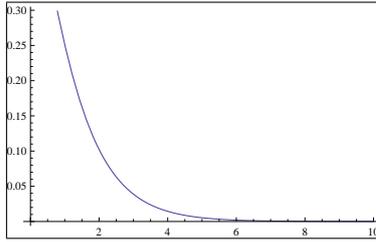


Fig.16 Convergence of error $e_1, t \in [0, 10]$

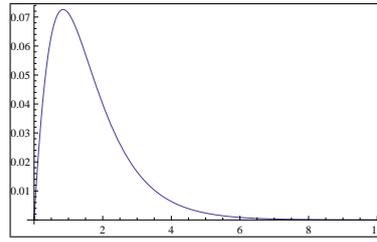


Fig.17 Convergence error of $e_2, t \in [0, 10]$

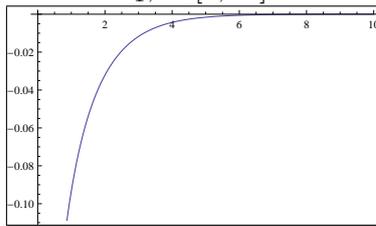


Fig.18 Convergence of error $e_3, t \in [0, 10]$

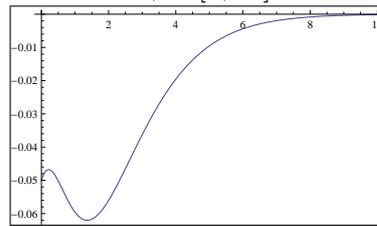


Fig.19 Convergence of error $e_4, t \in [0, 10]$

Conclusion and benefits

Since hyperchaotic systems have the characteristics of high capacity, high security and high efficiency, it has been studied with increasing interest in the fields of non-linear circuits, secure communications, lasers, control, synchronization, and so on. So we have studied Function Projective Synchronization behavior for this new hyper chaotic systems, which is of course more effective and useful in secure communication as FPS is more useful in secure communication as compare to others because of its unpredictability. The results are validated by numerical simulations using mathematica. It has more advantage over other synchronization to enhance security of communication and moreover as it is performed for hyperchaotic system, which makes it more useful.

REFERENCES

- [1] Pikovsky A, Rosenblum M, Kurths J. Synchronization: A Universal Concept Nonlinear Science. Cambridge University Press, 2002.
- [2] Glass L. Synchronization and rhythmic processes in physiology. Nature, 410:277,2001.
- [3] Strogatz S. Sync. Hyperion, 2003.
- [4] Pecora LM, Carroll TL. Synchronization in chaotic systems, Phys. Rev. Lett. 64(1990): 821–824.
- [5] Liao TL, Lin SH. Adaptive control and synchronization of Lorenz systems, J. Franklin Inst. 336(1999): 925937.
- [6] Chen SH, Lu J. Synchronization of an uncertain unified system via adaptive control, Chaos Solitons & Fractals 14(2002), 643–647.
- [7] Ge ZM, and Chen YS. Adaptive synchronization of unidirectional and mutual coupled chaotic systems, Chaos Solitons & Fractals 26(2005), 881–888.
- [8] Wang C, Ge SS. Synchronization of two uncertain chaotic systems via adaptive backstepping, Int. J. Bifurcat. Chaos 11(2001), 17431751.

- [9] Wang C, Ge SS. Adaptive synchronizaion of uncertain chaotic systems via adaptive backstepping design, *Chaos Solitons & Fractals* 12(2001), 1199–11206.
- [10] Tan X, Zhang J, Yang Y. Synchronizing chaotic systems using backstepping design, *Chaos Solitons & Fractals* 16(2003): 37–45.
- [11] Chen HK. Synchronization of two different chaotic system: a new system and each of the dynamical systems Lorenz, Chen and Lu, *Chaos Solitons & Fractals* 25(2005), 1049–1056.
- [12] Ho MC, Hung YC. Synchronization two different systems by using generalized active control, *Phys. Lett. A* 301(2002), 424–428.
- [13] Yassen MT. Chaos synchronization between two different chaotic systems using active control, *Chaos Solitons & Fractals* 23(2005), 131–140.
- [14] Park JH. Chaos synchronization between two different chaotic dynamical systems, *Chaos Solitons & Fractals* 27(2006), 549–554.
- [15] Chen HK. Global chaos synchronization of new chaotic systems via nonlinear control, *Chaos Solitons & Fractals* 23(2005), 1245–1251.
- [16] Ge ZM, Yu TC, Chen YS. Chaos synchronization of a horizontal platform system, *J. Sound Vibrat* 268(2003), 731–749.
- [17] Wen GL, Xu D. Observer-based control for full-state projective synchronization of a general class of chaotic maps in any dimension, *Phys. Lett. A* 330(2004), 420–425.
- [18] Rabinovich, Mikhail I. and Fabrikant, A. L. (1979). "Stochastic Self- Modulation of Waves in Nonequilibrium Media". *Sov. Phys. JETP* 50: 311.
- [19] Cenys, A., Tamasevicius, A. and Baziliauskas, A. (2003) Hyperchaos in coupled Colpitts oscillators, *Chaos, Solitons and Fractals*, Vol. 7, pp. 349-353.
- [20] Hua, M. and Xua, Z. Adaptive feedback controller for projective synchronization, *Nonlinear Analysis: Real World Applications* 9 (2008) 1253 –1260.
- [21] Liu, J., Chen, S. H. and Lu, J.A. Projective synchronization in a unified chaotic system and its control, *Acta Phys. Sin* 52 (2003) 1595-1599.
- [22] Mainieri, R. and Rehacek, J. Projective synchronization in three-dimensional chaotic systems, *Phys. Rev. Lett.* 82 (1999) 3042-3045.
- [23] Xin L. and Yong C. Function Projective Synchronization of Two Identical New Hyperchaotic Systems, *Commun. Theor. Phys. (Beijing, China)* 48 (2007) pp. 864-870.