ICTS INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

FONDATION AIRBUS GROUP
Mathematics of Complex Systems

Regularity of solutions of hydrodynamical equations

(joint work with S.S. Ray *et al*, ICTS-TIFR)

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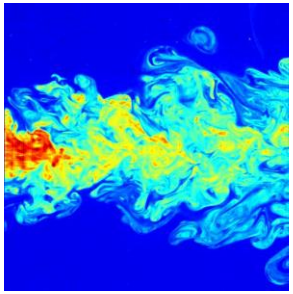

Indian Women and Mathematics 2015, University of Delhi – April 2, 2015

Regularity of solutions of hydrodynamical equations: *Problem motivation*

Applications ranging from

daily life - "what happens when cream is stirred in a cup of coffee ?"

to

serious technological concerns – "if two (or more) aircrafts were to land one behind the other, what is the optimum distance between them to be maintained so that they don't negatively *"Influence"* each other ?"

Question: Dependence of energy dissipation on viscosity ??

Regularity of solutions of hydrodynamical equations: *Clay Institute's Millennium-prize Problem*

- **RESEARCH AIM:** *Prove/disprove **smoothness and/or periodicity** of solutions to hydrodynamical equations (in particular, Navier-Stokes) under **smooth and/or periodic initial and boundary conditions**.*

APPROACHES

PURE / MATHEMATICAL

- Ladyzhenskaya, 1969
- Beale-Kato-Majda theorem
- Leray, 1934 (Acta Math.)
- Scheffer, 1993 (J. Geom. Anal.)
- Schnirelman, 1997 (Comm. Pure & Appl. Math.)
- **Otelbaev, 2013**
- Tao, 2014

. . .

APPLIED / COMPUTATIONAL

Statistical properties of turbulence from numerical simulations etc. –

- Kelvin
- Reynolds
- Kolmogorov (1941)
- von Neumann (1949)
- Kraichnan (1961, 1965)
- Frisch, Benzi, Bec, Succi (2k's)

. . .

Outline

- ***Mathematical formulation of dynamical equations***
  - Reduction to simpler models
  - Time evolution of solutions
- ***Computational methodology***
- ***Key results***
  - Burger’s equation
- ***Summary and future extensions***

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Mathematical formulation

Reduction to simpler models

Navier-Stokes' equations:
(incompressible)

$$\partial_t v + v \cdot \nabla v = -\nabla p + \nu \nabla^2 v,$$
$$\nabla \cdot v = 0$$

Euler equations:
(incompressible)

$$\partial_t v + v \cdot \nabla v = -\nabla p + \cancel{\nu \nabla^2 v},$$
$$\nabla \cdot v = 0$$

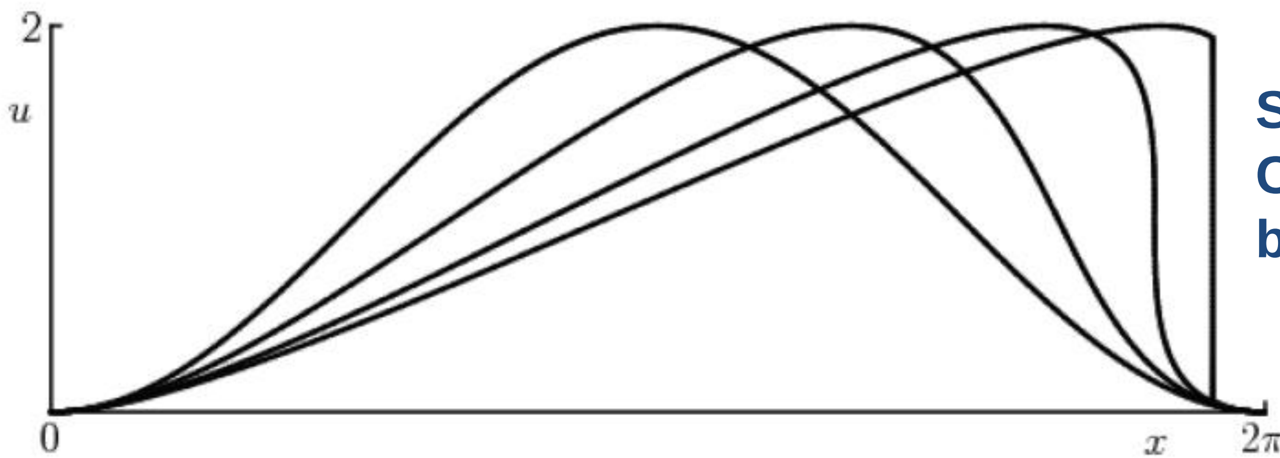
Inviscid Burger's equation:
(1-D)

$$\partial_t v + v \cdot \nabla v = \cancel{-\nabla p}$$

NOTE: The inviscid Burger's equation (here, being one-dimensional) is necessarily compressible, whereas the other two are not. Hence Euler and N-S can not admit shocks as their solutions!

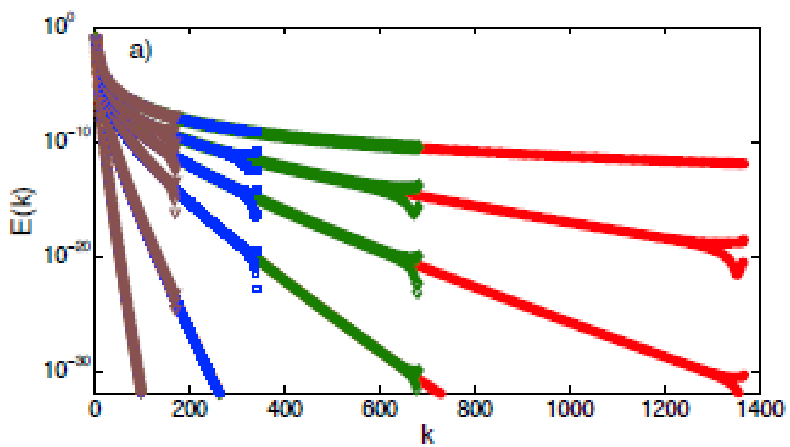
Time evolution of solutions
Burger’s/Euler’s equations

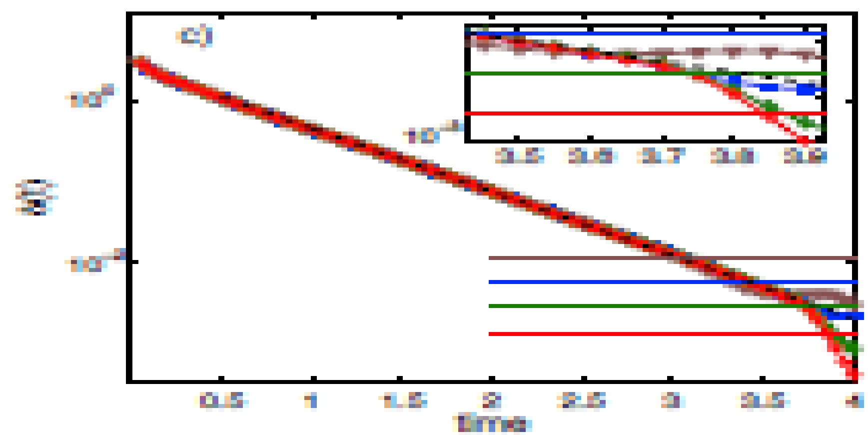
Real space



Shock occurrence ⟺ Complex singularity becomes real

Fourier (spectral) space





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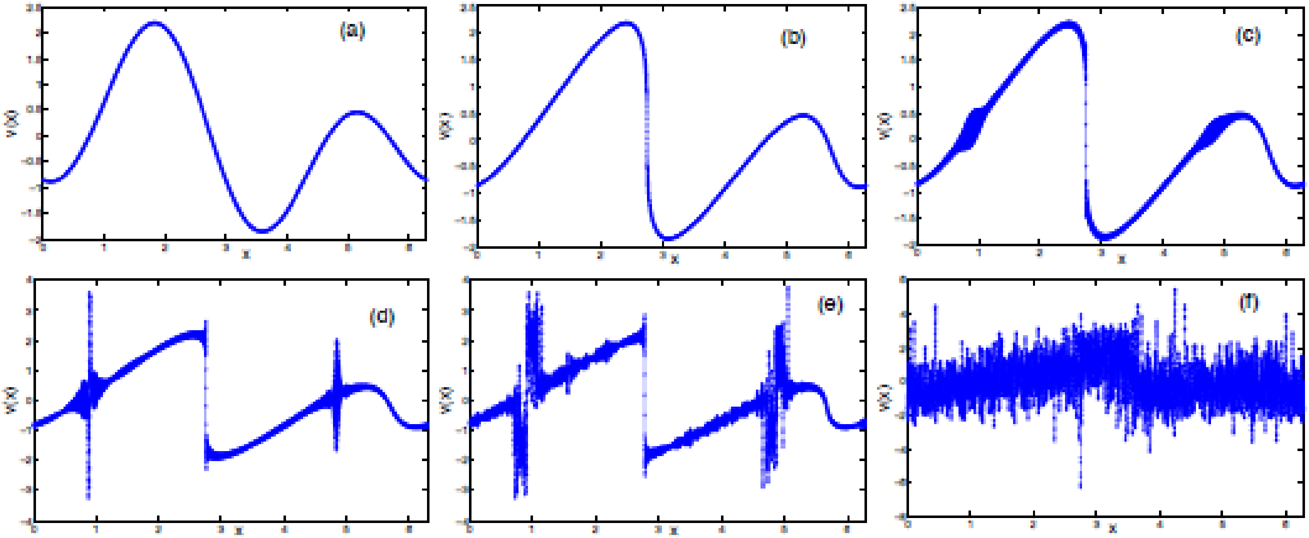
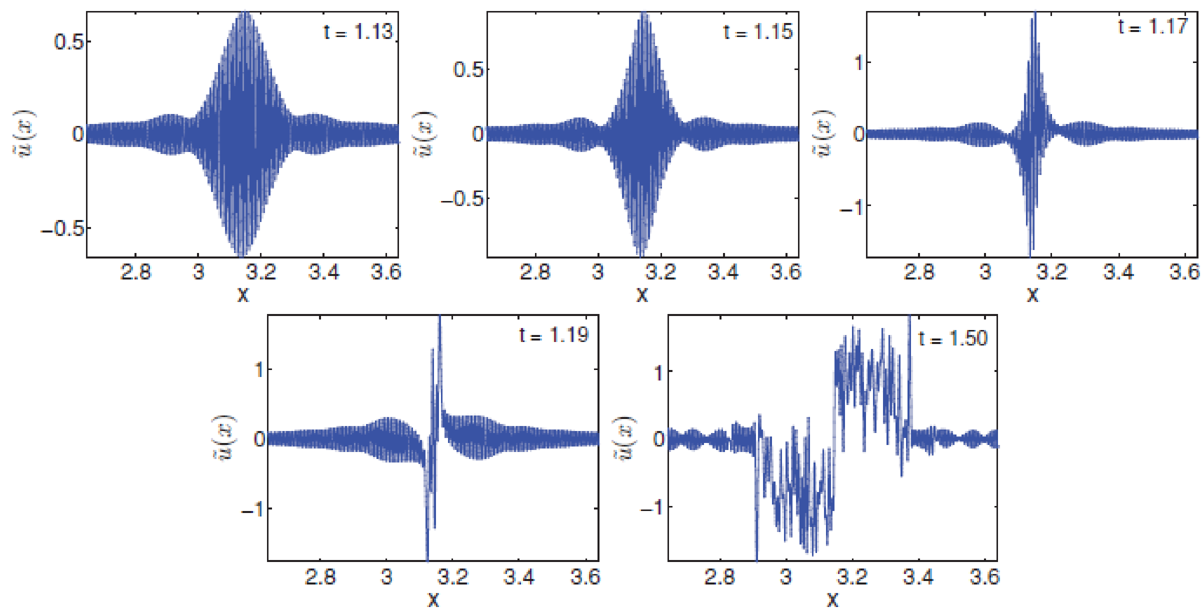
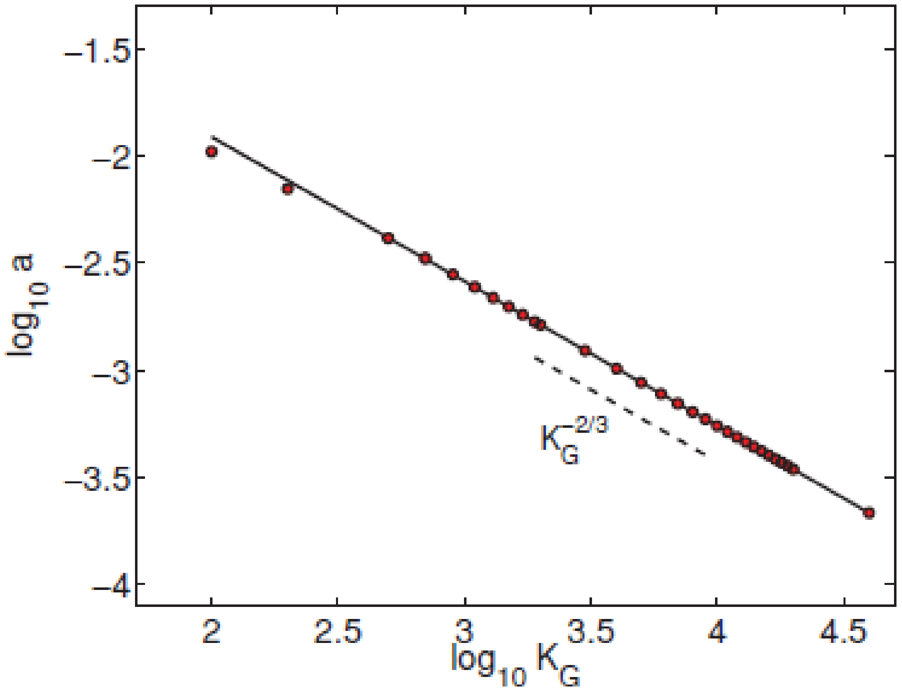
Key results
Burger's equation

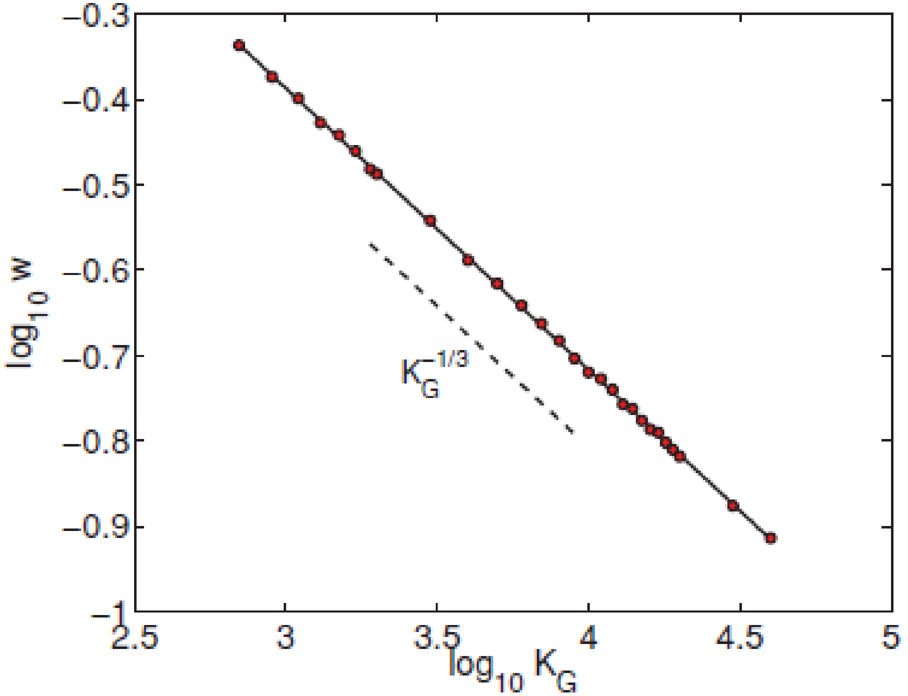




Key results

Burger’s equation (contd..)





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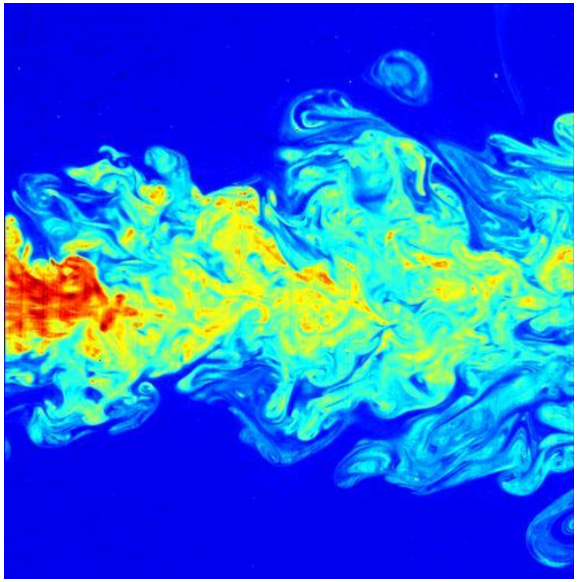
Summary and future extensions

- For the Euler equation, a finite-time blowup can exist only if the complex singularity hits the real axis ***sufficiently*** fast at the singularity time.

- Numerical results consistent with the possibility of a singularity.

- Higher-resolution studies needed to extend the time interval on which well-resolved power law behaviour happens.

Thank you for your attention !



Questions?