14TH DISCUSSION MEETING ON HARMONIC ANALYSIS ABSTRACTS

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
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ON THE DUAL OF THE DISCRETE HEISENBERG GROUP

Gerald B. Folland

HARISH-CHANDRA'S ROLE IN HARMONIC ANALYSIS ON LIE GROUPS

Alladi Sitaram

This is a historical survey.

FUNCTIONAL CALCULUS FOR RITT OPERATORS AND ANALYTIC SEMIGROUPS

Christian Lemerdy

HEISENBERG UNCERTAINTY INEQUALITY FOR GABOR TRANSFORM

Ashish Bansal

We discuss Heisenberg uncertainty inequality for groups of the form $K \ltimes \mathbb{R}^n$, K is a separable unimodular locally compact group of type I. We shall also discuss this inequality for Gabor transform for several classes of groups of the form $K \ltimes \mathbb{R}^n$.

SEMI-ORTHOGONAL PARSEVAL WAVELETS ASSOCIATED TO GMRAS ON LOCAL FIELDS OF POSITIVE CHARACTERISTICS

Niraj Kumar Shukla

In this article we establish theory of semi-orthogonal Parseval wavelets associated to generalized multiresolution analysis (GMRA) for the local field of positive characteristics (LFPC). By employing the properties of translation invariant spaces on the core space of GMRA we obtain a characterization of semi-orthogonal Parseval wavelets in terms of consistency equation for LFPC. As a consequence, we obtain a characterization of an orthonormal (multi)wavelet to be associated with an MRA in terms of multiplicity function as well as dimension function of a (multi)wavelet. Further, we provide characterizations of Parseval scaling functions, scaling sets and bandlimited wavelets together with a Shannon type multiwavelet for LFPC. This is a joint work with S.C. Maury and S. Mittal.

L^p–FOURIER TRANSFORM NORM FOR CERTAIN LIE GROUPS

Ali Baklouti

HILBERT SPACE VALUED GABOR FRAMES IN WEIGHTED AMALGAM SPACES

Anirudha Poria

In this talk, we prove that if the window function $\mathbf{g}$ is in the $\mathbb{H}$-valued weighted Wiener space $W_{\mathbb{H}}(L^\infty, L^1_w)$ then its canonical dual window $\boldsymbol{\gamma} = S_{\mathcal{G}}^{-1}\mathbf{g}$ is in the same space, where $\mathbb{H}$ be a separable complex Hilbert space and $S_{\mathcal{G}}$ is the $\mathbb{H}$-valued

Gabor frame operator. Also we show the continuity of the dual window $\boldsymbol{\gamma}$ if $\mathbf{g}$ is continuous. To prove that result we present the analogue of Walnut representation of the frame operator on $W_{\mathbb{H}}(L^p, L^q_v)$, $1 \leq p, q \leq \infty$ with generator in $W_{\mathbb{H}}(L^\infty, L^1_w)$ and make use of a resent version of Wiener's $1/f$ lemma.

FRAMES AND RIESZ BASES OF TWISTED SHIFT-INVARIANT SPACES

Saswata Adhikari

In this paper, twisted shift-invariant spaces in $L^2(\mathbb{R}^{2n})$ are introduced. Characterizations of orthonormal system and Bessel sequence of twisted translates in $L^2(\mathbb{R}^{2n})$ are obtained in terms of Weyl transform. These results appear to be totally different from the classical case of characterizations of orthonormal systems and Bessel sequences of translates in $L^2(\mathbb{R}^n)$. Further, characterizations of frames, Riesz basis in twisted shift-invariant spaces in $L^2(\mathbb{R}^{2n})$ are also obtained.

MULTIDIMENSIONAL FRAME AND DUAL FRAME WAVELET SYSTEMS

Anupam Gumber

The purpose of this research article is to study time-frequency localized functions, the sampling, approximation and reconstruction of such functions in the frame wavelet setting that can be applied not only in engineering branches such as signal and image processing but also in different areas of mathematics like harmonic analysis, approximation theory, partial differential equations, etc. Using the group theoretic approach based on the set of digits, we first investigate a finite collection of functions in a finite-dimensional Hilbert space that satisfy some localization properties in a region of the time-frequency plane, and then introduce a frame wavelet system in this setting with the help of invertible (expansive as well as non expansive) matrices. By expansive matrix, we mean all of its eigenvalues having a modulus larger than 1. Further, we provide characterizations for the frame wavelet system and it's dual having time-frequency localization properties in the multidimensional set up. As a consequence, characterizations for parseval frame wavelet system and orthonormal wavelet system are obtained. Finally, we discuss applications of spectral theorem to obtain some important results on eigenvalues and eigenvectors for the frame operator corresponding to the frame wavelet system. Along with the theory, we provide several examples to illustrate our results. This is a joint work with Niraj Kumar Shukla.

NUMERICAL RADII OF OPERATOR SPACES

Takashi Itoh

POTENTIAL OPERATORS ASSOCIATED WITH ORTHOGONAL EXPANSIONS

Krzysztof Stempak

In this survey talk, based on a series of joint papers with Adam Nowak, Riesz and Bessel potentials will be discussed in several settings of orthogonal expansions. These include continuous expansions (Hankel and modified Hankel transforms), as well as discrete Hermite, special Hermite and Laguerre expansions. Qualitatively sharp pointwise estimates of the corresponding potential kernels will be presented and applications to characterizations of $L_p - L_q$ estimates for potential operators will be overviewed.

HEISENBERG UNIQUENESS PAIRS FOR FOURIER TRANSFORM

Rajesh Srivastava

Let μ be a finite Borel measure which is supported on a curve Γ and absolutely continuous with respect to the arc length of Γ. Then for a given set Λ in $\mathbb{R}^2$, the pair (Γ, Λ) is called a Heisenberg uniqueness pair (HUP) for μ if its Fourier transform $\hat{\mu}$ vanishes on Λ, implies $\mu = 0$. This problem was first introduced by Hedenmalm and Montes-Rodríguez in 2011. They had shown that (hyperbola, some discrete set) is a HUP. There after, a considerable amount of work has been done on the HUP problem in the plane as well as in the Euclidean spaces. Eventually, this problem seems to be a variant of uncertainty principle for the Fourier transform.

HARMONIC AND POLYHARMONIC POLYNOMIALS ON THE HEISENBERG GROUP $\mathbb{H}_n$

Shivani Dubey

By iterating the Laplace operator, one can define so-called polyharmonic functions by operators Δ^m $(m \geq 2)$. The simplest polyharmonic functions are biharmonic functions which are defined as $\Delta^2 u$ $(= \Delta\Delta u) = 0$ in some domain. Historically, many investigations for the extension of harmonic functions are about biharmonic functions. The study of polyharmonic functions on $\mathbb{R}^n$ was first initiated by E. Almansi in 1899. The polynomial solution of Poisson equation and polyharmonic polynomials in $\mathbb{R}^n$ have been obtained by R. Z. Yeh. The classification of harmonic polynomials on the Heisenberg group was given by Korányi in 1982. In this article, we give an existence proof for a polynomial solution of the Poisson equation $L_0 u = q$ where q is a polynomial in the one dimensional Heisenberg Group. All the polynomial solutions of the polyharmonic equation $L_0^m u = 0$ in terms of harmonic polynomials are determined.

A CLASS OF BILINEAR MULTIPLIERS

Saurabh Shrivastava

In this talk we shall discuss the L^p-estimates for a class of bi-linear multiplier operators whose symbols are given by Littlewood-Paley decomposition.

A SHARP FORM OF MARCINKIEWICZ INTERPOLATION THEOREM FOR ORLICZ SPACES

Rama Rawat

Let $1 < p < q < \infty$. We give necessary and sufficient conditions for a weak type (p, p) and weak type (q, q) operator to be bounded from Orlicz space L_{ϕ_2} to Orlicz space L_{ϕ_1}.

AROUND A THEOREM OF PALEY AND WIENER.

Mithun Bhowmik

A Classical theorem due to Paley and Wiener says that if θ is a non-negative locally integrable function on $\mathbb{R}$ then there exists a non-zero $f \in L^2(\mathbb{R})$ vanishing for $x \geq x_0$ for some $x_0 \in \mathbb{R}$ such that $\hat{f}(y) = O(e^{-\theta(|y|)})$ for almost every y as $|y| \to \infty$ iff

$$\int_{\mathbb{R}} \frac{\theta(t)}{1+t^2} dt < \infty.$$

Variants of this result were later proved by Ingham, Levinson and Beurling. I shall talk about analogues of these results on Euclidean spaces and some non commutative groups.

HARDY'S INEQUALITY AND FRACTAL MEASURES

Senthil Raani KS

Extensions of the Plancherel theorem for measures supported on manifolds in $\mathbb{R}^n$ have been established by Agmon and Harmonder and for fractal measures by Strichartz. Hudson and Leckband proved Hardy-type inequality for fractal measures on $\mathbb{R}$. Inspired by these results, we discuss L^p asymptotics of Fourier transform of fractal measures on $\mathbb{R}^n$. In this talk, we concentrate on $1 \leq p \leq 2$ and discuss generalized Hardy-type inequality for fractal measures on $\mathbb{R}^n$ of dimension $0 \leq \alpha \leq n$ where α not necessarily an integer.

A GENERALIZED INTERPOLATION THEOREM OF MARCINKIEWICZ TYPE IN ORLICZ SPACES

Rajesh Kumar Singh

We establish a sharp extension of the Marcinkiewicz interpolation theorem in the framework of Orlicz spaces for more general class of operators. Namely, the class $\mathcal{W}(p, q, r; \mu, \nu)$ of quasilinear operators T such that

$$T : L_{p,r} \to L_{p,\infty}, \qquad \text{and} \qquad T : L_\infty \to L_\infty;$$

$$T : L_1 \to L_{1,\infty}, \qquad \text{and} \qquad T : L_{q,r} \to L_{q,\infty}.$$

Then we will see the necessary and sufficient conditions on Young's functions Φ_1, Φ_2 such that

$$T : L_{\Phi_2} \to L_{\Phi_1}.$$

ORE'S THEOREM FOR CYCLIC SUBFACTOR PLANAR ALGEBRAS AND APPLICATIONS

Sebastien Palcoux

We introduce the cyclic subfactors, generalizing the cyclic groups as the subfactors generalize the groups, and generalizing the natural numbers as the maximal subfactors generalize the prime numbers. On one hand, a theorem of O. Ore states that a finite group is cyclic if and only if its subgroups lattice is distributive, and on the other hand, every subgroup of a cyclic group is normal. Then, a subfactor planar algebra is called cyclic if all the biprojections are normal and form a distributive lattice. The main result shows in what sense a cyclic subfactor is singly generated, by generalizing one side of Ore's theorem as follows: if a subfactor planar algebra is cyclic then it is weakly cyclic (or w-cyclic), i.e. there is a minimal 2-box projection generating the identity biprojection. Some extensions of this result are discussed, and some applications of it are given in subfactors, quantum groups and finite group theories, for example, an upper bound for the minimal number of irreducible complex representations generating the left regular representation.

CHARACTERIZATION OF LOW-PASS FILTERS ON LOCAL FIELDS OF POSITIVE CHARACTERISTIC

Qaiser Jahan

In this talk, we will discuss about necessary and sufficient conditions on a function to be a low-pass filter on a local field K of positive characteristic associated to the scaling function for multiresolution analysis of $L^2(K)$. We use probability and martingale method to provide such a characterization.

SUPERIOR ORDER POINCARÉ-TYPE INEQUALITY RELATED TO THE JACOBI SEMIGROUP

Abdellatif BENTALEB

The aim objective of this note is to study the heat Jacobi semigroup generated by the operator $Lf(x) := (1-x^2)f'' + [(\beta-\alpha)-(\alpha+\beta+2)x]f', \quad \alpha, \beta > -1$ acting on the Hilbert space $\mathbb{L}^2([-1,+1],\mu)$ with respect to the normalized the Jacobi probability measure $\mu dx = c(1-x)^\alpha(a+x)*\beta dx$. We use some basic properties of the semigroups $\{exp\,(t\prod_{k=0}^{n}(L-k(k+\alpha+\beta+1)))\}_{t\geq 0}\,, n \geq 0$, to analyze a large family of geometric inequalities that does not exist in the literature and with which reinforced the (integral) Poincaré inequality.

COMPOSITION OPERATORS ON MODULATION SPACES AND APPLICATION TO SCHROEDINGER EQUATION

Ratnakumar P.K.

We discuss the nonlinear Schroedinger equation on modulation spaces $M_s^{p,1}(\mathbb{R}^n)$, with power type non linearity $F(u) = |u|^\alpha u, \alpha > 0$. A fundamental question in this context is the following: Does F maps, by composition, the modulation spaces to itself for all $\alpha > 0$? When α is an even integer this is known to be true, and relies on the algebra property of the modulations paces. But for other α, the problem was left open.

In this talk we, show that the answer to this question is negative. In fact, in the case of modulation space $M^{1,1}(\mathbb{R}^n)$, we also obtain a characterisation of non linearities F, that operate by composition. This work is to appear in JFA and is jointly with Divyang Bhimani.

MODULATION SPACES AND SHRÖDINGER EQUATIONS

Divyang Bhimani

For a complex function F on $\mathbb{C}$, we study the associated composition operator $T_F(f) := F(f)$ on modulation spaces $M^{p,q}(\mathbb{R}^d)$. We show, if T_F acts on $M^{p,1}(\mathbb{R}^d)$, then F is real analytic on $\mathbb{R}^2$; furthermore, the converse is true for $p = 1$. As an application, we point out that the standard method for the evolution of Shrödinger equations cannot be considered for nonlinearity of the form $u|u|^\alpha, \alpha \in (0,\infty) \setminus 2\mathbb{N}$. Finally, we will obtain some sufficient conditions for nonlinearity $fF(f)$ and $|f|$ to be in $M^{1,1}(\mathbb{R})$ whenever $f \in M^{1,1}(\mathbb{R})$ and F is a contraction on $\mathbb{C}$.

MATRIX-VALUED COMMUTING FAMILY OF DIFFERENTIAL OPERATORS ASSOCIATED WITH SYMMETRIC SPACES

Nobukazu Shimeno

I present commuting family of matrix-valued differential operators whose coefficients are given by elliptic functions.These operators are generalization of radial parts of invariant differential operators on certain homogeneous vector bundles over symmetric spaces.