Syllabus for Ph.D Programme

MATH16-01	Research Methodology
MATH16-02	Distribution Theory and Calculus on Banach Spaces
MATH16-03	Operator Theory and Function Spaces
MATH16-04	Geometric Function Theory
MATH16-05	Introduction to Operator Algebras
MATH16-06	Advanced Frame Theory
MATH16-07	Rings and Modules
MATH16-08	Group Rings
MATH16-09	Advanced Commutative Algebra
MATH16-10	Differential Manifolds
MATH16-11	Topological Structure
MATH16-12	Chaos Theory
MATH16-13	Ergodic Theory
MATH16-14	Singular Homology Theory
MATH16-15	Convex and Nonsmooth Analysis
MATH16-16	Multi-objective Optimization
MATH16-17	Parallel Iterative Methods for Partial Differential Equations
MATH16-18	Advanced Compressible Flow
MATH16-19	Elliptic Curves and Cryptography
MATH16-20	Lie Group and Lie Algebras
MATH16-21	Representation of Nilpotent Lie Group
MATH16-22	Univalent Functions
MATH16-23	Theory of Differential Subordination
MATH16-24	Harmonic Mappings in the Plane
MATH16-25	Operator Spaces
MATH16-26	Symmetries and Differential Equations
MATH16-27	Chaotic Dynamical Systems

MATH16-28 Minimal Ring Extensions and APD's

MATH16-29 Banach Spaces of Analytic Functions

MATH16-30 Banach Algebra Techniques in Operator Theory

MATH16-31 Conservation laws and Fluid Dynamics

MATH16-32 Methods in Fluid Dynamics

MATH16-33 Set-Valued Analysis

MATH16-34 Fixed Point Theorems in Non-Linear Analysis

MATH16-35 Applications of Fixed Point Theorems in Economics and Game Theory

MATH16-36 Introduction to Transformation Group

MATH16-37 Numerics of Partial Differential Equations

MATH16-38 Finite Difference Schemes for *K*-System Conservation Laws

MATH16-39 Computational Heat and Mass Transfer

MATH16-40 Uniform and Proximity structures on Topological Spaces

MATH16-41 Hyperspaces and Function spaces

MATH16-42 Introduction to Greedy Approximations

MATH16-01 : Research Methodology

Scientific research and literature survey. History of mathematics, finding and solving research problems, role of a supervisor, survey of a research topic, publishing a paper, reviewing a paper, research grant proposal writing, copyright issues, ethics and plagiarism.

Research tools. Searching google (query modifiers), MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, h-index, Google Scholar, ORCID, JStor, Online and open access journals, Virtual library of various countries

Scientific writing and presentation. Writing a research paper, survey article, thesis writing; LaTeX, PSTricks, Beamer, HTML and MathJaX

Software for Mathematics. Mathematica/Matlab/Scilab/GAP

Reference:

[1] J. Stillwell, Mathematics and its History, Springer International Edition, 4th Indian Reprint, 2005

[2] L. Lamport, LaTeX, a Document Preparation System, 2nd ed, Addison-Wesley, 1994.

[3] Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, Jean A. Dieudonne, How to Write Mathematics, American Mathematical Society, 1973.

[4] Nicholas J. Higham, Handbook of Writing for the Mathematical Sciences, Second Edition, SIAM, 1998.

[5] Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts, Mathematical Writing, Mathematical Association of America Washington, D.C., 1989.

[6] Frank Mittelbach, Michel Goossens, Johannes Braams, David Carlisle, Chris Rowley, The LaTeX Companion, 2nd edition (TTCT series), Addison-Wesley, 2004.

[7] Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel, Herbert Voss, The LaTeX Graphics Companion, 2nd edition (TTCT series), Addison-Wesley, 2004

[8] Mathtools documentation (http://mirrors.ctan.org/macros/latex/contrib/mathtools/mathtools.pdf)

[9] Pstricks documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

[10] MathJax documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

MATH16-02 : Distribution Theory and Calculus on Banach Spaces

Test functions and distributions, some operations with distributions, local properties of distributions, convolutions of distributions, tempered distributions and Fourier transform, fundamental solutions.

The Frechet derivative, chain rule and mean value theorems, implicit function theorem, extremum problems and Lagrange multipliers.

References.

[1] W. Cheney : Analysis for Applied Mathematics; Springer -Verlag, 2001.
[2] S. Kesavan : Topics in Functional Analysis and Applications; New Age International Publishers, 2008
[3] W. Rudin : Functional Analysis; Tata Mc-Graw Hill, 1991.
[4] Robert S. Strichartz : A guide to distribution theory and Fourier transforms; World Scientific Publishing Co., 2003.

MATH16-03 : Operator Theory and Function Spaces

Fredholm operators; semi-Fredholm operators; index of a Fredholm (semi- Fredholm) Operator; essential spectrum; Weyl spectrum and Weyl theorem; direct sums of operators, their spectra and numerical ranges; weighted shifts, their norms and spectral radii; normaloid, convexoid and spectraloid operators.

Invariant subspace problem; transitive, reductive and reflexive algebras; von-Neumann algebras.

Hardy spaces: Poisson's kernel; Fatou's theorem; zero sets of functions; multiplication, composition, Toeplitz and Hankel operators.

References.

[1] Vladimir V.Peller, Hankel operators and their applications, Springer, 2002.
[2] Nikolai L.Vasilevski, Commutative algebras of Toeplitz operators on Bergman space, Birkhauser, 2008.
[3] N.Young, An introduction to Hilbert space, Cambridge University Press, 1988.
[4] P.R.Halmos, A Hilbert space problem book, II Ed., D.VanNostrand Company, 1982.
[5] H.Radjavi and P.Rosenthal, Invariant subspaces, Springer Verlag, 1973.

MATH16-04 : Geometric Function Theory

Area theorem, growth, distortion theorems, coefficient estimates for univalent functions special classes of univalent functions. Lowner's theory and its applications; outline of de Banges proof of Bieberbach conjecture. Generalization of the area theorem, Grunsky inequalities, exponentiation of the Grunsky inequalities, Logarithmic coefficients. Subordination and Sharpened form of Schwarz Lemma

References.

[1] P. Duren, Univalent Functions, Springer, New York, 1983
[2] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983
[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht, Göttingen, 1975.
[4] M. Rosenblum, J. Rovnyak, **Topics** in Hardy Classes and Univalent Functions, Birkhauser Verlag, 1994
[5] D. J. Hallenbeck, T. H. MacGregor, Linear Problems and Convexity Techniques in Geometric Function Theory, Pitman Adv. Publ. Program, Boston-London-Melbourne,1984.
[6] I.Graham, G. Kohr, Geometric Function Theory in One and Higher Dimensions, Marcel Dekker, New York, 2003.

MATH16-04 : Introduction to Operator Algebras

Basic definitions and examples of Banach*-algebras, Spectrum of a Banach algebra element, L^1-algebras and Beurling algebras, Tensor products of Banach algebras, Multiplicative linear functional, The Gelfand representations, Fourier algebra, Functional calculus of in C*-algebras, Continuity and homomorphisms, Approximate identities in C*-algebras, Quotient algebras of C*-algebras, Representations and positive linear functional, Double Commutation Theorem, Enveloping von Neumann algebra of a C*-algebra, Tensor products of C*-algebras.

References.

[1] J.Diximier, C*-algebras, North-Holland Amersdem, 1977.
[2] R.V. Kadison and J.R.Ringrose, Fundamentals of the theory of operator algebras, Graduate studies in Mathematics, 15, AMS, Providence, 1997.
[3] E.Kaniuth, A course in commutative Banach algebras, Springer Verlag, 2008.
[4] M.Takesaki, Theory of Operator algebras, Springer Verlag, 2001.

MATH16-06 : Advanced Frame Theory

An overview on frames.B-Spline Symmetric B-Splines. Frames of translates. The canonical dual frame .Compactly supported generators.An application to sam-pling theory.Shift-Invariant Systems, Frame-properties of shift-invariant system, Representations of the frame operator. Gabor Frames in $L^2(R)$. Basic Gabor frame theory, Tight Gabor frames, The duals of a Gabor frame, Explicit construction of dual frame pairs, Popular Gabor conditions Representations of the Gabor frame operator and duality. Wavelet frames in $L^2(R)$.

References.

[1] O. Christensen, Frames and bases (An introductory course), Birkhauser, Boston (2008).
[2] I. Daubechies, Ten Lectures on wavelets, SIAM, Philadelphia (1992).
[3] R. Young, A introduction to non-harmonic Fourier series, Academic Press, New York (revised edition 2001).

MATH16-07: Rings and Modules

Essential and superfluous submodules, Decomposition of rings, Generating and cogenerating, Modules with composition series, Fitting Lemma, Indecomposable decompositions of modules, Projective modules and generators, Radicals of projective modules, Projective covers, Injective hulls, Cogenerators, Flat modules. Singular submodules, Localization and maximal quotient rings. Essential finite generation, Finite dimensionality, Uniform modules and Goldie rings. Regular rings, Strongly regular rings, Unit regular rings, Right π- regular rings. Baer rings,Rickart rings. Baer*rings, Rickart*rings.

References.

[1] A.F.Anderson and K.R.Fuller: Rings and categories of modules, Springer- Verlag,1991 (Relevant sections of Ch. 2,3,4,5).
[2] S.K.Berberian : Baer Vings,SpringerVerlag, New York ,1972 (Ch.1, sections 3, 4).
[3] K.R.Goodeari : Ring theory (Non singular rings and modules), Marcel Dekker,Inc. New York (Relevant sections of Ch. 1,2,3).
[4] K.R.Goodeari : Von Neumann regular rings,Pitman, London, 1979 (Ch. 1,3,4).
[5] T.Y.Lam: Lectures on Modules and rings, Springer Verlag, 1998(Ch. 3 ,section 7(d)).

MATH16-08: Group Rings

Twisted Group Rings, Tensor Products, Idempotents, Finite groups, Augmentation annihilators, Group algebra as injective modules, Linear identities. The Center, Finite conjugate groups, Chain conditions.

References.

[1] D. S. Passman The Algebraic structure of Group Rings, Dover Publications (Reprint edition). 2011.
[2] S. K. Sehgal, Topics in Group Rings, Marcel Dekker, New York, and Basel, 1978.
[3] I.B.S. Passi, Group Rings and their Augmentation Ideals Lecture Notes in Mathematics 715, Springer, New York, 1979.
[4] A. A. BOVDI, Group Rings Uzhgorod State University, 1978.
[5] D. S. Passman, Infinite Group Rings, Pure and Applied Math. 6, Marcel Dekkar, New York, 1971.
[6] P. Rihenboim, Rings and Modules, Interscience Tracts in Pure and Applied Mathematics, No.6, Interscience, New York, 1969.
[7] C.P. Milies and S.K. Sehgal, An Introduction to Group Rings, Kluwer Academic Publishers, Dordrecht, 2002.

MATH16-09: Advanced Commutative Algebra

Direct limit, Inverse limit, Graded rings and modules, Associated graded rings, I-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, UFD property of regular local rings, Hom functor, Tensor functor, I-torsion functor, Flat modules,

Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor, Minimal resolution, Regular sequences, Cohen-Macaulay rings and modules.

References:

[1] H. Matsumura, Commutative ring theory, Cambridge university press, 1989.

[2] Balwant Singh, Basic commutative algebra, World scientific publishing co., 2011.

[3] D. Eisenbud, Commutative algebra with a view towards algebraic geometry, Springer verlag, 1995.

[4] M.F. Atiyah & I.G. Macdonald, Introduction to commutative algebra, Addison Wesley, 1969.

MATH16-10 : Differential Manifolds

The derivative, continuously differentiable functions, the inverse function theorem, the implicit function theorem. Topological manifolds, partitions of unity, imbeddings and immersions, manifolds with boundary, submanifolds. Tangent vectors and differentials, Sard's theorem and regular values, vector fields and flows, tangent bundles, smooth maps and their differentials. Smooth manifolds, smooth manifolds with boundary, smooth sub-manifolds, construction of smooth functions.

References.

[1] G.E. Bredon, Topology and Geometry, Springer-verlag, 1993.

[2] L. Conlon, Differentiable Manifolds, Second Edition, Birkhauser,2003.

[3] A. Kosinski, Differential Manifolds, Academic Press, 1992.

[4] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. I; Publish or Perish, 1979.

MATH16-11 : Topological Structures

Dimension Theory: Definition and basic properties of the three dimension function inc. Inc. and dim, Characterization and subset theorems, equality of dim X and dim βX equality of Ind X and Ind βX.

Paracompactness: Paracompactness and full normality, presentation of paracompactness under mappings, Hanai-Moritastone theorem, products of paracompact spaces, countable paracompactness, strong paracompactness characterizations of strong paracompactness in regular spaces, products and subspaces of strongly paracompact spaces, pointwise paracompactness Arens Dugundji theorem, collectionwise normal spaces, Ding's example of a normal space which is not collectionwise normal.

Bitopological Spaces: Basic concepts, subspaces and products Separation and covering axioms.

References.

[1] R. Engelking: General Topology, Polish Scientific Publishers Warszawa, 2nd Ed., 1977.
[2] K. Nagami: DimensionTheory, Academic Press, New York, 1970.
[3] W.J. Pervin: Foundations of General Topology, Academic Press Inc., New York, 1964.
[4] S. Willard: General Topology, Addison-Wesley Publishing Co. Inc., 1970.

MATH16-12 : Chaos Theory

Topological transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for maps on I , Characterizing topological mixing in terms of topological transitivity for maps on I , Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on I .

Topological Entropy: Definitions, Entropy of interval maps, Horseshoes, Entropy of cycles, Continuity properties of the Entropy, Entropy of shift spaces, Entropy for circle maps, Various other definitions of Chaos and their interrelationships.

References.

[1] L. Alseda, J. Llibre, M. Misiurewicz, Combinatorial Dynamics and Entropy inDimension One, Advanced Series in Nonlinear Dynamics, 2000.

[2] L. S. Block and W. A. Coppel, Dynamics in One dimension, Springer, 1992.

[3] R. L. Devaney, A First Course in Chaotic Dynamical Systems, Westview Press, 1992.

[4] D. Hanselman and B. Littlefiels, Mastering MATLAB, Pearson Education,2005.

[5] Clark Robinson, Dynamical Systems, Stability, Symbolic Dynamics and Chaos, CRC press, 1999.

[6] S. Ruette, Chaos for continuous interval maps: A survey of relationship between various sorts of chaos, 2003.

[7] Introduction to Dynamical Systems, Brin and Stuck, Cambridge Univ. Press, 2002

MATH16-13 : Ergodic Theory

Measure preserving transformations and examples, Recurrence, Poincare's Recurrence theorem, Ergodicity, ergodicity of shift transformations

Ergodic theorems of Birkhoff and Von Neuman, Mixing, Weak-mixing and their characterizations, the isomorphism problem: conjugacy, Spectral equivalence, Transformations with discrete spectrum, Entropy, Kolmogorov-Sinai theorem, K-systems examples of calculation of entropy, Unique ergodicity, uniformly distributed sequences, applications to Diophantine approximation.

References.

[1] P. R. Halmos, Lectures on Ergodic Theory, American Mathematical Society, 2006

[2] M. G. Nadkarni, Basic Ergodic Theory, Birkhauser Verlag, 1998.

[3] Peter Walters, An Introduction to Ergodic Theory, Springer.

MATH16-14 : Singular Homology Theory

Singular complex and homology groups, functorial properties, relative homology groups, the Eilenberg-Steenrod axioms of homology theory.Long exact sequences. The reduced homology groups, the Mayer-Vietoris sequence. Homology of spheres. The degree of self maps of n-sphere, The Brouwer's fixed point theorem, Hairy Ball Theorem, Lusternik-Schnirelmann Theorem, Jordan-Brouwer Separation Theorem, Invariance of Domain.

References:

[1] E H Spanier, Algebraic Topology, Springer Verlag, 1989.

[2] G E Bredon, Topology and geometry, Springer Verlag, 2005.

[3] A Dold, Lectures on Algebraic Topology, Springer-Verlag, Second Edition 1980.

[4] J JRotman, An Introduction to Algebraic Topology, Springer Verlag, 1988.

[5] M.J. Greenberg and J.R. Harper, Algebraic Topology- A first course, Addison-Wesley Publishing Company, Inc. 1981.

[6] W S Massey, A Basic Course in Algebraic topology, Springer- Verlag, 1991.

MATH16-15 : Convex and Nonsmooth Analysis

Convex sets, Convexity-preserving operations for a set, Relative Interior, Asymptotic cone, Separation theorems, Farkas Lemma, Conical approximations of convex sets, Bouligand tangent and normal cones. Convex functions of several variables, Affine functions, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties, Sublinear functions, Support function, Norms and their duals, Polarity. Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem, Calculus rules with subdifferentials, Subdifferential as a multifunction, monotonicity and continuity properties of the subdifferential, Subdifferential and limits of gradients.

References.

[1] Convex, Analysis and Minimization Algorithms I, Jean-Baptiste Hiriart-Urruty and Claude Lemarechal, Springer- Verlag, Berlin, 1996.
[2] Convex Analysis and Nonlinear Optimization : Theory and Examples, Jonathan M. Borwein Adrian and S. Lewis, CMS Books in Mathematics, Springer Verlag, New York, 2006.
[3] Convex Analysis, R. Tyrrell Rockafellar, Priceton University Press, Princeton, New Jersey, 1997.

MATH16-16 : Multi-objective Optimization

Multiple Objective Linear Programming Problem, Multiple Criteria Examples, Utility Functions,Non Dominated Criteria Vectors and Efficient Points, Point Estimate Weighted Sums Approach, Optimal Weighting Vectors, Scaling and Reduced Feasible Region Methods, Vector Maximum Algorithm. Formulation of the Multiple Objective Model, Method of Solutions, Augmented Goal Programming, Interactive Multiple Objective Methods. Multiple Objective Linear Fractional Programming. Multiple Objective Non linear Programming Problem, Efficiency and Non- Dominance, Weakly and Strictly Efficient Solutions, Proper Efficiency and Proper Non- Dominance. Weighted Sum Scalarization : (Weak) Efficiency, Proper Efficiency, Optimality Conditions. Scalarization Techniques: The €-Constraint Method, The Hybrid Method, The Elastic Constraint Method and Benson's Method.

References:

[1] Ralph E.Steuer : Multi Criteria Optimization: Theory, Computation, and Application, John Wiley and Sons, 1986. Chapters-1, 6, 7, 8, 9, 12.
[2] James P. Ignizio : Linear Programming in Single and Multiple Objective Systems, Prentice-Hall Inc. , Englewood Cliffs, N.J 1981. Chapters- 16, 17, 20.
[3] Matthias Ehrgott: Multicriteria Optimization, Springer, Berlin, Heidelberg, 2005, Second Edition, Chapters- 2, 3,4.

MATH16-17 : Parallel Iterative methods for Partial Differential Equations

Speedup; efficiency; Amdahl's law; point and block parallel relaxation algorithms (Jacobi, Gauss-Seidel, SOR); triangular matrix decomposition; quadrant interlocking factorisation method; red-black ordering; application to elliptic BVPs; parallel ADI algorithms; parallel conjugate-gradient method; parallel multi-grid method; parallel domain decomposition method.

The alternating group explicit method for two point BVPs (natural, derivative, mixed, periodic) and their convergence analysis; the MAGE and NAGE methods; the computational complexity of the AGE method; the Newton-AGE method.

Parabolic equation: AGE algorithm for diffusion-convection equation and its convergence analysis; stability analysis of more general scheme; CAGE method; AGE method for fourth order parabolic equation.

Hyperbolic equation: Group explicit method for first and second order hyperbolic equations; GER, GEL, GAGE, GEU, GEC algorithms; stability analysis of GE method; AGE iterative method for first and second order hyperbolic equations.

Elliptic equation: Douglas-Rachford algorithm; BLAGE iterative algorithm with different boundary conditions; AGE-DG algorithm; parallel implementation.

This course consists of theory paper and computer practical.

References.

[1] Y. Saad, Iterative Methods for Sparse Linear Systems, SIAM, Philadelphia (2003).
[2] L.A. Hageman and D.M. Young, Applied Iterative Methods, Dover publication, New York (2004).
[3] D.M. Young, Iterative Solution of Large Linear Systems, Academic Press, New York (1971).
[4] Jianping Zhu, Solving Partial Differential Equations on Parallel Computers, World Scientific, New Jersey (1994).
[5] D.J. Evans, Group Explicit Methods for the Numerical Solution of Partial Differential Equations, Gordon and Breach Science publisher, Amsterdam (1997).

MATH16-18 : Advanced Compressible Flows

One-dimensional gas flow (with perfect and van der Waals gas, gravitation, viscosity, heat addition, and conduction), Diffusion, Shock waves (discontinuity surface, jump condition, strength, thickness, reflection, structure, heat addition and MHD effects), Detonation and Deflagration waves, Methods of solution of compressible flow problems. Dimensional analysis and similarity method, Self-similar motion of spherical, cylindrical and plane waves in a gas. Two dimensional subsonic and supersonic flow with linearized theory, Two dimensional subsonic potential flows(Rayleigh-Janzen Method), Two dimensional supersonic flow with method of characteristics. Anisentropic rotational flow of inviscid compressible fluid.

References.

1. Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
2. Fluid Mechanics, L.D.Landau and E.M.Lifshitz, Pragamon Press, 1989
3. Introduction to the theory of compressible flow, S.I.Pai, D. Van Nostrand Company, 1958
4. Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Ya. B. Zel'dovich, and Yu.P. Raizer, Academic Press, 1966.
5. Introductory Fluid Mechanics, J.Katz , Cambridge University Press, 2010

MATH16-19 : Elliptic Curves and Cryptography

Finite field arithmetic, Geometry and arithmetic of elliptic curves, torsion points, Elliptic curves over finite fields,Determination of number of points on elliptic curves,Discrete Logarithm Problem, Elliptic curve cryptography - including key agreement and key trans- port, Other applications such as factoring, primality testing, Elliptic curves over Q, Elliptic curves over C, complex multiplication, Divisors, Isogenies, Pairings and cryptography from pairings.

References:

[1] Lawrence C. Washington, Elliptic Curves, Number Theory and Cryptography, CRC Press, 2008.

[2] Darrel Hankerson, Alfred Menezes, Scott Vanstone, Guide to Elliptic curve Cryptog- raphy, Springer, 2004.

[3] Ian F. blake, Gadiel Seroussi, Nigel p. Smart, Advances in Elliptic curve cryptography, London University Press, 2005.

MATH16-20 : Lie Groups and Lie Algebras

Unit -I : Differential Manifolds Topological manifolds, Charts, Atlases and smooth structure, Smooth maps and diffeomorphism, Partitions of Unity, Tangent space, Tangent map, Vector fields and 1-forms.

Unit -II : Lie Groups Definition and examples, Linear Lie groups, Lie group homomorphism, Lie algebra and the exponential map, Adjoint representation, Homogeneous spaces, Baker-Campbell-Housdorff formula.

Unit -III : Lie Algebras Definition and examples, Classical Lie algebras, Solvable and nilpotent Lie algebras, Lie and Engel theorems, Semisimple and reductive algebras, Semisimplicity of Classical Lie algebras, Killing form and Cartan criterion, Cartansubalgebra, root decomposition and root systems, Weyl group and Weyl chambers, Dynkin diagrams, Classsification of simple Lie algebras.

Unit -IV : Partial Differential Equations on Manifolds Partial differential operators and formal adjoints, Sobolev spaces in R^n, Elliptic estimates in R^n , Elliptic regularity, Fredholm theory and spectral theory of Laplacian.

Suggested Texts:

1. S. Kumaresan, Differential Geometry and Lie Groups, Hindustan Book Agency.

2. Alexander KirillovJr, An Introduction to Lie Groups and Lie Algebras, Cambridge University Press.

3. James Humphreys, Introduction to Lie Algebras and Representation Theory, Springer.

4. Brian Hall, Lie Groups, Lie Lagebras, and Representations: An Elementary Introduction, Second Edition, Springer.

5. J. M. Lee, Manifolds and Differential Geometry, Graduate Studies in Mathematics vol 107, AMS.

6. Liviu I Nicolaescu, Lectures on Geometry of Manifolds, Second Edition, World Scientific.

7. A. C. Pipkin, A Course on Integral Equations, Text in Applied Mathematics Series, Springer.

MATH16-21 : Representation of Nilpotent Lie Group

Basic facts about Lie groups and Lie algebras, Nilpotent Lie groups, Coadjoint orbits and the dual of g, Some generalities on representations, Elements of kirillov theory, Proof of basic theorems, subgroups of condimension 1 and representations.

References

1. L.J. Corwin and F.D. Grenleaf, Representations of nilpotent Lie groups and their applications, Cambridge University Press, 1990
2. V.S. Varadarajan, Lie groups, Lie algebras and their representations, Prentice-Hall, 1974.
3.

MATH16-22 : Univalent Functions

Univalent functions, area theorems, Bieberbach theorem and itsapplications, subclasses of starlike and convex functions and theirgeneralizations, functions with positive real part, typically realfunctions.

Close-to-convex functions and the functions of bounded boundaryrotation, bounded functions, radius problems and Koebe domains,combination and convolutions of univalent functions, Integrals andintegral inequalities, meromorphic functions.

References.

[1]] A. W. Goodman, Univalent Functions I & II, Mariner, Florida, 1983.

[2] P. Duren, Univalent Functions, Springer, New York, 1983

[3] Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht,Göttingen, 1975.

MATH16-23 : Theory of Differential Subordination

Jack-Miller-Mocanu Lemma, Admissible functions and fundamentaltheorems, open door lemma and integral existence theorem, first orderdifferential subordination, Briot-Bouquet differential subordinations,and its generalizations and applications, integral operator, subordination preserving integral operators .

Second order differential subordinations, integral operatorspreserving functions with positive real part, bounded functions,averaging operators, Hypergemetric functions, Schwarzian derivative,applications to starlikeness and convexity.

References

[1] S. S. Miller and P. T. Mocanu, Differential Subordinations. Theoryand. Applications, Marcel Dekker Inc., New York, Basel, 2000.

[2] T. Bulboac˘a, Differential Subordinations and Superordination:Recent Results, Cluj-Napoca,2005.

MATH16-24 : Harmonic Mappings in the Plane

Harmonic mappings, Argument principle, Dirichlet problem, criticalpoints of harmonic mappings, Lewy's theorem, Heinz's theorem, Rado'stheorem

The Rado-Kneser-Choquet theorem, Shear construction, structure ofconvex mappings, covering theorems and coefficient bounds

Harmonic self mappings of the disk, normalization and normality ofharmonic univalent functions, Harmonic Koebe functions andcoefficient conjectures, extremal problems, typically real andstarlike functions,

problems and conjectures in planar harmonicmappings.

Text:

[1] P.L. Duren, Harmonic Mappings in the Plane, Cambridge Univ.Press, Cambridge, 2004.

[2] D. Bshouty and A. Lyzzaik, Problems and Conjectures in PlanarHarmonic Mappings, J. Analysis, Volume 18 (2010), 69–81.

MATH16-25 : Operator Spaces

Operator Spaces (concrete and abstract), Completely bounded maps, subspaces, quotients, products, Dual spaces, conjugates, mapping spaces, opposite, representation theorem, The min and max quantizations, Arveson-Wittstock theorem, Column and Row Hilbert spaces. Projective tensor product, injective tensor product and Haagerup tensor product.

References

1. Blecher, D. P. and Merdy, C. Le., Operator algebras and their modules-an operator space approach. London Mathematical Society Monographs, New series, vol. 30, The Clarendon Press, Oxford University Press, Oxford, 2004.
2. Effros, E. G. and Ruan, Z. J., Operator spaces, Claredon Press-Oxford, 2000.
3. Pisier, G., Introduction to operator space theory, Cambridge University Press, 2003.

MATH16-26 : Symmetries and Differential Equations

Lie Groups of Transformations: Groups, Groups of Transformations, One-Parameter Lie Group of Transformations, Examples of One-Parameter Lie Groups of Transformations.

Infinitesimal Transformations: First Fundamental Theorem of Lie, Infinitesimal Generators, Invariant Functions, Canonical Coordinates, Examples of Sets of Canonical Coordinates, Invariant Surfaces, Invariant Curves, and Invariant Points.

Extended Transformations (Prolongations): Extended Group Transformations-One, Dependent and One Independent Variable, Extended Infinitesimal Transformations-One Dependent and One Independent Variable, Extended Transformations-One Dependent and n Independent Variables, Extended Infinitesimal Transformations-One, Dependent and n Independent Variables, Extended Transformations and Extended Infinitesimal Transformations-m Dependent and n Independent Variables.

Ordinary Differential Equations: Invariance of an Ordinary Differential Equation, First Order ODE's, Determining Equation for Infinitesimal Transformations of a First Order ODE, Determination of First Order ODE's Invariant Under a Given Group, Second and Higher Order ODE's, Reduction of Order by Differential

Invariants, Examples of Reduction of Order, Determining Equations for Infinitesimal Transformations of an nth Order ODE, Determination of nth Order ODE's Invariant Under a Given Group, Applications to Boundary Value Problems for ODE's.

Partial Differential Equations: Invariance of a Partial Differential Equation, Invariant Solutions, Mapping of Solutions to Other Solutions from Group Invariance of a PDE, Determining Equations for Infinitesimal Transformations of a k^{th} Order PDE, Invariance for Systems of PDE's, Determining Equations for Infinitesimal Transformations of a System of PDE's, Applications to Boundary Value Problems for PDE's.

References:

(1). George W. Bluman, J. D. Cole, Similarity methods for differential equations, Springer New York (Verlag), 1974.
(2). George W. Bluman, SukeyukiKumei, Symmetries and Differential Equations, Springer New York, 1989.

MATH16-27 : Chaotic Dynamical Systems

Theory and Application of Chaos in Dynamical systems, One dimensional map, Examples of Dynamical Systems, Stability of fixed points, Orbits, Graphical Analysis, Fixed and Periodic points, Quadratic family, Transition to Chaos.

Bifurcations of Chaotic Systems, Dynamics of a quadratic map, Saddle node Bifurcation, Period Doubling Bifurcation, Transcritical Bifurcation, Pitchfork Bifurcation.

Lyapunov Exponents, chaotic orbits, conjugacy and logistic map, Transition graphs and fixed points, Basin of attraction. Lorenz equations, strange attractors, Lorenz map, Simple properties of Lorenz equations, Chaos in Hamiltonian Systems, and Control and Synchronism of chaos.

Equilibria in Nonlinear Systems, Nonlinear Sinks and Sources, Saddles, Stability, Closed orbit and Limit Sets, Poincare map, Applications in physics, engineering and biology.

References:

(1). R. L. Deveny, A First Course in Chaotic Dynamical Systems, 2nd Edition, Westview Press.
(2). Steven H. Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Westview Press.
(3). Kathleen T. Alligood, Tim D. Sauer, James A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer-Verlag, New York.
(4). L. Douglas Kiel and Euel W. Elliott, Chaos Theory in the Social Sciences, University of Michigan Press.
(5). M.W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and an Introduction to Chaos, 3rd Edition, Academic Press, USA.

MATH16-28 : Minimal Ring Extensions and APD's

Contents: Introduction to Minimal Ring extensions, Minimal Ring Homomorphisms, Overring, Finitely many intermediate rings property and related results, Finitely many subrings property and related results, Composites, Kaplansky Transform, Direct Products and ƛ-extensions, μ-extensions, P-extensions, i-domains. Results on minimal field extension.

Commutative Perfect Rings and almost Perfect Rings, Properties of Almost perfect domains, Valuation overrings of APDs, connection of APDs with other classes of domains, examples of APDs.

References:

1. Commutative Rings: New Research, John Lee, Nova Science Publication Inc., New York.
2. Multiplicative Ideal Theory in Commutative Algebra, J. W. Brewer, S. Glaz, W. J. Heinzer, B. M. Olberding, Springer, 2006
3. Commutative ring theory, H. Matsumura, Cambridge university press, 1989.
4. Basic commutative algebra, Balwant Singh, World scientific publishing co., 2011.

MATH16-29 : Banach Spaces of Analytic Functions

Analytic and Harmonic Functions in the unit disc: Cauchy and Poisson kernels, boundary values, Fatou's Theorem, H^pspaces.
The space H^1: The HelsonLowdenslager Approach, Szego's theorem, Dirichlet Algebras.
Factorization of H^pFunctions: Inner and outer functions, Blaschke products and singular functions, Factorisation theorem. The Shift operator : The shift operator on H^2.Invariant subspaces for H^2 on the half plane, the shift on L^2 the vector valued case, representations on $H\infty$.

Text book: K. Hoffman, Banach Spaces of Analytic Functions, Dover Publications, 2007.

MATH16-30 : Banach Algebra Techniques in Operator Theory

Revision of Banach spaces and Geometry of Hilbert Space.Basic theory of Banach Algebras, The Disk Algebra.Multiplication operators and maximal abelian algebras. The Bilateral shift operator. C* algebras.The GelfandNaimark Theorem.Spectral Theorem.Functional Calculus.The Unilateral Shift Operator.Topelitz operators.The Spectrum of self-adjoint and analytic Toeplitz Operators.

R.G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics 179, Springer, 1998

MATH16-31 : Conservation laws in Fluid Dynamics

Hyperbolic system of conservation laws, breakdown of smooth solution, genuine nonlinearity, weak solutions and jump condition, Riemann problem, entropy conditions, Convection, diffusion and heat transfer, two-phase flow, boundary layer flow, Free and Moving boundary problems.

Reference

1. Hyperbolic system of conservation laws and mathematical theory of shock waves, Peter D. Lax, SIAM, 1973
2. Quasilinear Hyperbolic Systems, Compressible Flows and Waves, V.D.Sharma, Chapman and Hall/ CRC, 2010
3. Free and Moving Boundary Problems, J. Crank, Oxford university press, New York, 1984
4. Boundary Layer Theory, H.Schlichting, K. Gersten, Springer, 2000
5. Thermo-Fluid Dynamics of Two-Phase Flow, M. Ishii, T.Hibiki, Springer, 2011

MATH16-32 : Methods in Fluid Dynamics

Characteristics methods, Similarity methods, Self-similar solution and the method of Lie-group invariance, Perturbation methods, Homotopy perturbation methods, Homotopy analysis method, Adomian decomposition method, Variational method, Numerical method.

References

6. Similarity and Dimensional Method in Mechanics, L.I.Sedov, Mir Publisher, 1982
7. Symmetries and Differential Equation, G.W. Bluman and S. Kumei, Springer, 1989
8. Beyond Perturbation: Introduction to the Homotopy Analysis Method, S. Liao, Chapman and Hall/ CRC, 2004
9. Partial Differential Equation and Soliton Wave Theory, Abdul-MajidWaswas, Springer, 2009
10. Numerical Approximation of Hyperbolic System of Conservation Laws, E. Godlweski, P.A. Raviart, Springer, 1996
11. Fundamental of finite element method in heat and fluid flow, R.W.Lewis, P. Nithiarasu, K.N. Seetharamu, John-Wiley and Sons, 2004

MATH16-33 : Set-Valued Analysis

Order relations, Cone properties related to the topology and the order, Convexity notions for sets and set-valued maps, Solution concepts in vector optimization, Vector optimization problems with variable

ordering structure, Solution concepts in set-valued optimization, Solution concepts based on vector approach, Solution concepts based on set approach, Solution concepts based on lattice structure, The embedding approach by Kuroiwa, Solution concepts with respect to abstract preference relations, Set-valued optimization problems with variable ordering structure, Approximate solutions of set-valued optimization problems, Relationships between solution concepts

Continuity notions for set-valued maps, Continuity properties of set-valued maps under convexity assumptions, Lipschitzproperties for single-valued and set-valued maps, Clarke's normal cone and subdifferential, Limiting cones and generalized differentiability, Approximate cones and generalized differentiability

References

1. Akhtar A. Khan, Christiane Tammer, Constantin Zălinescu, **Set-Valued Optimization: An Introduction with Applications**, Springer Verlag, 2015.
2. Regina S. Burachik and Alfredo N. Iusem, **Set-Valued Mappings and Enlargements of Monotone Operators**, Springer Verlag, 2008.
3. Guang-ya Chen, XuexiangHuang and Xiaogi Yang, **Vector Optimization: Set-valued and Variational Analysis**, Springer Verlag, 2005.

MATH16-34 : Fixed Point Theorems in Non-Linear Analysis

Contractions, Banach Contraction Principle, Theorem of Edelstein, Picard–Lindelof Theorem.Non expansive Maps, Schauder'sTheorem for non–expansive maps, Continuation Methods for Contractive and non–expansive mappings.Some Applications of The Banach Contraction Principle, Some Extensions of Banach Contraction Principle for Single – Valued Mappings, Generalized distances, Some Extensions of Banach Contraction Principle under Generalized Distances, Multivalued versions of Banach Contraction Principle.

References :

[1] S. Almezel, Q. H. Ansari and M. A. Khamsi; Topics in Fixed Point Theory, Springer 2014.

[2] R. P. Agarwal, M. Meehan, D. O' Regan; Fixed Point Theory And Applications, Cambridge University Press 2004.

MATH16-35 : Applications of Fixed Point Theorems in Economics and Game Theory

Sperner's Lemma, TheKnaster – Kuratowski –Mazurkiewicz Lemma, Brouwer's Fixed Point Theorem, The Fan – Browder Theorem, Kakutani's Theorem. The maximum Theorem, Set with convex sections and a minimax Theorem, Variational inequalities, Price equilibrium and complementarity, Equilibrium of excess demand correspondences, Nash equilibrium of games and abstract economics, Walrasian equilibrium of an economy.

Reference :

[1] K.C.Border; Fixed Point Theorems with Applications to Economics and Game Theory; Cambridge University Press 1985.

MATH16-36 : Introduction to Transformation Groups

Definition and fundamental properties oftopological Groups, Examples oftopological groups, subgroups,Isotropygroups,Isomorphism,Semi-direct products and Direct products, the Classical groups, Characteristic functions on compact groups.

Transformation groups and its fundamental properties, Examples of transformation groups, Group actions, Fixed point sets, Orbits and orbit spaces. Homogeneous spaces and equivariant maps, Induced transformation groups.

REFERENCES:

1. P.J. Hissins, Introduction to Topological Groups, L M S (Lecture Notes Series), Cambridge University Press, 1975.
2. M. L. Curtis, Matrix Groups, Springer, 1984.
3. G. E. Bredon, Introduction to Compact transformation Groups, Academic Press, 1972.
4. T. B. Singh, Elements of Topology, CRC Press (Taylor and Francis Group), 2013.

MATH16-37 : Numerics of Partial Differential Equations

Finite Difference Methods for Parabolic, Hyperbolic and Elliptic PDEs of two and Three dimensions and their Consistency, Stability and Convergence, Dispersion and Dissipation analysis of PDEs and its Finite Difference Schemes, Artificial Dissipation, More Dissipation, Discontinuous Solutions, Finite Difference Schemes for the systems of Parabolic and Hyperbolic PDEs, Courant FriedrichsLewy condition for systems. Analysis of well-posed initial value problem of Parabolic, Hyperbolic systems, Kreiss matrix Theorem, Convergence estimates for Parabolic and Hyperbolic PDEs.

References

[1] John C. Strikwerda, *Finite Difference Schemes and Partial Differential equations*, SIAM, Philadelphia (2004).

[2] J.W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer-Verlag New York (1995).

[3]Quarteroni, A and Valli, A. *Numerical Approximation of Partial Differential Equations*, Springer, (1997).

[4]Ueberrhuber, C. W. , *Numerical Computation: Methods, Software and Analysis*, Springer, (1997).

[5]Axelsson, O. *Iterative Solution Methods,* Cambridge University Press, (1994).

MATH16-38 : Finite Difference Schemes for *K*-System Conservation Laws

Theory of Scalar and *K* - System of Conservation Laws, Finite Difference Schemes for Conservation Laws. Difference Schemes for Scalar Conservation Laws: Godunov Scheme, TVD Scheme, Flux-Limiter Methods, Slope-Limiter Methods, Modified Flux Method. Finite Difference Schemes for *K*-System Conservation Laws, High Resolution Schemes for Linear *K*-System conservation Laws, Flux-Limiter Schemes for Linear *K*-System Conservation Laws, Slope Limiter Schemes for Linear *K*-System Conservation Laws, A Modified Flux Scheme for Linear *K*-System Conservation Laws, Approximate Reimann solvers, Difference Schemes for two dimensional Conservation Laws.

References

[1] J.W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer-Verlag New York (1995).

[2] Kroner, D, *Numerical Schemes for Conservation Laws*, John Wiley (1997).

[3]LeVeque, R.J, *Numerical Methods for Conservation Laws*, Birkhauser (1992).

[4]LeVeque, R. J, *Finite Volume methods for Hyperbolic Problems*, Cam-

bridge University Press (2002).

[5] Godlewski and Raviart, P, *Numerical Approximation of Hyperbolic Systems of Conservation Laws*, Springer(1995).

[6] John C. Strikwerda, *Finite Difference Schemes and Partial Differential equations*, SIAM, Philadelphia (2004).

MATH16-39 : Computational Heat and Mass Transfer

Introduction and basics of heat and mass transfer, Modes of heat transfer, Fourier's law, Conductivity, Diffusivity, Analogy between heat and mass transfer, Mass diffusion, Fick's Law, Transient mass diffusion, Steady and transient heat conduction, 1-D and 2-D Heat conduction, General heat conduction equation, Boundary and initial conditions, Heat generation, Introduction to convection: Fundamentals, Velocity and thermal boundary layer, Laminar, Turbulent flows, Conservation equations for mass, momentum and energy, Solution of boundary layer equations, Analogy between heat and momentum transfer, Non-dimensional numbers, Numerical methods: Solution of heat and mass transfer equations using finite difference and finite volume methods, Different explicit and implicit methods of finite differences, Different finite volume schemes for steady and transient Convection-Diffusion equations, Methods for solving finite difference and finite volume discretization equations, Consistency, stability and convergence of finite difference methods.

References

[1]F.P.Incropera and D.P.Dewitt, *Fundamental of Heat and Mass Transfer*, Wiley, USA, 1990.

[2]Yunus A. Cengel, *Heat Transfer*, McGra-Hill, NewYork.USA, 2004.

[3] J.P.Holmann, *Heat Transfer*, McGra-Hill, NewYork. USA, 2009.

[4] S.V. Patanker, *Numerical Heat Transfer and Fluid Flow*, Taylor and Francis, Hemisphere Pub.Comp., USA, 2004.

[5]H.K.Versteeg and W. Malalasekhera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Pearson, 2007

MATH16-40 : Uniform and Proximity structures on Topological Spaces

Uniform Spaces [1]: Denition of Uniform spaces, Discrete uniformity, Trivial uniformity, Base for a uniformity, Separated Uniformities, Totally bounded Uniformities, Uniform Continuities, Product uniformities, Induced Uniformities. **15 L**

Uniform Topology [1]: Uniform neighbourhood, Closure in Uniform spaces, Uniformization of Compact Hausdor spaces, Cauchy Sequence, Cauchy Filters. **5L**

Proximity Space [3]: Kuratowski closure axioms, Cech Closure axioms, Definition of proximity, Basic proximity, L-proximity, separated proximity, weakly regular spaces, compatible topology,s-axioms, Fine L-proximity, subspace proximity, Examples of proximity, Functionally separating proximity, Uniformity induced-proximity, proximal continuity, Clusters, Clans and Bunches, Wallman topology, Bases and subbases for a proximity. **20L**

References

[1] I.M.James,Topological and uniform spaces,Springer-Verlag, New York Berlin Heidelberg,(1987).

[2] SomashekharNaimpally, Proximity Approach to Problems in Topology and Analysis, OldenbourgVerlag, Munich,Germany, (2009).

Suggested Readings

[1] John L. Kelly, General Topology, Springer-Verlag, New York Berlin Heideberg (1957).

[2] Stephen Willard, General Topology, Dover Publication, Inc, Mineola, New York (1970).

MATH16-41 : Hyperspaces and Function spaces

Hyperspace Topologies :

Topology for Hyperspaces [1]: The general notion of a Hyperspace, Vietoris topology for CL(X), Base for a Hyperspace topology, Topological Invariance, Specified Hyperspaces, The Hausdorff Metric. **10 L**

Hyperspace Topologies [2]: Hit-and-miss topology, Far set, upper-far topology, Vietoris and Proximal topologies, Fell topology, Hausdorff Metric topology, Wijsman topologies, Lower proximal locally finite topology,Locallyfnite topology, Poppe'sΔ-topologies, Uniformly discrete Hypertopology, Bounded topologies. **10 L**

Hyperspace Topologies on Function Spaces [2]: Uniform convergence on compacta, equicontinuity, K-spaces, Proximal set-open topologies, Leader convergence and simple Leader convergence in function

spaces, Quasi uniform convergence, Nearness convergence, Wijsman convergence, Proximal graph topologies on function spaces. **20 L**

References

[1] Alejandro Wanes and Sam B. Nadler, Jr. Hyperspaces: Fundamentals and Recent Advances, Marcel Dekker, Inc. New York (1999).

[2] SomashekharNaimpally, Proximity Approach to Problems in Topology and Analysis, OldenbourgVerlag, Munich,Germany, (2009).

Suggested Readings

[1] James Dugundji, Topology, Allyn and Bacon, Inc., Boston (1966)

[2] John L. Kelly, General Topology, Springer-Verlag, New York Berlin Heideberg (1957).

[3] R. Lowen, Approach Spaces: The missing link in the Topology-Uniformity-Metric Triad, Clarendon Press, Oxford (1997).

MATH16-42 : Introduction to Greedy Appromixations

Greedy approximation, definition and examples. Quasi-greedy and almost greedy approximation. Lebesgue-type inequalities for greedy approximation. Saturation property of greedy-type algorithms. Approximation in compressed sensing: Equivalence of three approximation properties of the compressed sensing matrix, Construction of a good matrix, Exact recovery of sparse signals.

References

1. M. Fabian, P. Habala, P. Hajek, V.M. Santalucia, J. Pelant and V. Zizler, Functional Analysis and Infinite-Dimensional Geometry, Springer-Verlag, New York, 2001.

2. V. Temlyakov, Greedy Approximation, Cambridge University Press, 2011.