M.Phil. / Ph.D. Coursework Syllabus 2021-22

The courses MATH20-R01 Research Methodology and MATH20-R02 Research and Publication Ethics are compulsory. Apart from this a research scholar is required to study any of the three courses from the nine courses MATH21-R03 to MATH21-R11.

Total credits of the course work is 16.

MATH21-R01: RESEARCH METHODOLOGY

Total Marks: **50 (Theory: 35, Internal Assessment: 15)**
Duration of Examination: **2 Hrs.**
Workload: **2 Lectures per week.** Credits: **2**

Scientific Research and Literature Survey: History of mathematics, Overview of scientific research, Selection of a research topic and a research problem, Literature survey of the topic and a problem, Role of a supervisor, Studying, reviewing and publishing a paper, Funding agencies, Writing a research proposal.

Scientific Writing and Presentation: Writing a research paper and thesis by using LaTeX; Presentation tools and skills: Beamer as a tool for paper and thesis presentations, Oral and poster presentation.

Software for Mathematics: Mathematica / Matlab.

References

1. **Katz Victor J.,** *A History of Mathematics: An Introduction*, 3rd edition, Addison-Wesley, 2009.
2. **Kitsakorn Locharoenrat**, *Research Methodologies for Beginners*, Pan Stanford Publishing Pte. Ltd., Singapore, 2017.
3. **Nicholas J. Higham**, *Handbook of Writing for the Mathematical Sciences,* SIAM, 1998.
4. **Donald E. Knuth, Tracy Larrabee** & **Paul M. Roberts**, *Mathematical Writing*, Mathematical Association of America, 1989.
5. **Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer** & **Jean A.Dieudonné**, *How to Write Mathematics*, American Mathematical Society, 1973.
6. **Leslie Lamport**, *LaTeX, a Document Preparation System,* Pearson, 2008.
7. **Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel** & **Herbert Voss,** *The LaTeX Graphics Companion,* Addison-Wesley, 2008.
8. **Paul Wellin, Sam Kemin and Richard Gaylord**, An Introduction to Programming with Mathematica, Cambridge University Press, 3rd edition, UK, 2005.

MATH21-R02: RESEARCH AND PUBLICATION ETHICS

Total Marks: **50 (Theory: 35, Internal Assessment: 15)**
Duration of Examination: **2 Hrs.**
Workload: **2 Lectures per week.** Credits: **2**

Philosophy and Ethics: Introduction to philosophy: Definition, nature and scope, concept, branches; Ethics: Definition, moral philosophy, nature of moral judgements and reactions.

Scientific Conduct: Ethics with respect to science and research, Intellectual honesty and research integrity; Scientific misconducts: Falsification, Facbrication and Plagiarism(FFP); Redundant publications: Duplicate and overlapping publications, salami slicing; Selective reporting and misrepresentation of data.

Publication Ethics: Definition, intoduction and importance of publication ethics; Best practices / standards setting initiatives and guidelines: COPE, WAME, etc.; Conflicts of interest; Publication misconduct: Definition, concept, problems that led to unethical behaviour and vice versa, types; Violation of publication ethics, authorship and contributorship; Identification of publication misconduct, complaints and appeals; Predatory publishers and journals.

Open Access Publishing: Open access publications and initiatives; SHEPRA/RoMEO online resource to check publisher copyright & self-archiving policies; Software tool to identify predatory publishers developed by SPPU; Journal finder/journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.

Publication Misconduct: Subject specific ethical issues, FFP, authorship; Conflict of interest; Complaints and appeals: examples and fraud from India and abroad.

Software tools: Use of plagiarism software like Turnitin, Urkund and other open source software tools.

Database and Research Metrics: Databases: Indexing databases; Citation databases: Web of science, Scopus, etc.; Research Metrics: Impact factor of journal as per Journal Citation Report, SNIP, SJR, IPP, CiteScore; Metrics: *h*-index, *g* index, *i*10 index, altmetrics.

References

1. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)
2. **Kitsakorn Locharoenrat**, *Research Methodologies for Beginners*, Pan Stanford Publishing Pte. Ltd., Singapore, 2017.
3. **Anthony, M., Graziano, A.M. and Raulin, M.L.,** Research Methods: A *Process of Inquiry*, Allyn and Bacon, 2009.
4. Committee on Publication Ethics- COPE (https://publicationethics.org/)

MATH21-R03: ADVANCED COMMUTATIVE ALGEBRA

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Workload: **4 Lectures per week**
Credits: **4**

Localization of rings and its properties, Integral extensions, Discrete valuation rings, Dedekind domains, Graded rings and modules, Associated graded rings, *I*-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, Hom functor, Tensor functor, *I*-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor.

References

1. **M.F. Atiyah & I.G. MacDonald**, *Introduction to Commutative Algebra*, CRC Press, 2018.
2. **David Eisenbud**, *Commutative Algebra with a View Toward Algebraic Geometry*, Springer-Verlag, 1995.
3. **Hideyuki Matsumura**, *Commutative Ring Theory*, Cambridge, 1989.
4. **Balwant Singh**, *Basic Commutative Algebra*, World Scientific, 2011.

MATH21-R04: TOPICS IN ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Stone−Weierstrass theorem, Contraction principle, Non-expansive maps and Browder fixed point theorem; Integration of vector functions— Bochner integrability. Definition and elementary properties of vector measures.

Differential calculus in normed linear spaces, Gâteaux and Fréchet derivative of functions, Mean value theorems, Chain rule, Higher order derivatives, Taylor's formula, Local and global inverse function theorems, Implicit function theorem, Extremum problems and Lagrange multipliers.
Introduction to the theory of distributions – test functions, convolutions, Schwartz space, tempered distributions.

Sobolev spaces, Extension operators, Trace results, Sobolev Inequalities, Compactness.

References

1. H. Brezis, *Functional Analysis, Sobolev spaces and partial differential equations,* Springer 2011

2. Ward Cheney, *Analysis for Applied Mathematics,* Springer-Verlag, 2013.

3. J. Diestel and J.J. Uhl, Vector Measures, Mathematical Surveys and Monographs 15, AMS 1977.

4. Pavel Drábek & Jaroslav Milota, *Methods of Nonlinear Analysis: Applications to Differential Equations*, Birkhäuser, Berlin 2013.

5. Lawrence C. Evan., *Partial Differential Equations*, Univ. of California, Berkeley, 1998.

6. Srinivasan Kesavan, *Topics in Functional Analysis and Applications*, Wiley Eastern Ltd., 1989.

7. Walter Rudin, *Principles of Mathematical Analysis*, McGraw Hill, 1978.

8. Walter Rudin, Functional Analysis, McGraw Hill Education Europe, 2007

MATH21-R05: ADVANCED FUNCTIONAL ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Topological Vector Spaces. Types of topological vector spaces, Separation properties, Linear mappings, boundedness and continuity, Quotient spaces, Examples, Banach–Steinhaus theorem, Open mapping theorem, Closed graph theorem, Hahn–Banach Theorem on topological vector spaces, Weal topologies, Weak*-topology of a dual space, Compact convex sets, Extreme points, Milman's theorem, vector-valued integration, Vector-valued holomorphic functions.

Banach Algebras. Definition and examples of Banach algebras and *-Banach algebras, Complex homomorphisms, Spectrum, Symbolic calculus, Group of invertible elements, Ideals and quotient algebras, Gelfand transform, Applications to non-commutative Banach algebras, Spectral theorem, Symbolic calculus for normal operators, Characterization of C^*-algebras, Unbounded operators.

References

1. **Eberhard Kaniuth**, *A Course in Commutative Banach Algebras*, Springer, 2009.
2. **Walter Rudin**, *Functional Analysis*, Tata McGraw-Hill Education, 2006.
3. **H. H. Schaefer & M. P. Wolf**, *Topological Vector Spaces*, Springer, 2012.

MATH21-R06: TOPOLOGY AND MIXING

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Topological Transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for continuous interval maps, Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on intervals**,** Various other definitions of chaos and their interrelationships.

Topological Entropy: Definition using open covers, Examples and properties, Bowen's definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov−Sinai theorem, Topological entropy of an expansive homeomorphism, of the two sided shift, of the topological Markov chain, of any homeomorphism of the unit circle, of any homeomorphism of closed unit interval, an upper bound for the topological entropy of a diffeomorphism of a finite dimensional Riemannian manifold.

References

1. **Lluís Alsedà, Jaume Llibre & Michał Misiurewicz**, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
2. **Louis S. Block & William A. Coppel**, *Dynamics in One Dimension*, Springer, 2014.
3. **Michael Brin & Garrett Stuck**, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.
4. **Robert L. Devaney**, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
5. **Clark Robinson**, *Dynamical Systems, Stability, Symbolic Dynamics and Chaos*, CRC press, 1998.
6. **S. Ruette**, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
7. **Peter Walters**, *An Introduction to Ergodic Theory*, Springer, 2000.

MATH21-R07: CONVEX AND NONSMOOTH ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Convex sets, Convexity-preserving operations for a set, Relative interior, Asymptotic cone, Extreme points, Face, Projection operator, Separation theorems, Bouligand tangent and normal cones.

Convex functions, Closedness, Affinity, Epigraphical hull and lower-bound function of a set, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties; Sublinear functions, Support function, Calculus of support functions, Norms and their duals, Polarity.

Subdifferential of convex functions, Geometric construction, interpretation and properties of subdifferentials, Minimality conditions, Mean-value theorem; Calculus rule with subdifferentials.

References

1. **Jonathan M. Borwein & Adrian S. Lewis**, *Convex Analysis and Nonlinear Optimization: Theory and Examples*, CMS Books in Mathematics, Springer, 2006.
2. **Jean-Baptiste Hiriart-Urruty & Claude Lemaréchal**, *Fundamentals of Convex Analysis*, Springer, 2004.
3. **Boris S. Mordukhovich & Nguyen Mau Nam**, *An Easy Path to Convex Analysis and Applications*, Morgan & Claypool, 2014.
4. **R. Tyrrell Rockafellar**, *Convex Analysis*, Princeton University Press, 1997.
5. **C. Zălinenscu,** *Convex Analysis in General Vector Spaces*, World Scientific, 2002.

MATH21-R08: HYPERBOLIC SYSTEM OF CONSERVATION LAWS AND BOUNDARY LAYER THEORY

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Hyperbolic system of conservation laws: Fundamental concepts and examples, Scalar and system of conservation laws, Riemann Problem, Entropy condition, Classical and non-classical shocks, Similarity method.
Boundary layer theory: Laminar boundary layer, Turbulent flow, Turbulent boundary layer; Heat and Mass transfer, conduction, convection and radiation; Thermal boundary layer; Modeling and method of solution of the problems.

References

1. **G. B. Whitham**, *Linear and Nonlinear Waves*, John Wiley, 1999.
2. **Vishnu D. Sharma**, *Quasilinear Hyperbolic Systems, Compressible Flows and Waves*, CRC, 2010.
3. **Philippe G. LeFloch**, *Hyperbolic Systems of Conservation Laws: The Theory of Classical and Nonclassical Shock Waves*, Springer Basel AG, 2002.
4. **Hermann Schlichting & Klaus Gersten**, *Boundary-Layer Theory*, Springer, 2017.
5. **Tuncer Cebeci**, *Analysis of Turbulent Flows*, Elsevier, 2004.
6. **J.P. Holman & Souvik Bhattacharyya**, *Heat Transfer in SI Units*, Tata McGraw-Hill, 2011.
7. **George W. Bluman & Sukeyuki Kumei**, *Symmetries and Differential Equations*, Springer, New York, 1996.
8. **Eleuterio F. Toro**, *Riemann Solvers and Numerical Methods for Fluid Dynamics*: *A Practical Introduction*, Springer, 2009.

MATH21-R09: PARTIAL DIFFERENTIAL EQUATIONS: THEORY AND NUMERICS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Maximum principles for second order linear parabolic, elliptic and hyperbolic partial differential equations; Weak solutions for second order linear parabolic, elliptic and hyperbolic partial differential equations; Lax–Milgram theorem, Local existence, uniqueness and regularity results for second order linear parabolic, elliptic and hyperbolic partial differential equations.

Dispersion and dissipation analysis of PDEs and its finite difference schemes, Discontinuous solutions; Finite difference schemes for systems of parabolic and hyperbolic PDEs; Analysis of well-posed initial value problem of parabolic and hyperbolic systems, Convergence estimates for parabolic and hyperbolic PDES, Finite difference schemes for curved boundaries of elliptic PDEs.

References

1. **Lawrence C. Evans**, *Partial Differential Equations*, American Mathematical Society, 2010.
2. **Robert C. McOwen**, *Partial Differential Equations: Methods and Applications*, Pearson Education, 2003.
3. **Gerald B. Folland**, *Introduction to Partial Differential Equations*, Prentice-Hall of India, 2001.
4. **Michael Renardy & Robert C. Rogers**, *An Introduction to Partial Differential Equations*, Springer-Verlag, 2004.
5. **John C. Strikwerda**, *Finite Difference Schemes and Partial Differential Equations*, SIAM, 2004.
6. **J. W. Thomas**, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 1995.

MATH21-R10: ERGODIC THEORY

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Measure preserving transformations and examples, Recurrence, Poincare's Recurrence theorem, Ergodicity, ergodicity of shift transformations.

Ergodic theorems of Birkhoff and Von Neuman, Mixing, Weak-mixing and their characterizations, the isomorphism problem: conjugacy, Spectral equivalence, Transformations with discrete spectrum, Entropy, Kolmogorov-Sinai theorem, *K*-systems examples of calculation of entropy, Unique ergodicity, uniformly distributed sequences, applications to Diophantine approximation.

References

1. **P. R. Halmos,** *Lectures on Ergodic Theory*, Dover Publications 2017.
2. **M. G. Nadkarni,** *Basic Ergodic Theory*, Hindustan Book Agency 2013.
3. **Peter Walters,** *An Introduction to Ergodic Theory*, Springer, 2000.

MATH21-R11: INTRODUCTION TO MINIMAL SURFACES

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Revisiting Multivariable calculus: Inverse and implicit function theorems, Serret-Frenet formula for curves, Parametric surfaces, Isothermal parameters, Gauss Map, Gaussian Curvature, Mean curvature, Area functional, Relationship between conformal and complex analytic maps, Harmonic function, Riemann surface and covering surfaces (only rudimentary discussion).

Surfaces that locally minimize area in Euclidean space (minimal surfaces), Harmonic coordinates in isothermal parameters, Examples of Minimal Surfaces.

Manifold theory (rudimentary introduction), Minimal Surface with boundary: Plateau problem, Geodesics (rudimentary), Complete surfaces, Riemannian manifolds (rudimentary introduction).

Minimal Surfaces and isothermal parametrizations, Bernstein's Theorem, Gauss map in local coordinates , Gauss map for minimal surface with some examples, Complete minimal surfaces, Weierstrass-Enneper Representations of Minimal Surfaces.

References
Main Texts of the Course

1. **Johannes C C Nitsche** , Lectures on Minimal Surfaces Vol 1 , Cambridge University Press, 2011.

Additional Texts of the Course
1. **Manfredo Do Carmo,** *Differential Geometry of Curves and Surfaces,* Second Edition, Dover Publications, 2016.
2. **A T Fomenko and A Tuzhilin,** *Elements of Geometry and Topology of Minimal Surfaces in three-dimensional Space,* AMS, 1991.
3. **John Oprea** , *The Mathematics of Soap Films: Explorations with Maple* , STML Vol 10, AMS, 2000.
4. **Robert Osserman ,** *A Survey of Minimal Surfaces*, Dover Publications, 2014.