M.PHIL./ PH.D. COURSEWORK SYLLABUS

The course MATH19-R01 Research Methodology is compulsory. Apart from this a research scholar is required to study any of the three courses from the nine courses MATH19-R02 to MATH19-R10. Total credits of the course work is 16 and credits for each course is 4.

Total Marks in Each Course: **100,** Duration of Examination for Each Course: **3 Hrs.**
Qualifying Marks for Each Course: **55 (Internal + Final taken together)**

MATH19-R01: RESEARCH METHODOLOGY

Total Marks: **100 (Theory: 40, Practical: 30, Internal Assessment: 30)**
Duration of Examination: **3 Hrs. (Theory: 2 Hrs., Practical: 1Hr.)**
Lab/Theory: **4 Lectures per week**

Scientific Research and Literature Survey: History of mathematics, finding and solving research problems, Role of a supervisor, Survey of a research topic, Publishing a paper, Reviewing a paper, Funding agencies, Research grant proposal writing, Copyright issues, Ethics and plagiarism.

Research Tools: MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, *h*-index, Google Scholar, ORCID, JStor, Online and open access journals.

Scientific Writing and Presentation: Writing a research paper, survey article, thesis writing; LaTeX, PSTricks, Beamer and HTML.

Software for Mathematics: Mathematica/Matlab/Scilab/GAP.

References

1. **Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel** & **Herbert Voss,** *The LaTeX Graphics Companion*, Addison-Wesley, 2008.
2. **Nicholas J. Higham**, *Handbook of Writing for the Mathematical Sciences*, SIAM, 1998.
3. **Donald E. Knuth, Tracy Larrabee** & **Paul M. Roberts**, *Mathematical Writing*, Mathematical Association of America, 1989.
4. **Leslie Lamport**, *LaTeX, a Document Preparation System*, Pearson, 2008.
5. **Frank Mittelbach, Michel Goossens, Johannes Braams, David Carlisle** & **Chris Rowley**, *The LaTeX Companion*, Pearson, 2004.
6. **Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer** & **Jean A. Dieudonné**, *How to Write Mathematics*, American Mathematical Society, 1973.
7. *Mathtools documentation* (http://mirrors.ctan.org/macros/latex/contrib/mathtools/mathtools.pdf)
8. *Pstricks documentation* (http://tug.org/PSTricks/main.cgi?file=doc/docs)

MATH19-R02: ADVANCED COMMUTATIVE ALGEBRA

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Localization of rings and its properties, Integral extensions, Discrete valuation rings, Dedekind domains, Graded rings and modules, Associated graded rings, *I*-adic completion, Krull's intersection theorem, Hensel's lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, Hom functor, Tensor functor, *I*-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor.

References

1. **M.F. Atiyah & I.G. MacDonald**, *Introduction to Commutative Algebra*, CRC Press, 2018.
2. **David Eisenbud**, *Commutative Algebra with a View Toward Algebraic Geometry*, Springer-Verlag, 1995.
3. **Hideyuki Matsumura**, *Commutative Ring Theory*, Cambridge, 1989.
4. **Balwant Singh**, *Basic Commutative Algebra*, World Scientific, 2011.

MATH19-R03: TOPICS IN ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Uniform convergence and differentiation, Stone-Weierstrass theorem, Contraction principle, Non-expansive maps and Browder fixed point theorem; Integration of vector functions—Bochner integrability.

Differential calculus in normed linear spaces, Gâteaux and Fréchet derivative of functions, Mean value theorems, Chain rule, Higher order derivatives, Taylor's formula, Local and global inverse function theorems, Implicit function theorem, Extremum problems and Lagrange multipliers.

Spherical distance in the extended complex plane, Uniform convergence and local uniform convergence with respect to this metric for sequence of meromorphic functions, Normality of families of meromorphic functions and various characterizations.

Criterions for normality of families of holomorphic functions and their applications to Montel's theorem, Miranda's theorem and Bloch's theorem; Criterions for normality of families of meromorphic functions and their applications to Montel's theorem, Zalcman's theorem and Gu's theorem.

References

1. **Ward Cheney**, *Analysis for Applied Mathematics*, Springer-Verlag, 2013.
2. **Chi-Tai Chuang**, *Normal Families of Meromorphic Functions*, World Scientific, 1993.
3. **John B. Conway**, *Functions of One Complex Variable*, Narosa, New Delhi, 2002
4. **Pavel Drábek & Jaroslav Milota**, *Methods of Nonlinear Analysis: Applications to Differential Equations*, Birkhäuser, Berlin 2013.
5. **Walter Rudin**, *Principles of Mathematical Analysis*, McGraw Hill, 1976.

MATH19-R04: ADVANCED FUNCTIONAL ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Topological Vector Spaces. Types of topological vector spaces, Separation properties, Linear mappings, boundedness and continuity, Quotient spaces, Examples, Banach–Steinhaus theorem, Open mapping theorem, Closed graph theorem, Hahn-Banach Theorem on topological vector spaces, Weal topologies, Weak*-topology of a dual space, Compact convex sets, Extreme points, Milman's theorem, vector-valued integration, Vector-valued holomorphic functions.

Banach Algebras. Definition and examples of Banach algebras and *-Banach algebras, Complex homomorphisms, Spectrum, Symbolic calculus, Group of invertible elements, Ideals and quotient algebras, Gelfand transform, Applications to non-commutative Banach algebras, Spectral theorem, Symbolic calculus for normal operators, Characterization of C^*-algebras, Unbounded operators.

References

1. **Eberhard Kaniuth**, *A Course in Commutative Banach Algebras*, Springer, 2009.
2. **Walter Rudin**, *Functional Analysis*, Tata McGraw-Hill Education, 2006.
3. **H. H. Schaefer & M. P. Wolf**, *Topological Vector Spaces*, Springer, 2012.

MATH19-R05: TOPOLOGY AND MIXING

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Topological Transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for continuous interval maps, Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on intervals**,** Various other definitions of chaos and their interrelationships.

Topological Entropy: Definition using open covers, Examples and properties, Bowen’s definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov-Sinai theorem, Topological entropy of an expansive homeomorphism, of the two sided shift, of the topological Markov chain, of any homeomorphism of the unit circle, of any homeomorphism of closed unit interval, an upper bound for the topological entropy of a diffeomorphism of a finite dimensional Riemannian manifold.

References

1. **Lluís Alsedà, Jaume Llibre & Michał Misiurewicz**, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
2. **Louis S. Block & William A. Coppel**, *Dynamics in One Dimension*, Springer, 2014.
3. **Michael Brin & Garrett Stuck**, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.
4. **Robert L. Devaney**, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
5. **Clark Robinson**, *Dynamical Systems, Stability, Symbolic Dynamics and Chaos*, CRC press, 1998.
6. **S. Ruette**, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.

7. **Peter Walters**, *An Introduction to Ergodic Theory*, Springer, 2000.

MATH19-R06: CONVEX AND NONSMOOTH ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Convex sets, Convexity-preserving operations for a set, Relative interior, Asymptotic cone, Extreme points, Face, Projection operator, Separation theorems, Bouligand tangent and normal cones.

Convex functions, Closedness, Affinity, Epigraphical hull and lower-bound function of a set, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties; Sublinear functions, Support function, Calculus of support functions, Norms and their duals, Polarity.

Subdifferential of convex functions, Geometric construction, interpretation and properties of subdifferentials, Minimality conditions, Mean-value theorem; Calculus rule with subdifferentials.

References

1. **Jonathan M. Borwein & Adrian S. Lewis**, *Convex Analysis and Nonlinear Optimization: Theory and Examples*, CMS Books in Mathematics, Springer, 2006.
2. **Jean-Baptiste Hiriart-Urruty & Claude Lemaréchal**, *Fundamentals of Convex Analysis*, Springer, 2004.
3. **Boris S. Mordukhovich & Nguyen Mau Nam**, *An Easy Path to Convex Analysis and Applications*, Morgan & Claypool, 2014.
4. **R. Tyrrell Rockafellar**, *Convex Analysis*, Princeton University Press, 1997.
5. **C. Zălinenscu,** *Convex Analysis in General Vector Spaces*, World Scientific, 2002.

MATH17-R07: HYPERBOLIC SYSTEM OF CONSERVATION LAWS AND BOUNDARY LAYER THEORY

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Hyperbolic system of conservation laws: Fundamental concepts and examples, Scalar and system of conservation laws, Riemann Problem, Entropy condition, Classical and non-classical shocks, Similarity method.
Boundary layer theory: Laminar boundary layer, Turbulent flow, Turbulent boundary layer; Heat and Mass transfer, conduction, convection and radiation; Thermal boundary layer; Modeling and method of solution of the problems.

References

1. **G. B. Whitham**, *Linear and Nonlinear Waves*, John Wiley, 1999.
2. **Vishnu D. Sharma**, *Quasilinear Hyperbolic Systems, Compressible Flows and Waves*, CRC, 2010.
3. **Philippe G. LeFloch**, *Hyperbolic Systems of Conservation Laws: The Theory of Classical and Nonclassical Shock Waves*, Springer Basel AG, 2002.
4. **Hermann Schlichting & Klaus Gersten**, *Boundary-Layer Theory*, Springer, 2017.
5. **Tuncer Cebeci**, *Analysis of Turbulent Flows*, Elsevier, 2004.
6. **J.P. Holman & Souvik Bhattacharyya**, *Heat Transfer in SI Units*, Tata McGraw-Hill, 2011.

7. **George W. Bluman & Sukeyuki Kumei**, *Symmetries and Differential Equations*, Springer, New York, 1996.

8. **Eleuterio F. Toro**, *Riemann Solvers and Numerical Methods for Fluid Dynamics*: *A Practical Introduction*, Springer, 2009.

MATH19-R08: PARTIAL DIFFERENTIAL EQUATIONS: THEORY AND NUMERICS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Maximum principles for second order linear parabolic, elliptic and hyperbolic partial differential equations; Weak solutions for second order linear parabolic, elliptic and hyperbolic partial differential equations; Lax-Milgram theorem, Local existence, uniqueness and regularity results for second order linear parabolic, elliptic and hyperbolic partial differential equations.

Dispersion and dissipation analysis of PDEs and its finite difference schemes, Discontinuous solutions; Finite difference schemes for systems of parabolic and hyperbolic PDEs; Analysis of well-posed initial value problem of parabolic and hyperbolic systems, Convergence estimates for parabolic and hyperbolic PDES, Finite difference schemes for curved boundaries of elliptic PDEs.

References

1. **Lawrence C. Evans**, *Partial Differential Equations*, American Mathematical Society, 2010.
2. **Robert C. McOwen**, *Partial Differential Equations: Methods and Applications*, Pearson Education, 2003.
3. **Gerald B. Folland**, *Introduction to Partial Differential Equations*, Prentice-Hall of India, 2001.
4. **Michael Renardy & Robert C. Rogers**, *An Introduction to Partial Differential Equations*, Springer-Verlag, 2004.
5. **John C. Strikwerda**, *Finite Difference Schemes and Partial Differential Equations*, SIAM, 2004.
6. **J. W. Thomas**, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 1995.

MATH19-R09: OPERATORS AND FUNCTION THEORY ON THE UNIT CIRCLE

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Review of topics dealing with the basic properties of the Lebesgue and Hardy spaces on the unit circle and their inter-connections, Review of Beurling's theorem with an elementary proof, Review of basic operator theory on Hilbert spaces, Review of definition and basic properties of Banach algebras.

The Helson-Lodenslager theorem on L^p, Generalisations of the Helson-Lowdenslager theorem, Beurling's theorem as a trivial corollary to the Helson-Lowdenslager theorem, Maximality of the algebra of bounded analytic functions as a weak star closed sub-algebra via the Helson-Lowdenslager theorem, De Branges' generalization of Beurling's theorem, Shifts of finite multiplicity and their invariant subspaces on Hardy spaces, Shifts of finite multiplicity on sub-Hardy Hilbert spaces.

Basic properties of algebras of functions on the unit circle that shall include the Disk Algebra, the algebra of bounded analytic functions, the algebra of essentially bounded functions and some of their important sub-algebras; Wermer's Maximality theorem, Closed and maximal ideals of the algebras $C(T)$; Classifying closed ideals of the Disk Algebra via invariant subspaces, Properties of maximal ideals of the algebra of bounded analytic functions, Bohr's inequality on the disk algebra and its application to the von Neumann inequality conjecture on non-unital Banach algebras, Uniform algebras: definition and simple examples, Bohr's inequality on uniform algebras.

References

1. **Ronald G. Douglas,** *Banach Algebra Techniques in Operator Theory*, Springer, 1998.
2. **John B. Garnett,** *Bounded Analytic Function*, Springer, 2007.
3. **Henry Helson,** *Harmonic Analysis*, Hindustan Book Agency, 2019.
4. **Kenneth Hoffman,** *Banach Spaces of Analytic Functions*, Dover Publications, 1990.
5. **Gershon Kresin & Vladimir G. Maz'ya**, *Sharp Real-Part Theorems*: *A Unified Approach*, Lecture Notes in Mathematics, Volume 1903, Springer, 2007.
6. **Nikolai K. Nikolski,** *Operators, Functions and Systems*: *An Easy Reading, Volume* 1: *Hardy, Hankel, and Toeplitz*, American Mathematical Society, 2002.
7. **Donald Sarason**, *Sub-Hardy Hilbert Spaces in the Unit Disk*, Wiley, 1994.

MATH19-R10: INTRODUCTION TO SEVERAL COMPLEX VARIABLES

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

Definitions of holomorphic function and equivalence of these definitions, Cauchy formula, Cauchy's inequality, identity theorem, open mapping theorem, Weierstrass's theorem, Montel's theorem, Holomorphic mapping.

Hartogs's phenomenon, Weierstrass preparation theorem, division theorem and their consequences, Zero set of holomorphic functions, Analytic set, regular and singular points of an analytic set.

Automorphism group of bounded domains, Poincaré theorem: The ball and the polydisc are not biholomorphic.

Domains of Holomorphy: The continuity principle, Hartogs convexity, Domain of holomorphy, Weak domain of holomorphy, Pseudoconvexity, plurisubharmonic functions, Holomorphic convexity.

References

1. **Klaus Fritzsche & Hans. Grauert**, *From Holomorphic Functions to Complex Manifolds*, Springer-Verlag, 2002.
2. **Lars Hörmander**, *An Introduction to Complex Analysis in Several Variables*, North-Holland, 1990.
3. **Raghavan Narasimhan**, *Several Complex Variables*, University of Chicago Press, 1995.
4. **R. Michael Range**, *Holomorphic Functions and Integral Representations in Several Complex Variables*, Springer-Verlag, 1986.