EC DATED: 09.05.2013

AC DATED: 07.05.2013

FOUR YEAR

UNDERGRADUATE PROGRAMME

(Effective from Academic Year 2013-2014 onwards)

DEPARTMENT OF MATHEMATICS

UNIVERSITY OF DELHI

DELHI

Four Year Under-graduate ProgrammeWith Multiple Exit Options

About the Program

Curriculum

The overall structure

Category	*No. of Courses*		
	Exit After 2 Yrs	***Exit After 3 Yrs***	***Exit After 4 Yrs***
Foundation Courses (FC)	11	11	11
Departmental Courses (DC1)	8	14	20
Outside Department Courses (DCII)	2	4	6
Application Courses (AC)	3	5	5
Co-Scholastic Courses	4	6	8
TOTAL	**28**	**40**	**50**

About the Foundation Courses

FOUNDATION COURSE (FC)

Course no.	***Title***	***C***	***P***	***T/LB***	***Marks***		
					CE	**ESE**	**Total**
FC 1	Language, Literature, and Creativity - I (Hindi/MIL/Sanskrit /Arabic /Persian / Indian Literature)	4	1		35	40	75
FC 2	Language, Literature, and Creativity - II (English),Watch the video	4	1		35	40	75
FC 3	Information Technology	2	1		35	40	75
FC 4	Business, Entrepreneurship, and Management	2	1		35	40	75
FC 5	Governance and Citizenship	2	1		35	40	75
FC 6	Philosophy, Psychology, Communication and Life Skills	2	1		35	40	75
FC 7	Geographic and Socio-Economic Diversity	2	1		35	40	75
FC 8	Science and Life	2	1		35	40	75
FC 9	Indian History and Culture	2	1		35	40	75
FC 10	Building Mathematical Ability	2	1		35	40	75
FC 11	Environment and Public Health	2	1		35	40	75

C:-Class, **P**:-Presentation, **T**:-Tutorial, **LB**:-Lab, **CE**:-Continuous Evaluation, **ESE**:-End Semester Exam

DEPARTMENTAL COURSES (DCI)

Course no.	*Title*	*C*	*P*	*T/LB*	*Marks*			
					CE	**ESE**	**PRAC**	**Total**
I.1	Calculus-I	4	1	0/4	25	75	50	150
I.2	Algebra-I	4	1	1/0	25	75	0	100
II.1	Analysis-I (Real Analysis)	4	1	1/0	25	75	0	100
II.2	Differential Equations-I	4	1	0/4	25	75	50	150
III.1	Analysis-II (Real Functions)	4	1	1/0	25	75	0	100
III.2	Numerical Methods	4	1	0/4	25	75	50	150
IV.1	Calculus-II (Multivariate Calculus)	4	1	1/0	25	75	0	100
IV.2	Probability & Statistics	4	1	0/4	25	75	50	150
V.1	Algebra-II(Group Theory-I)	4	1	1/0	25	75	0	100
V.2	Analysis-III (Riemann Integration & Series of Functions)	4	1	1/0	25	75	0	100
V.3	Diff. Eqns.-II (P.D.E. & System of ODE)	4	1	0/4	25	75	50	150
VI.1	Algebra-III (Ring Theory & Linear Algebra-I)	4	1	1/0	25	75	0	100
VI.2	Analysis-IV (Metric Spaces)	4	1	1/0	25	75	0	100
VI.3	Calculus of Variations & Linear Programming	4	1	0/4	25	75	50	150
VII.1	Algebra-IV (Group Theory-II)	4	1	1/0	25	75	0	100
VII.2	Differential Equations-III	4	1	0/4	25	75	50	150
VII.3	Research Methodology							
VIII.1	Analysis-V (Complex Analysis)	4	1	0/4	25	75	50	150
VIII.2	Algebra –V(Ring Theory & Linear Algebra –II)	4	1	1/0	25	75	0	100
VIII.3	Research (Dissertation)							

C:-Class, **P**:-Presentation, **T**:-Tutorial, **LB**:-Lab, **CE**:-Continuous Evaluation, **ESE**:-End Semester Exam., **PRAC**:-Practical

About the Applied Courses

APPLIED COURSES (AC)

Course no.	*Title*	*C*	*P*	*T/LB*	*Marks*		
					CE	**ESE**	**Total**
ACL	Language Course	2	1	0/2	75	0	75
Maths AC 1	C++ Programming	2	1	0/2	75	0	75
Maths AC 2	Mathematical Finance	2	1	0/2	75	0	75
Maths AC 3	Cryptography and Network Systems	2	1	0/0	75	0	75
Maths AC 4	Discrete Mathematics	2	1	0/0	75	0	75

ACL:-Applied Course Language, **C**:-Class, **P**:-Presentation, **T**:-Tutorial, **LB**:-Lab, **CE**:-Continuous Evaluation, **ESE**:-End Semester Exam **PRAC**:-Practical

Syllabi of Applied Courses (AC)

About DC II

DCII COURSES OFFERED BY THE DEPARTMENT FOR STUDENTS OUTSIDE THE DEPARTMENT

Course no.	*Title*	*C*	*P*	*T/LB*	*Marks*			
					CE	**ESE**	**PRAC**	**Total**
Maths DC II 1	Calculus	4	1	0/0	25	75	0	100
Maths DC II 2	Linear Algebra	4	1	0/0	25	75	0	100
Maths DC II 3	Differential Equations & Mathematical Modeling	4	1	0/3	20	60	20	100
Maths DC II 4	Numerical Methods	4	1	0/3	20	60	20	100
Maths DC II 5	Real Analysis	4	1	0/0	25	75	0	100
Maths DC II 6	Abstract Algebra	4	1	0/0	25	75	0	100

C:-Class, **P**:-Presentation, **T**:-Tutorial, **LB**:-Lab, **CE**:-Continuous Evaluation, **ESE**:-End Semester Exam. **PRAC**:-Practical

Syllabi of DCII Courses Offered By The Department For Students Outside The Department

Semester-wise Schedule of Courses

Semester	Foundation Course				DC I			DC II Outside Department	AC	IMBH/CA
I	FC	FC	FC	FC	I.1	I.2	-	-	-	1-IMBH
II	FC	FC	FC	-	II.1	II.2	-	-	ACL	1-IMBH
III	FC	FC	-	-	III.1	III.2	-	DC II	Maths AC 1	1-CA
IV	FC	FC	-	-	IV.1	IV.2	-	DC II	Maths AC 2	1-CA
V	-	-	-	-	V.1	V.2	V.3	DC II	Maths AC 3	1-CA
VI	-	-	-	-	VI.1	VI.2	VI.3	DC II	Maths AC 4	1-CA
VII	-	-	-	-	VII.1	VII.2	VII.3	DC II	-	1-CA
VIII	-	-	-	-	VIII.1	VIII.2	VIII.3	DC II	-	1-CA

Teaching Hours

	Foundation Course	DC-I	DC-II	Applied Course	IMBH/NCC/NSS/ Sport/CA
Periods per week	For 14 weeks every semester: Total 3 periods Lectures – 2 periods Class presentation-I period In addition, two week for field work/project work/trip-related activity as required by the course curriculum. In the case of language, Literature and Creativity course, Lectures 4 periods, Class Presentation – I period (5 periods)	For 14 weeks every semester: Total:5 periods Lectures – 4 periods Class presentation – I period. Practicals (wherever applicable)-4 periods Tutorials* (wherever applicable) – as per requirement of course. In addition, two week for field work/project work/trip-related activity as required by the course curriculum. * No tutorials shall be conducted for the courses having practical component	For 14 weeks every semester Total: 5 periods Lectures -4 periods Class presentation –I period Practicals (wherever applicable) – 4 periods In addition, two week for field work/ project work/ trip-related activity as required by the course curriculum.	For 16 weeks Total:3 periods Practical/ hands-on experience/ project work	For 16 weeks Total : 2 periods
Maxi-mum marks	Maximum 75 marks, with 40 marks for end semester examination and 35 marks for continuous evaluation of project work	Maximum 100 marks, with 75 marks for end semester examination and 25 marks for internal assessment. Where DC-I courses have a Practical component, these papers shall have maximum 150 marks, with 75 marks for end semester examination and 25 marks for internal assessment and 50 marks for the	Maximum 100 marks, with 75 marks for end semester examination and 25 marks for internal assessment. Where DC-II courses have a Practical component, these papers shall have maximum 150 marks, with 75 marks for end	Maximum 75 marks. Student will be continuously evaluated by the teacher(s) concerned. However, Applied Language Courses in the first year shall have an end semester examination of 40 marks and continuous evaluation of 35 marks	Not applicable

		Practical (25 marks for continuous evaluation and 25 marks for end semester examination). The paper on research methodology (Semester 7) shall carry 100 marks. The Project that starts in semester 7 and continues in semester 8 shall carry 100 marks.	semester examination and 25 marks for internal assessment and 50 marks for the Practical (25 marks for continuous evaluation and 25 marks for end semester examination)		
Duration of end semester theory exami-nation	2 hours	3 hours	3 hours	2 hours (only for Applied Language Courses)	Not applicable

BUILDING MATHEMATICAL ABILITY

1. Overview

This course is designed to provide a hands-on experience to the student for gaining insight into the world of data for building models; and for providing a glimpse into the practical power of the world of numbers. These models are relevant for some of the needs and challenges of India. A significant portion of the learning process is based on open e-books that are highly interactive and that transfer knowledge into practice. This course aims to seamlessly blend theory and practice where often practical knowledge precedes theoretical insights. Additionally, the project and application oriented hands on learning process would introduce the joys and advantages of working cohesively and productively in groupsto relate mathematics to real world problems.

2. Objective and Expected outcome

- Learning mathematics by connecting it to real life problems linked to society
- Extending blackboard teaching to solve practical problems using mathematics
- Triggering analytical thinking
- Encourage experimentation and hands-on projects mode of learning
- Appreciation of role of data, learn how to collect data and analyze it
- Correlate real world observations with the theoretical knowledge
- Mathematical analysis of data for simple quantitative inference

3. Themes & Sub-themes

I. Numbers **6 lectures**

Prime numbers

Interesting properties of prime numbers without proofs

Ramanujan's work on the Prime Number Theorem

Euclid's division algorithm

Mathematical illustration through intuitive examples

Visualisation through tiling analogy

Encryption and Prime numbers

Gentle introduction of 2 x 2 matrices

Constructing the RSA Algorithm

Project Themes

Ramanujan's work

Implementations of RSA algorithms

Other methods of encryption

II. Data and patterns — 6 lectures

Historical perspective and importance of data

Kepler's law for planetary orbits from Tycho Brahe's astronomical observation

Ramanujan's work on Prime numbers through data

Data collection techniques

Formulation of problem – goals, targets

Methods to collect data – questionnaire, observations, recording, etc.

How much of data is enough for the given problem

Population and sample

Project Themes

Collecting and organising data in various situations through practical methods, from the internet and from other sources.

Use of spreadsheets for practical work related to the above concepts.

III. Statistics — 8 lectures

Organisation of data

Frequency table

Grouping

Visualisation of data

Pictorially displaying data: dot plots, bar graphs, line graphs, pie charts

Misinterpretation of data by distorting the figures: Scaling and axis manipulation,

Line graphs and cropping

Analysis of data

Mean, median, mode, variance, standard deviation.

Histogram, skewed distribution

Comparing two distributions

Project Themes

Statistical analysis of daily life data

Statistical analysis of stock market data

Statistical analysis of weather data

Statistical analysis of data for better governance

IV. Probability — 4 lectures

Interpreting probability, Sample Space, Events

Understanding the tossing of a coin and throwing of dice for large number of trials

Probability in a situation where there are equally-likely outcomes

Probability of two independent events

Project Themes

Compute probabilities from insufficient information
Validity of computed probability

4. **Suggestive Projects**
   - Algorithmic approach of Sieve of Erastosthene's.
   - Ramanujan's theory of prime numbers: Use of prime numbers in coding and decoding of messages.
   - Bertrnad's postulate
   - Download http://pib.nic.in/prs/2011/latest31mar.pdf. Analyse various information that have been extracted from the Census, 2011. Understand as to how these information have been presented.
   - Visit the census site of India http://www.censusindia.gov.in/Census_Data_2001/Census_Data_Online/Language/Statement3.htm
     Depict the information given there in a pictorial form.
   - Prepare a questionnaire to collect information about money spent by your friends in a month on activities like traveling, movies, recharging of the mobiles, etc. and draw interesting conclusions.
   - Check out the local newspaper and cut out examples of information depicted by graphs. Draw your own conclusions from the graph and compare it with the analysis given in the report.
   - Analysis of population migration data – positive and negative influence on urbanization
   - Each day newspaper tells us about the maximum temperature, minimum temperature, humidity. Collect the data for a period of 30 days and represent it graphically. Compare it with the data available for the same time period for the previous year.
   - Draw a career graph of a cricketer (batting average for a batsman and bowling average for a bowler). Conclude the best year of his career. It may be extended for other players also – tennis, badminton, athlete.
   - Share market data analysis – correlation and extreme fluctuation
   - Vehicle registration data – correlating with pollution and number of accidents
   - Visit a village near Delhi and collect data of various crops over past few years from the farmers. Also collect data about temperature variation and rain over the period for a particular crop. Try to find the effect of temperature and rain variations on various crops.
   - How safe are privately owned public transport versus government owned public transport? Collect the data from archives about accidents of Blue Line buses and compare with those of DTC buses. Verify whether DTC buses are significantly safer.
   - Visit Kirana shops near your home and collect the data of sale of certain commodaties over a month. Try to figure out the stock of a particular commodity which should be in the store in order to maximize the profit.

- Mendelian Genetics: Genes are molecular units of heredity and carry certain information. They occur in pairs. Gregor Mendel studied about the inheritance in pea plants. One of the characteristics about their inheritance is smooth (S) and wrinkled (W). Suppose a plant has a heterogeneous gene that contains both the characteristic S and W. Find the probability of having a heterogeneous offspring (SW) or a homogeneous offspring (SS or WW) in the first generation, second genration, third generation, ….
- Choose any week of your ongoing semester. Collect data for the past 10 – 15 years for the amount of rainfall received in Delhi during that week. Predict amount of rainfall for the current year.
- Gambler's fallacy
- Stock price movement using the Binomial Distribution
- Weather prediction (prediction of monsoon from past data)
- Risk assessments by insurance firms from data
- Predicting stock market crash
- Predicting outcome of election – exit polls
- Predicting mortality of infants
- Studying various games that use the concept of Probability – Lotto, Throwing dice, KhulJaSimSim, etc
- Data from iterative map for population growth – dynamics from plots
- Statistical analysis of alphabets/ words appearing in a given text
- Statistical methods for drug testing
- Modelling population growth through data
- Modelling spread of disease through data
- Validation of existing models through data

5. Reading List

Printed Material

Berlinghoff, W. P., Grant, K. E., & Skrien, D. (2001). *A Mathematics Sampler: Topics for the Liberal Arts* (4 ed.). Ardsley House.

Maki, D. P. (2006). *Mathematical Modeling and Computer Simulation* (1 ed.). Thomsons Brooke Cole.

Parks, H. M. (2007). *A Mathematical View of Our World* (1 ed.). Thomsons Brooks Cole.

Staszkow, R., & Bradshaw, R. (2004). *The Mathematical Palette* (3 ed.). Thomsons Brooks Cole.

Tannenbaum, P. (2010). *Excursions in Modern Mathematics.* Pearson.

E-resources:

http://chandra.harvard.edu/edu/formal/icecore/The_Astronomers_Tycho_Brahe_and_Johannes_Kepler.pdf
http://onlinestatbook.com/2/introduction/introduction.html
http://onlinestatbook.com/2/graphing_distributions/graphing_distributions.html
http://davidmlane.com/hyperstat/A16252.html
http://www.math.uah.edu/stat/prob/index.html
http://onlinestatbook.com/2/probability/probability.html
http://www.ted.com/talks/hans_rosling_shows_the_best_stats_you_ve_ever_seen.html
http://stattrek.com/probability/what-is-probability.aspx?Tutorial=AP
http://stattrek.com/probability/probability-rules.aspx?Tutorial=AP
http://www.mathspace.com/nsf_probstat/teacher_inservice_workshops/prob_review_count_no_answers2.pdf
http://www.cimt.plymouth.ac.uk/projects/mepres/alevel/discrete_ch14.pdf
http://www.opentextbookstore.com/mathinsociety/index.html
http://heja.szif.hu/ANM/ANM-000926-A/anm000926a/node2.html

Structures of DC-I,DC-II, AC Mathematics Courses

DC-I STRUCTURE*

Semester	No. of Papers	Papers	Page No.
1	2	I.1 Calculus-I	19-21
		I.2 Algebra-I	22-23
2	2	II.1 Analysis-I (Real Analysis)	24-25
		II.2 Differential Equations-I	26-29
3	2	III.1 Analysis-II (Real Functions)	30-31
		III.2 Numerical Methods	32-35
4	2	IV.1 Calculus-II (Multivariate Calculus)	36-37
		IV.2 Probability & Statistics	38-42
5	3	V.1 Algebra-II(Group Theory-I)	43-44
		V.2 Analysis-III (Riemann Integration & Series of Functions)	45-46
		V.3 Differential Equations -II (P.D.E. &System of ODE)	47-50

6	**3**	**VI.1 Algebra-III (Ring Theory & Linear Algebra-I)** **VI.2 Analysis-IV (Metric Spaces)** **VI.3 Calculus of Variations & Linear Programming**	**51-52** **53-54** **55-57**
7	**2+1**	**VII.1 Algebra-IV (Group Theory-II)** **VII.2 Differential Equations-III** **VII.3 Research**	**58** **59-60** **61**
8	**2+1**	**VIII.1 Analysis-V (Complex Analysis)** **VIII.2 Algebra –V(Ring Theory & Linear Algebra –II)** **VIII.3 Research (Dissertation)**	**62-65** **66-67** **68**

DC-II STRUCTURE

Semester	No. of Papers	Papers	Page No.
3	1	**Calculus**	**71-72**
4	1	**Linear Algebra**	**73-74**
5	1	**Differential Equations & Mathematical Modeling**	**75-77**
6	1	**Numerical Methods**	**78-80**
7	1	**Real Analysis**	**81-83**
8	1	**Abstract Algebra**	**84-85**

APPLIED COURSES STRUCTURE

Semester	No./ of Papers	Papers	Page No.
3	1	C++ Programming	87-90
4	1	Mathematical Finance	91-93
5	1	Cryptography and Network Systems	94-96
6	1	Discrete Mathematics	97-99

DISCIPLINE COURSES- I

MATHEMATICS

This course aims to create a solid foundation for assimilation of mathematical concepts and structures and build mathematical skills like creative, logical and analytical thinking. The syllabus has been designed to ensure that as the course progresses systematically, it provides a firm grounding in core mathematics subjects including Calculus, Analysis, Algebra,Differential Equations and Modelling real life problems. Tools such as Mathematica /SPSS/Maxima shall be used to enhance the understanding of fundamental mathematical concepts. The study of this course shall be of immense help to those who would like to persue a career in fields like economics, physics, engineering, management science, computer science, operational research, mathematics and several others.

DC-I STRUCTURE*

Semester	No. of Papers	Papers	Page No.
1	2	**I.1 Calculus-I** **I.2 Algebra-I**	19-21 22-23
2	2	**II.1 Analysis-I (Real Analysis)** **II.2 Differential Equations-I**	24-25 26-29
3	2	**III.1 Analysis-II (Real Functions)** **III.2 Numerical Methods**	30-31 32-35
4	2	**IV.1 Calculus-II (Multivariate Calculus)** **IV.2 Probability & Statistics**	36-37 38-42
5	3	**V.1 Algebra-II** (Group Theory-I) **V.2 Analysis-III (Riemann Integration & Series of Functions)** **V.3 Differential Equations-II** (P.D.E. &System of ODE)	43-44 45-46 47-50
6	3	**VI.1 Algebra-III** (Ring Theory & Linear Algebra-I) **VI.2 Analysis-IV (Metric Spaces)** **VI.3 Calculus of Variations & Linear Programming**	51-52 53-54 55-57

7	**2+1**	**VII.1 Algebra-IV** (Group Theory-II) **VII.2 Differential Equations-III** VII.3 Research	**58** **59-60** **61**
8	**2+1**	**VIII.1 Analysis-V** (Complex Analysis) **VIII.2 Algebra –V(Ring Theory & Linear Algebra –II)** **VIII.3 Research (Dissertation)**	**62-65** **66-67** **68**

*Each Practical will be of four classes. There is only one paper containing practicals in each semester.The practical paper may contain practicals from other papers of the same semester. Out of 14 weeks in a semester the courses are designed for 12 weeks and two weeks are given for revision/problem sessions/difficulties of students etc.

I.1 :Calculus I

Total marks: 150 (Theory: 75, Practical: 50, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student),
Use of Scientific Calculators is allowed.

1st Week:
Hyperbolic functions, Higher order derivatives, Applications of Leibnitz rule.
[2]: Chapter 7 (Section 7.8)

2nd Week:
The first derivative test, concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test, limits at infinity, graphs with asymptotes.
[1]: Chapter 4 (Sections 4.3, 4.4)

3rd Week:
Graphs with asymptotes, L'Hopital's rule, applications in business, economics and life sciences.
[1]: Chapter 4 (Sections 4.5, 4.7)

4th Week:
Parametric representation of curves and tracing of parametric curves, Polar coordinates and tracing of curves in polar coordinates
[1]: Chapter 9 (Section 9.4)
[2]: Chapter 11(Section 11.1)

5th Week:
Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \mathrm{Sin}^n x\,dx$, $\int \mathrm{Cos}^n x\,dx$, $\int \tan^n x\,dx$, $\int \mathrm{Sec}^n x\,dx$, $\int (\log x)^n dx$, $\int \mathrm{Sin}^n x \mathrm{Cos}^m x\,dx$
[2]: Chapter 8 (Sections 8.2-8.3, pages 532-538)

6th Week:
Volumes by slicing; disks and washers methods, Volumes by cylindrical shells.
[2]: Chapter 6 (Sections 6.2-6.3)

7th Week:
Arc length, arc length of parametric curves, Area of surface of revolution
[2]: Chapter 6 (Sections 6.4-6.5)

8th Week:

Techniques of sketching conics, reflection properties of conics

[2]: Chapter 11 (Section 11.4)

9th Week:

Rotation of axes and second degree equations, classification into conics using the discriminant

[2]: Chapter 11 (Section 11.5) (Statements of Theorems 11.5.1 and 11.5.2)

10th Week:

Introduction to vector functions and their graphs, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions.

[1]: Chapter 10 (Sections 10.1-10.2)

11th Week:

Modeling ballistics and planetary motion, Kepler's second law.

[1]: Chapter 10 (Section 10.3)

12th Week:

Curvature, tangential and normal components of acceleration.

[1]: Chapter 10 (Section 10.4)

[2]: Chapter 13 (Section 13.5)

Practical / Lab work to be performed on a computer:

Modeling of the following problems using Matlab / Mathematica / Maple etc.

1. Plotting of graphs of function of type $ax, a \in R, [x]$ (greatest integer function), $\sqrt{ax+b}$, $|ax+b|$, $c \pm |ax+b|$, x^n (n even and odd positive integer), x^{-n} (n even and odd positive integer), $x^{\frac{1}{n}}$ (n a positive integer) $\frac{|x|}{x}$ for $x \neq 0$, $\sin\frac{1}{x}$ for $x \neq 0$, $x\sin\frac{1}{x}$ for $x \neq 0$, $e^{\pm\frac{1}{x}}$, e^{ax+b}, $\log(ax+b)$, $1/(ax+b)$, $\sin(ax+b)$, $\cos(ax+b)$, $|\sin(ax+b)|$, $|\cos(ax+b)|$. Discuss the effect of a and b on the graph.

2. Plotting the graphs of polynomial of degree 4 and 5, the derivative graph, the second derivative graph and comparing them.
3. Sketching parametric curves.
4. Tracing of conics in Cartesian coordinates.
5. Obtaining surface of revolution of curves.
6. Sketching ellipsoid, hyperboloid of one and two sheets,

elliptic cone, elliptic paraboloid, hyperbolic paraboloid
using Cartesian co-ordinates.

7. To find numbers between two real numbers and ploting of finite and infinite subset of R.
8. Matrix operations (addition, multiplication, inverse, transpose, determinant, rank, eigenvectors, eigenvalues, Characteristic equation and verification of Cayley Hamilton equation, system of linear equations)
9. Graph of Hyperbolic functions.
10. Computation of limit, differentiation and integration of vector functions.
11. Complex numbers and their representations, operations like addition, multiplication, division, modulus. Graphical representation of polar form.

REFERENCES:

1. M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus (3^{rd} Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
2. H. Anton, I. Bivens and S. Davis, Calculus (7^{th} Edition), John Wiley and sons (Asia), Pt Ltd., Singapore, 2002.

I.2: Algebra I

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per student per week)

(1st& 2nd Weeks)
Polar representation of complex numbers, nth roots of unity, De Moivre's theorem for rational indices and its applications.
[1]: Chapter 2

(3rd, 4th& 5th Weeks)
Equivalence relations, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers , Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

[2]: Chapter 2 (Section 2.4),
Chapter 3,
Chapter 4 (Sections 4.1 upto 4.1.6, 4.2 upto 4.2.11, 4.4(till 4.4.8), 4.3.7 to 4.3.9)
Chapter 5 (5.1.1, 5.1.4).

(6th, 7th& 8th Weeks)
Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation Ax = b, solution sets of linear systems, applications of linear systems, linear independence.

(9th& 10th Weeks)
Introduction to linear transformations, matrix of a linear transformation, inverse of a matrix, characterizations of invertible matrices.

(11th& 12th Weeks)
Subspaces of R^n, dimension of subspaces of R^n and rank of a matrix, Eigen values, Eigen Vectors and Characteristic Equation of a matrix.

[3]: Chapter 1 (Sections 1.1-1.9),
Chapter 2 (Sections 2.1-2.3, 2.8-2.9),
Chapter 5 (Sections 5.1, 5.2).

REFERENCES:

1. TituAndreescu and DorinAndrica, Complex Numbers from A to …. Z, Birkhauser, 2006.
2. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory(3rd Edition), Pearson Education (Singapore) Pvt. Ltd., Indian Reprint, 2005**.**
3. David C. Lay, Linear Algebra and its Applications (3rd Edition), Pearson Education Asia, Indian Reprint, 2007.

II.1: Analysis I

Total marks:100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per student per week)

1st Week

Review of Algebraic and Order Properties of *R,* δ-neighborhood of a point in *R,* Idea of countable sets, uncountable sets and uncountability of *R.*
[1]: Chapter 1 (Section 1.3),
Chapter 2 (Sections 2.1, 2.2.7,2.2.8)

2nd& 3rd Week

Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Suprema and Infima, The Completeness Property of *R,* The Archimedean Property, Density of Rational (and Irrational) numbers in *R,* Intervals.
[1]: Chapter 2 (Sections 2.3, 2.4, 2.5.)

4th Week

Limit points of a set, Isolated points, Illustrations of Bolzano-Weierstrass theorem for sets.
[1]:Chapter 4(Section 4.1)

5th Week

Sequences, Bounded sequence, Convergent sequence, Limit of a sequence.
[1]: Chapter 3 (Section 3.1)

6th& 7th Week

Limit Theorems, Monotone Sequences, Monotone Convergence Theorem.
[1]: Chapter 3 (Sections 3.2, 3.3)

8th&9th Week

Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano Weierstrass Theorem for Sequences.Cauchy sequence, Cauchy's Convergence Criterion.
[1]: Chapter 3 (Sections 3.4, 3.5)

10th, 11th& 12th Week

Infinite series, convergence and divergence of infinite series, Cauchy Criterion,Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's nth root test, Integral test, Alternating series, Leibniz test, Absolute and Conditional convergence.
[2]: Chapter 6 (Section 6.2)

REFERENCES:

1. R.G. Bartle and D. R. Sherbert, *Introduction to Real Analysis* (3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
2. Gerald G. Bilodeau , Paul R. Thie, G.E. Keough, *An Introduction to Analysis,* Jones & Bartlett, Second Edition, 2010.
3. Brian S. *Thomson*, Andrew. M. *Bruckner*, and Judith B. *Bruckner*, *Elementary Real Analysis,* Prentice Hall, 2001.

II.2 :Differential Equations -I

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)

(1st , 2nd& 3rd Weeks)

Differential equations and mathematical models.General, particular, explicit, implicit and singular solutions of a differential equation.Exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equation and Bernoulli equations, special integrating factors and transformations.

[2] Chapter 1 (section 1.1, 1.2, 1.4),
[3] Chapter 2 (section 2.1-2.4)

(4th, 5th& 6th Weeks)

Introduction to compartmental model, exponential decay model, lake pollution model (case study of Lake Burley Griffin), drug Assimilation into the blood (case of a single cold pill, case of a course of cold pills), exponential growth of population, limited growth of population, limited growth with harvesting.

[1]Chapter 2 (section 2.1, 2.2, 2.5-2.7),
Chapter 3 (section 3.1-3.3)

(7th, 8th& 9th Weeks)

General solution of homogeneous equation of second order, principle of super position for homogeneous equation, Wronskian: its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters.

[2] Chapter 3 (Section 3.1-3.3, 3.5)

(10th, 11th& 12th Weeks)

Equilibrium points, Interpretation of the phase plane, predatory-prey model and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.
Ref.:
[1]Chapter 5 (Section 5.1-5.3, 5.7) Chapter 6 (Section 6.1-6.4)

REFERENCES:

[1]Belinda Barnes and Glenn R. Fulford, *Mathematical Modeling with case studies, A differential equation approach using Maple and Matlab,* 2nd edition, Taylor and Francis group, London and New York 2009.

[2] C.H. Edwards and D.E. Penny, *Differential equations and boundary value problems Computing and modelling,* Pearson Education India, 2005.

[3] S.L. Ross, *Differential equations* 3rd edition, John Wiley and Sons, India,2004.

Suggestive Reading

[1] Martha L Abell, James P Braselton, *Differential equations with MATHEMATICA,* 3rd Edition, Elsevier Academic Press, 2004.

LIST OF PRACTICALS
(*MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE*)

1. Plotting of second order solution family of differential equation.
2. Plotting of third order solution family of differential equation.
3. Growth model (exponential case only).
4. Decay model (exponential case only).
5. Any two of the following:
   a) Lake pollution model (with constant/seasonal flow and pollution concentration).
   b) Case of single cold pill and a course of cold pills.
   c) Limited growth of population (with and without harvesting).
6. Any two of the following:
   a) Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   b) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   c) Battle model (basic battle model, jungle warfare, long range weapons).
7. Plotting of recursive sequences.
8. Find a value of N that will make the following inequality holds for all $n > N$:

   (i) $|\sqrt[n]{0.5} - 1| < 10^{-3}$, (ii) $|\sqrt[n]{n} - 1| < 10^{-3}$,

(ii) $(0.9)^n < 10^{-3}$, (iv) $2^n/n! < 10^{-7}$ etc.

9. Study the convergence of sequences through plotting.

10. Verify Bolzano Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.

11. Study the convergence/divergence of infinite series by plotting their sequences of partial sum.

12. Cauchy's root test by plotting nth roots.

13. Ratio test by plotting the ratio of nth and n+1th term.

14. For the following sequences $< a_n >$, given $\in > 0$ and $p \in N$,

Find $m \in N$ such that $(i) \mid a_{m+p} - a_m \mid < \in, \quad (ii) \mid a_{2m+p} - a_{2m} \mid < \in$

(a) $a_n = \frac{n+1}{n}$ for $(\in = \frac{1}{2^k}, p = 10^j, j = 1,2,3,4....), k = 0,1,2,5...$

(b) $a_n = \frac{1}{n}$ for $(\in = \frac{1}{2^k}, p = 10^j, j = 1,2,3,4....), k = 0,1,2,5...$

(c) $a_n = 1 + \frac{1}{2!} + ... + \frac{1}{\mathrm{n!}}$ for $(\in = \frac{1}{2^k}, p = 10^j, j = 1,2,3...), k = 0,1,2,..$

(d) $a_n = \frac{(-1)^n}{n}$ for $(\in = \frac{1}{2^k}, p = 10^j, j = 1,2,3,4....), k = 0,1,2,5...$

(e) $a_n = 1 - \frac{1}{2} + \frac{1}{3} - .... + \frac{(-1)^{\mathrm{n}}}{\mathrm{n}}$ for $(\in = \frac{1}{2^k}, p = 10^j, j = 1,2,3...), k = 0,1,2,..$

15. For the following series $\sum a_n$, calculate

$(i) \left| \frac{a_{n+1}}{a_n} \right|, (ii) \left| a_n \right|^{\frac{1}{n}}$, for $n = 10^j, j = 1,2,3,.....$, and identify the

convergent series:

(a) $a_n = \left(\frac{1}{n}\right)^{1/n}$ (b) $a_n = \frac{1}{n}$ (c) $a_n = \frac{1}{n^2}$ (d) $a_n = \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

(e) $a_n = \frac{n!}{n^n}$ (f) $a_n = \frac{n^3 + 5}{3^n + 2}$ (g) $a_n = \frac{1}{n^2 + n}$ (h) $a_n = \frac{1}{\sqrt{n+1}}$

(j) $a_n = \cos n$ (k) $a_n = \frac{1}{n \log n}$ (l) $a_n = \frac{1}{n(\log n)^2}$

III.1:Analysis- II(Real Functions)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per student per week)

1st Week
Limits of functions (ε-δ approach), sequential criterion for limits, divergence criteria.
[1] Chapter 4, Section 4.1

2nd Week
Limit theorems, one sided limits.
[1]Chapter 4, Section 4.2, Section 4.3 (4.3.1 to 4.3.4)

3rd Week
Infinite limits & limits at infinity.
[1]Chapter 4, Section 4.3 (4.3.5 to 4.3.16)

4th Week
Continuous functions, sequential criterion for continuity & discontinuity.
[1] Chapter 5, Section 5.1

5th Week
Algebra of continuous functions.
[1]Chapter 5**,** Section 5.2

6th Week
Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem.
[2]Art.18.1, 18.2, 18.3, 18.5, 18.6

7th Week
Uniform continuity, non-uniform continuity criteria, uniform continuity theorem.
[1]Chapter 5, Section 5.4 **(**5.4.1 to 5.4.3)

8th Week
Differentiability of a function at a point & in an interval, Carathéodory's theorem, algebra of differentiable functions.

[1]Chapter 6, Section 6.1 **(**6.1.1 to 6.1.7)

9th& 10th Week

Relative extrema, interior extremumtheorem.Rolle's theorem, Mean value theorem, intermediate value property of derivatives - Darboux's theorem. Applications of mean value theorem to inequalities & approximation of polynomials Taylor's theorem to inequalities.

[1] Chapter 6, Section 6.2 (6.2.1 to 6.2.7, 6.2.11, 6.2.12)

11th&12th Week

Cauchy's mean value theorem.Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder, application of Taylor's theorem to convex functions, relative extrema. Taylor's series &Maclaurin's series expansions of exponential & trigonometric functions, $\ln(1+x)$, $\frac{1}{(ax+b)}$ &$(1+x)^n$.

[1] Chapter 6, Section 6.3 (6.3.2) Section 6.4 (6.4.1 to 6.4.6)

REFERENCES:

1. **R. G. Bartle & D.R. Sherbert,** Introduction to Real Analysis, John Wiley & Sons (2003)
2. **K. A. Ross,** Elementary Analysis: The Theory of Calculus, Springer (2004).

Suggestive Readings

1. **A. Mattuck,** Introduction to Analysis, Prentice Hall (1999).
2. **S.R.Ghorpade&B.V.Limaye**, A Course in Calculus and Real Analysis – Springer (2006).

III.2:Numerical Methods

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)
Use of Scientific Calculators is allowed.

Algorithms, Convergence, Errors: (I & II weeks)
Relative, Absolute, Round off, Truncation.
[1] 1.1, 1.2
[2] 1.3 (pg 7-8)

Transcendental and Polynomial equations: (III & IV weeks)
Bisection method, Newton's method, Secant method.Rate of convergence of these methods.
[2] 2.2, 2.3, 2.5, 2.10

System of linear algebraic equations (V & VI weeks)
Gaussian Elimination and Gauss Jordan methods.Gauss Jacobi method, Gauss Seidel method and their convergence analysis.
[2] 3.1, 3.2, 3.4

Interpolation: (VII & VIII weeks)
Lagrange and Newton's methods.Error bounds.
Finite difference operators. Gregory forward and backward difference interpolation.
[2] 4.2, 4.3, 4.4

Numerical Integration: (IX & X weeks)
Trapezoidal rule, Simpson's rule, Simpsons $3/8^{th}$ rule, Boole's Rule.
Midpoint rule, Composite Trapezoidal rule, Composite Simpson's rule.
[1] 6.4, 6.5 (pg 467- 482)

Ordinary Differential Equations: XI & XII weeks)
Euler's method.Runge-Kutta methods of orders two and four.
[1] 7.2 (pg 558 - 562), 7.4

REFERENCES:

1. Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New age International Publisher, India, 6th edition, 2007.

SUGGESTED READING:

1. C. F. Gerald and P. O. Wheatley, App;ied Numerical Analysis, Pearson Education, India,7th edition,(2008)
2. Uri M. Ascher and Chen Greif : A first course in Numerical Methods, PHI Learning Private Limited, (2013).
3. John H. Mathews and Kurtis D. Fink: Numerical Methods using Matlab, 4th Edition, PHI Learning Private Limited(2012).

LIST OF PRACTICALS

Practical / Lab work to be performed on a computer: Use of computer aided software (CAS), for example Matlab / Mathematica / Maple / Maxima etc., for developing the following Numerical programs:

(i) Bisection Method
(ii) Secant Method
(iii) Newton Raphson Method
(iv) Gauss-Jacobi Method
(v) Gauss-Seidel Method
(vi) Lagrange Interpolation
(vii) Newton Interpolation
(viii) Composite Simpson's Rule
(ix) Composite Trapezoidal Rule
(x) Euler's Method
(xi) RungeKutta Method of order 2 and 4.

Illustrations of the following :

1. Let $f(x)$ be any function and L be any number. For given a and $\in > 0$, find a $\delta > 0$ such that for all x satisfying $0 < |x - a| < \delta$, the inequality $|f(x) - L| < \in$ holds. For examples:

(i) $f(x) = x + 1, L = 5, a = 4, \in = 0.01$

(ii) $f(x) = \sqrt{x+1}, L = 1, a = 0, \in = 0.1$

(iii) $f(x)=x^2, L=4, a=-2, \in=0.5$

(iv) $f(x)=\frac{1}{x}, L=-1, a=-1, \in=0.1$

2. Discuss the limit of the following functions when x tends to 0: $\pm\frac{1}{x}, \sin\frac{1}{x}, \cos\frac{1}{x}, x\sin\frac{1}{x}, x\cos\frac{1}{x}, x^2\sin\frac{1}{x}, \frac{1}{x^n}(n\in N), |x|, [x], \frac{1}{x}\sin x$.

3. Discuss the limit of the following functions when x tends to infinity : $e^{\frac{1}{x}}, e^{-\frac{1}{x}}, \sin\frac{1}{x}, \frac{1}{x}e^{x}, \frac{1}{x}e^{-x}, \frac{x}{1+x}, x^2\sin\frac{1}{x}, \frac{ax+b}{cx^2+dx+e}(\mathrm{a}\neq 0\neq \mathrm{c})$

4. Discuss the continuity of the functions at $x=0$ in practical 2.

5. Illustrate the geometric meaning of Rolle's theorem of the following functions on the given interval :

(i) x^3-4x on $[-2, 2]$, (ii) $(x-3)^4(x-5)^3$ on $[3, 5]$ etc.

6. Illustrate the geometric meaning of Lagrange's mean value theorem of the following functions on the given interval:

(i) $\log x$ on $\left[1/2, 2\right]$, (ii) $\mathrm{x}(x-1)(x-2)$ on $\left[0, 1/2\right]$,

(iii) $2x^2-7x+10$ on $[2, 5]$ etc.

7. For the following functions and given $\in>0$, if exists, find $\delta>0$ such that $|f(x_1)-f(x_2)|<\in$ whenever $|x_1-x_2|<\delta$, and discuss uniform continuity of the functions:

(i) $f(x)=\frac{1}{x}$ on $[0, 5], \in=\frac{1}{2^j}, j=0,1,2,3,...$

(ii) $f(x)=\frac{1}{x}$ on $(0, 5], \in=\frac{1}{2^j}, j=0,1,2,3,...$

(iii) $f(x)=x^2$ on [-1, 1], $\in=\frac{1}{2^j}, j=0,1,2,3,...$

(iv) $f(x)=\sin x$ on $(0, \infty), \in=\frac{1}{2^j}, j=0,1,2,3,...$

(v) $f(x) = \sin x^2$ on $(0, \infty), \in = \frac{1}{2^j}, j = 0,1,2,3,...$

(vi) $f(x) = \frac{x}{1+x^2}$ on R, $\in = \frac{1}{2^j}, j = 0,1,2,3,...$

(vii) $f(x) = x^3$ on $[0, 1], \in = \frac{1}{2^j}, j = 0,1,2,3,...$

8. Verification of Maximum –Minimum theorem, boundedness theorem & intermediate value theorem for various functions and the failure of the conclusion in case of any of the hypothesis is weakened.
9. Locating points of relative & absolute extremum for different functions
10. Relation of monotonicity & derivatives along with verification of first derivative test.
11. Taylor's series - visualization by creating graphs:
    a. Verification of simple inequalities
    b. Taylor's Polynomials – approximated up to certain degrees
    c. Convergence of Taylor's series
    d. Non-existence of Taylor series for certain functions
    e. Convexity of the curves

Note: For any of the CAS Matlab / Mathematica / Maple / Maxima etc., the following should be introduced to the students.
Data types-simple data types, floating data types
arithmetic operators and operator precedence,
variables and constant declarations, expressions, input/output,
relational operators, logical operators and logical expressions,
control statements and loop statements, arrays.

IV.1:Calculus-II (Multivariate Calculus)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)
Use of Scientific Calculators is allowed.

1st Week:

Functions of several variables, limit and continuity of functions of two variables

[1]: Chapter 11 (Sections 11.1(Pages 541-543), 11.2)

2nd Week:

Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability.

[1]: Chapter 11 (Section 11.3, 11.4)

3rd Week:

Chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes

[1]: Chapter 11 (Sections 11.5, 11.6)

4th Week:

Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, Definition of vector field, divergence and curl

[1]: Chapter 11(Sections 11.7 (Pages 598-605), 11.8(Pages 610-614))
Chapter 13 (Pages 684-689)

5th Week:

Double integration over rectangular region, double integration over nonrectangular region

[1]: Chapter 12 (Sections 12.1, 12.2)

6th Week:

Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions

[1]: Chapter 12 (Sections 12.3, 12.4 (Pages 652-655))

7th Week:

Volume by triple integrals, cylindrical and spherical co-ordinates.

[1]: Chapter 12 (Sections 12.4(Pages 656-660), 12.5)

8th Week:

Change of variables in double integrals and triple integrals.

[1]: Chapter 12 (Section 12.6)

9th Week:

Line integrals, Applications of line integrals: Mass and Work.

[1]: Chapter 13 (Section 13.2)

10th Week:

Fundamental theorem for line integrals, conservative vector fields, independence of path.

[1]: Chapter 13 (Section 13.3)

11th Week:

Green's theorem, surface integrals, integrals over parametrically defined surfaces.

[1]: Chapter 13 (Sections 13.4(Page 712–716), 13.5(Page 723–726, 729-730))

12th Week:

Stokes' theorem, The Divergence theorem.

[1]: Chapter 13 (Section 13.6 (Page 733–737), 13.7 (Page 742–745)

REFERENCES:

1. M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus (3rd Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.

SUGGESTED READING:

2. E. Marsden, A. J. Tromba and A. Weinstein, *Basic multivariable calculus*, Springer (SIE), Indian reprint, 2005.

IV.2: PROBABILITY AND STATISTICS

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)
Use of Calculators is allowed

1st Week

Sample space, Probability axioms, Real random variables (discrete and continuous).

2nd Week

Cumulative distribution function, Probability mass/density functions, Mathematical expectation.

3rd Week

Moments, Moment generating function, Characteristic function.

4th Week

Discrete distributions: uniform, binomial, Poisson, Geometric, Negative Binomial distributions.

5th Week

Continuous distributions: Uniform, Normal, Exponential, Gamma distributions
[1]Chapter 1 (Section 1.1, .3, 1.5-1.9)
[2]Chapter 5 (Section 5.1-5.5,5.7), Chapter 6 (Sections 6.2-6.3,6.5-6.6)

6th Week

Joint cumulative distribution Function and its properties, Joint probability density functions – marginal and conditional distributions

7th Week

Expectation of a function of two random variables, Conditional expectations, Independent random variables, Covariance and correlation coefficient.

8th Week

Bivariate normal distribution, Joint moment generating function.

9th Week

Linear regression for two variables, The rank correlation coefficient.

[1]Chapter 2 (Section 2.1, 2.3-2.5)
[2]Chapter 4 (Exercise 4.47), Chapter 6 (Sections 6.7), Chapter 14 (Section 14.1, 14.2), Chapter 16 (Section 16.7)

10thWeek

Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers.

11thWeek

Central Limit Theorem for independent and identically distributed random variables with finite variance.

12thWeek

Markov Chains, Chapman – Kolmogorov Equations, Classification of states.
[2] Chapter 4 (Section 4.4)
[3] Chapter 2 (Sections 2.7), Chapter 4 (Section 4.1-4.3)

REFERENCES:

1. Robert V. Hogg, Joseph W. Mc Kean and Allen T. Craig. Introduction of Mathematical Statistics, Pearson Education, Asia, 2007
2. Irvin Miller and Marylees Miller, John E. Freund's Mathematical Statistics with Applications (7thEdn), Pearson Education, Asia, 2006.
3. Sheldon Ross, Introduction to Probability Models (9th Edition), Academic Press, Indian Reprint, 2007

PRACTICALS /LAB WORK TO BE PERFORMED ON A COMPUTER USING SPSS/EXCEL/Mathematica etc.

1. Calculation of
   (i) Arithmetic Mean, geometric Mean, harmonic Mean
   (ii) Variance

2. Fitting of
   (i) Binomial Distribution
   (ii) Poisson Distribution
   (iii) Negative Binomial Distribution
   (iv) Normal Distribution

3. Calculation of
   (i) Correlation coefficients
   (ii) Rank correlation

4. Fitting of polynomials
5. Regression curves
6. Illustrations of Central limit theorem.

7. Draw the following surfaces and find level curves at the given heights:

   (i) $f(x,y)=10-x^2-y^2;\quad z=1,z=6,z=9$,

   (ii) $f(x,y)=x^2+y^2;\quad z=1,z=6,z=9$, (iii) $f(x,y)=x^3-y;\quad z=1,z=6$,

   (iv) $f(x,y)=x^2+\frac{y^2}{4};\quad z=1,z=5,z=8$

   (v) $f(x,y)=4x^2+y^2;\quad z=0,z=1,z=3,z=5$,

   (vi) $f(x,y)=2-x-y;\quad z=-6,z=-4,z=-2,z=0,z=2,z=4,z=6$.

8. Draw the following surfaces and discuss whether limit exits or not as (x,y) approaches to the given points. Find the limit, if it exists:

   (i) $f(x,y)=\frac{x+y}{x-y};\quad (x,y)\to(0,0)$ and $(x,y)\to(1,3)$,

   (ii) $f(x,y)=\frac{x-y}{\sqrt{x^2+y^2}};\quad (x,y)\to(0,0)$ and $(x,y)\to(2,1)$,

   (iii) $f(x,y)=(x+y)e^{xy};\quad (x,y)\to(1,1)$ and $(x,y)\to(1,0)$,

(iv) $f(x,y)=e^{xy}$; $(x,y)\to(0,0)$ and $(x,y)\to(1,0)$,

(v) $f(x,y)=\dfrac{x-y^2}{x^2+y^2}$; $(x,y)\to(0,0)$,

(vi) $f(x,y)=\dfrac{x^2+y}{x^2+y^2}$; $(x,y)\to(0,0)$,

(vii) $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2}$; $(x,y)\to(0,0)$ and $(x,y)\to(2,1)$,

(viii) $f(x,y)=\dfrac{x^2-y}{x+y}$; $(x,y)\to(0,0)$ and $(x,y)\to(1,-1)$.

9. Draw the tangent plane to the following surfaces at the given point:

(i) $f(x,y)=\sqrt{x^2+y^2}$ at $(3,1,\sqrt{10})$, (ii) $f(x,y)=10-x^2-y^2$ at $(2,2,2)$,

(iii) $x^2+y^2+z^2=9$ at $(3,0,0)$,

(iv) $z=\arctan x$ at $(1,\sqrt{3},\dfrac{\pi}{3})$ and $(2,2,\dfrac{\pi}{4})$,

(v) $z=\log|x+y^2|$ at $(-3,-2,0)$.

10. Use an incremental approximation to estimate the following functions at the given point and compare it with calculated value:

(i) $f(x,y)=3x^4+2y^4$ at $(1.01, 2.03)$, (ii) $f(x,y)=x^5-2y^3$ at $(0.98, 1.03)$,

(iii) $f(x,y)=e^{xy}$ at $(1.01, 0.98)$,

(iv) $f(x,y)=e^{x^2y^2}$ at $(1.01, 0.98)$.

11. Find critical points and identify relative maxima, relative minima or saddle points to the following surfaces, if it exist:

(i) $z=x^2+y^2$, (ii) $z=y^2-x^2$, (iii) $z=1-x^2-y^2$, (iv) $z=x^2y^4$.

12. Draw the following regions **D** and check whether these regions are of **Type I** or **Type II**:

(i) $D=\{(x,y)\,|\,0\le x\le 2, 1\le y\le e^x\}$,

(ii) $D=\{(x,y)\,|\,\log y\le x\le 2, 1\le y\le e^2\}$,

(iii) $D=\{(x,y)\,|\,0\le x\le 1, x\le y\le 1\}$,

(iv) The region D is bounded by $y=x^2-2$ and the line $y=x,$

(v) $D=\{(x,y)\,|\,0\le x\le 1, x^3\le y\le 1\}$,

(vi) $D=\{(x,y)\,|\,0\le x\le y^3, 0\le y\le 1\}$,

(vii) $D=\left\{(x,y)\,|\,0\le x\le \frac{\pi}{4}, \cos x\le y\le \sin x\right\}$.

V.1:Algebra-II (Group Theory -I)

Total marks: 100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)

(1st& 2nd Weeks)
Symmetries of a square, Dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups.

(3rd Week)
Subgroups and examples of subgroups, centralizer, normalizer, center of a group, product of two subgroups.

(4th& 5thWeeks)
Properties of cyclic groups, classification of subgroups of cyclic groups.
[1]: Chapters 1, Chapter 2, Chapter 3 (including Exercise 20 on page 66 and Exercise 2 on page 86), Chapter 4.

(6th, 7th& 8th Weeks)
Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.

(9th& 10th Weeks)
External direct product of a finite number of groups, normal subgroups, factor groups, Cauchy's theorem for finite abelian groups.
[1]: Chapter 5 (till end of Theorem 5.7), Chapter 7 (till end of Theorem 7.2, including Exercises 6 and 7 on page 168), Chapter 8 (till the end of Example 2), Chapter 9 (till end of Example 10, Theorem 9.3 and 9.5).

(11th& 12th Weeks)

Group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms, First, Second and Third isomorphism theorems.
[1]: Chapter 6 (till end of Theorem 6.2), Chapter 10.

REFERENCES:

1. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), NarosaPublishing House, New Delhi, 1999.(IX Edition 2010)

SUGGESTED READING:

1. Joseph J. Rotman, An Introduction to the Theory of Groups (4^{th} Edition), Springer Verlag, 1995.

V .2: ANALYSIS- III (RIEMANN INTEGRATION & SERIES OF FUNCTIONS)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)

1st Week

Riemann integration; inequalities of upper and lower sums; Riemann conditions of integrability.
[1] Chapter 6 (Art. 32.1 to 32.7)

2nd& 3rd Week

Riemann sum and definition of Riemann integral through Riemann sums; equivalence of two definitions; Riemann integrability of monotone and continuous functions, Properties of the Riemann integral; definition and integrability of piecewise continuous and monotone functions. Intermediate Value theorem for Integrals; Fundamental theorems of Calculus.
[1] Chapter 6 (Art. 32.8, 32.9, 33.1, 33.2, 33.3, 33.4 to 33.8, 33.9, 34.1, 34.3)

4th Week.

Improper integrals; Convergence of Beta and Gamma functions.
[3] Chapter 7 (Art. 7.8)

5th, 6th& 7th Week

Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions.
[2] Chapter 8, Section 8.1, Section 8.2 (8.2.1 – 8.2.2), Theorem 8.2.3, Theorem 8.2.4 and Theorem 8.2.5

8th& 9th Week

Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test
[2] Chapter 9, Section 9.4 (9.4.1 to 9.4.6)

10^{th}, 11^{th}& 12^{th} Week

Limit superior and Limit inferior. Power series, radius of convergence, Cauchy Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass Approximation Theorem.
[1] Chapter 4, Art. 26 (26.1 to 26.6), Theorem 27.5

REFERENCES:

1. K.A. Ross, Elementary Analysis: The Theory of Calculus, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.
2. R.G. Bartle D.R. Sherbert, Introduction to Real Analysis (3^{rd} edition), John Wiley and Sons (Asia) Pvt. Ltd.., Singapore, 2002.
3. Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett (Student Edition),2011.

V.3:DIFFERENTIAL EQUATIONS-II (PDE & SYSTEM OF ODE)

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)

Section – 1 (1st , 2nd & 3rd Weeks)

Partial Differential Equations – Basic concepts and Definitions, Mathematical Problems. First-Order Equations: Classification, Construction and Geometrical Interpretation. Method of Characteristics for obtaining General Solution of Quasi Linear Equations. Canonical Forms of First-order Linear Equations. Method of Separation of Variables for solving first – order partial differential equations.
[1]: Chapter 1:1.2, 1.3
[1]: Chapter 2: 2.1-2.7

Section – 2 (3-weeks)

Derivation of heat equation, Wave equation and Laplace equation.Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order Linear Equations to canonical forms.
[1]: Chapter 3: 3.1, 3.2, 3.5, 3.6
[1]: Chapter 4: 4.1-4.5

Section – 3 (3-weeks)

The Cauchy problem, the Cauchy-Kowaleewskaya theorem, Cauchy problem of an infinite string. Initial Boundary Value Problems, Semi-Infinite String with a fixed end, Semi-Infinite String with a Free end, Equations with non-homogeneous boundary conditions, Non-Homogeneous Wave Equation. Method of separation of variables – Solving the Vibrating String Problem, Solving the Heat Conduction problem
[1]: Chapter 5: 5.1 – 5.5, 5.7
[1]: Chapter 7: 7.1, 7.2, 7.3, 7.5

Section -4 (3-weeks)

Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients, Basic Theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two Equations in two unknown functions, The method of successive approximations, the Euler method, the modified Euler method, The Runge- Kutta method.

[2]: Chapter 7: 7.1, 7.3, 7.4,
[2]:Chapter 8: 8.3, 8.4-A,B,C,D

REFERENCES:

[1]: TynMyint-U and LkenathDebnath, *Linear Partial Differential Equations for Scientists and Engineers,* 4th edition, Springer, Indian reprint, 2006
[2]: S. L. Ross, *Differential equations,* 3rd Edition, John Wiley and Sons, India,2004

Suggested Reading

[1]: Martha L Abell, James P Braselton, *Differential equations with MATHEMATICA,* 3rd Edition, Elsevier Academic Press, 2004.

LIST OF PRACTICALS
(*MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE*)

1. Solution of Cauchy problem for first order PDE.

2. Plotting the characteristics for the first order PDE.

3. Plot the integral surfaces of a given first order PDE with initial data.

4. Solution of wave equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ for any 2 of the following associated conditions:

(a) $u(x,0) = \emptyset(x), u_t(x,0) = \psi(x),\ x \in \mathbb{R},\ t > 0.$

(b) $u(x,0) = \emptyset(x),\ u_t(x,0) = \psi(x),\ u(0,t) = 0,\ x \in (0,\infty),\ t > 0.$

(c) $u(x,0) = \emptyset(x),\ u_t(x,0) = \psi(x),\ u_x(0,t) = 0,\ x \in (0,\infty),\ t > 0.$

(d) $u(x,0) = \emptyset(x), u_t(x,0) = \psi(x),\ u(0,t) = 0, u(l,t) = 0,\ 0 < x < l,\ t > 0.$

5. Solution of one-Dimensional heat equation $u_t = \kappa\, u_{xx}$, for a homogeneous rod of length l.

That is - solve the IBVP:

$$u_t = \kappa u_{xx}, \qquad 0 < x < l, \qquad t > 0,$$

$$u(0,t) = 0, u(l,t) = 0, t \geq 0,$$

$$u(x,0) = f(x), 0 \leq x \leq l$$

6. Solving systems of ordinary differential equations.

7. Approximating solution to Initial Value Problems using any of the following approximate methods:

(a) The Euler Method

(b) The Modified Euler Method.

(c) The Runge-Kutta Method.

Comparison between exact and approximate results for any representative differential equation.

8. Draw the following sequence of functions on given the interval and discuss the pointwise convergence:

(i) $f_n(x) = x^n$ for $x \in \mathrm{R}$, (ii) $f_n(x) = \frac{x}{n}$ for $x \in \mathrm{R}$,

(iii) $f_n(x) = \frac{x^2 + nx}{n}$ for $x \in \mathrm{R}$, (iv) $f(x) = \frac{\sin nx + n}{n}$ for $x \in \mathrm{R}$

(v) $f_n(x) = \frac{x}{x+n}$ for $x \in \mathrm{R}, x \geq 0$, (vi) $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \in \mathrm{R}$,

(Vii) $f_n(x) = \frac{nx}{1+nx}$ for $x \in \mathrm{R}, x \geq 0$,

(viii) $f_n(x) = \frac{x^n}{1+x^n}$ for $x \in \mathrm{R}, x \geq 0$

9. Discuss the uniform convergence of sequence of functions above.

VI .1: ALGEBRA -III (RINGS AND LINEAR ALGEBRA- I)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)

(1st& 2nd Weeks)
Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring.

(3rd& 4th Weeks)
Ideals, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals.

(5th& 6th Weeks)
Ring homomorphisms, properties of ring homomorphisms, Isomorphism theorems I, II and III, field of quotients.
[2]: Chapter 12, Chapter 13, Chapter 14, Chapter 15.

(7th& 8th Weeks)
Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces.

(9th& 10thWeeks)
Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations.

(11th& 12th Weeks)
Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix.
[1]: Chapter 1 (Sections 1.2-1.6, Exercise 29, 33, 34, 35), Chapter 2 (Sections 2.1-2.5).

REFERENCES:

1. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra* (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.

2. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi, 1999.

SUGGESTED READING:

1. S Lang, Introduction to Linear Algebra (2nd edition), Springer, 2005
2. Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007
3. S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999.
4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra 2nd Ed., Prentice-Hall Of India Pvt. Limited, 1971

VI.2:ANALYSIS-IV (METRIC SPACES)

Total marks: 100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)

(1st Week)
Metric spaces: definition and examples.
[1] Chapter1, Section 1.2 (1.2.1 to 1.2.6).

(2nd Week)
Sequences in metric spaces, Cauchy sequences.
[1] Chapter1, Section 1.3, Section 1.4 (1.4.1 to 1.4.4)

(3rd Week)
Complete Metric Spaces.
[1] Chapter1, Section 1.4 (1.4.5 to 1.4.14 (ii)).

(4th Week)
Open and closed balls, neighbourhood, open set, interior of a set
[1] Chapter2, Section 2.1 (2.1.1 to 2.1.16)

(5th& 6thWeeks)
Limit point of a set, closed set, diameter of a set, Cantor's Theorem.
[1] Chapter2, Section 2.1 (2.1.17 to 2.1.44)

(7th Week)
Subspaces, dense sets, separable spaces.
[1] Chapter2, Section 2.2, Section 2.3 (2.3.12 to 2.3.16)

(8th Week)
Continuous mappings, sequential criterion and other characterizations of continuity.
[1] Chapter3, Section 3.1

(9th Week)
Uniform continuity
[1] Chapter3, Section3.4 (3.4.1 to 3.4.8)

(10th Week)

Homeomorphism, Contraction mappings, Banach Fixed point Theorem.
[1] Chapter3, Section 3.5 (3.5.1 to 3.5.7(iv)), Section 3.7 (3.7.1 to 3.7.5)

(11th Week)

Connectedness, connected subsets of **R,** connectedness and continuous mappings.
[1]Chapter4, Section 4.1 (4.1.1 to 4.1.12)

(12th Week)

Compactness, compactness and boundedness, continuous functions on compact spaces.
[1] Chapter5, Section 5.1 (5.1.1 to 5.1.6), Section 5.3 (5.3.1 to 5.3.11)

REFERENCES:

[1] SatishShirali&Harikishan L. Vasudeva, Metric Spaces, Springer Verlag London(2006) (First Indian Reprint 2009)

SUGGESTED READINGS:

[1] S. Kumaresan, Topology of Metric Spaces, Narosa Publishing House, Second Edition 2011.
[2] G. F. Simmons, Introduction to Topology and Modern Analysis, Mcgraw-Hill, Edition 2004.

VI.3:CALCULUS OF VARIATIONS AND LINEAR PROGRAMMING

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)

(1st&2nd Week)
Functionals, Some simple variational problems, The variation of a functional, A necessary condition for an extremum, The simplest variational problem, Euler's equation, A simple variable end point problem.
[1]: Chapter 1 (Sections 1, 3, 4 and 6).

(3rd&4th Week)
Introduction to linear programming problem, Graphical method of solution, Basic feasible solutions, Linear programming and Convexity.
[2]: Chapter 2 (Section 2.2), Chapter 3 (Sections 3.1, 3.2 and 3.9).

(5th& 6th Week)
Introduction to the simplex method, Theory of the simplex method, Optimality and Unboundedness.
[2]: Chapter 3 (Sections 3.3 and 3.4).

(7th& 8th Week)
The simplex tableau and examples, Artificial variables.
[2]: Chapter 3 (Sections 3.5 and 3.6).

(9th&10th Week)
Introduction to duality, Formulation of the dual problem, Primal-dual relationship, The duality theorem, The complementary slackness theorem.
[2]: Chapter 4 (Sections 4.1, 4.2, 4.4 and 4.5).

(11th&12th Week)
Transportation problem and its mathematical formulation, Northwest-corner method, Least-cost method and Vogel approximation method for determination of starting basic solution, Algorithm for solving transportation problem, Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.
[3]: Chapter 5 (Sections 5.1, 5.3 and 5.4)

PRACTICAL/LAB WORK TO BE PERFORMED ON A COMPUTER: (MODELLING OF THE FOLLOWING PROBLEMS USING EXCEL SOLVER/LINGO/MATHEMATICA, ETC.)

(i) Formulating and solving linear programming models on a spreadsheet using excel solver.
[2]: Appendix E and Chapter 3 (Examples 3.10.1 and 3.10.2).
[4]: Chapter 3 (Section 3.5 with Exercises 3.5-2 to 3.5-5)

(ii) Finding solution by solving its dual using excel solver and giving an interpretation of the dual.
[2]: Chapter 4 (Examples 4.3.1 and 4.4.2)

(iii) Using the excel solver table to find allowable range for each objective function coefficient, and the allowable range for each right-hand side.
[4]: Chapter 6 (Exercises 6.8-1 to 6.8-5).

(iv) Formulating and solving transportation and assignment models on a spreadsheet using solver.
[4]: Chapter 8 (**CASE 8.1**: Shipping Wood to Market, **CASE 8.3**: Project Pickings).

From the Metric space paper, exercises similar to those given below:

1. Calculate d(x,y) for the following metrics

   (i) X=**R,** d(x,y)=|x-y|, (ii) X=$\mathbf{R^3}$, d(x,y)= $(\sum(x_i-y_i)^2)^{1/2}$

   x: 0, 1, π, e x: (0,1,-1), (1,2,π), (2,-3,5)
   y: 1, 2, ½, √2 y: (1, 2, .5), (e,2,4), (-2,-3,5)

   (iii) X=C[0,1], d(f,g)= sup |f(x)-g(x)|

   f(x): x^2 , sin x, tan x
   g(x): x , |x|, cos x

2. Draw open balls of the above metrics with centre and radius of your choice.
3. Find the fixed points for the following functions

   f(x)=x^2 , g(x)= sin x, h(x)= cos x in X=[-1, 1],

   f(x,y)= (sin x, cos y), g(x,y) = (x^2 , y^2) in X= { (x,y): $x^2+y^2\leq 1$},

under the Euclidean metrics on **R** and $\mathbf{R}^2$ respectively.

4. Determine the compactness and connectedness by drawing sets in R^2.

REFERENCES:

[1]. I. M. Gelfand and S. V. Fomin, *Calculus of Variations*, Dover Publications, Inc., New York, 2000.

[2]. Paul R. Thie and Gerard E. Keough, *An Introduction to Linear Programming and Game Theory*, Third Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2008.

[3]. Hamdy A. Taha, *Operations Research*: *An Introduction*, Ninth Edition, Prentice Hall, 2011.

[4]. Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research*, Ninth Edition, McGraw-Hill, Inc., New York, 2010.

SUGGESTED READING:

[1].R. Weinstock, *Calculus of Variations*, Dover Publications, Inc. New York, 1974.

[2].M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, *Problems and Exercises in the Calculus of Variations*, Mir Publishers, Moscow, 1975.

[3].Mokhtar S. Bazaraa, John J. Jarvis and Hanif D, Sherali, *Linear Programming and Network Flows*, Fourth Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2010.

[4].G. Hadley, *Linear Programming*, Narosa Publishing House, New Delhi, 2002.

VII.1:ALGEBRA-IV (GROUP THEORY -II)

Total marks:100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)

(1st, 2nd& 3rd Weeks)

Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Characteristic subgroups, Commutator subgroup and its properties.

[1]: Chapter 6, Chapter 9 (Theorem 9.4), Exersices1-4 on page168, Exercises 52, 58 on page Pg 188.

(4th, 5th& 6th Weeks)

Properties of external direct products, the group of units modulo n as an external direct product, internal direct products, Fundamental Theorem of finite abelian groups.

[1]: Chapter 8, Chapter 9 (Section on internal direct products), Chapter 11.

(7th, 8th& 9th Weeks)

Group actions, stabilizers and kernels, permutation representation associated with a given group action, Applications of group actions: Generalized Cayley's theorem, Index theorem.

(10th, 11th& 12th Weeks)

Groups acting on themselves by conjugation, class equation and consequences, conjugacy in S_n, *p*-groups, Sylow's theorems and consequences, Cauchy's theorem, Simplicity of A_n for $n \geq 5$, non-simplicity tests.

[2]: Chapter 1 (Section 1.7), Chapter 2 (Section 2.2), Chapter 4 (Section 4.1-4.3, 4.5-4.6).
[1]: Chapter 25.

REFERENCES:

1. Joseph A. Gallian, Contemporary Abstract Algebra (4th Ed.), Narosa Publishing House, 1999.
2. David S. Dummit and Richard M. Foote, Abstract Algebra (3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd, Singapore, 2004

VII.2: DIFFERENTIAL EQUATIONS-III (TRANSFORMS & BOUNDARY VALUE PROBLEMS)

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)

(1st, 2nd& 3rd Weeks)

Introduction: power series solution methods, series solutions near ordinary point, series solution about regular singular point. Special functions: Bessel's equation and function, Legendre's equation and function.
[1] Chapter 8: 8.1-8.3
[2]Chapter 8: 8.6,8.9

(4th, 5th& 6th Weeks)

Sturm Theory: Self-Adjoint equation of the second order, Abel's formula, Sturm separation and comparison theorems, method of Separation of variables: The Laplace and beam equations, non-homogeneous equation.
[3] Chapter 11: 11.8
[2] Chapter 7: 7.7, 7.8

(7th , 8th& 9th Weeks)

Boundary Value Problem: Introduction, maximum and minimum Principal, Uniqueness and continuity theorem, Dirichlet problem for a circle, Neumann Problem for a circle.
[2] Chapter 9: 9.1-9.4, 9.6

(10th,11th& 12th Weeks)

Integral transform-introduction, Fourier transforms, properties of Fourier transforms, convolution Theorem of Fourier transforms, Laplace transforms, properties of Laplace transforms, convolution theorem of Laplace transforms
[2] Chapter 12: 12.1-12.4, 12.8-12.10

REFERENCE :

1.C.H. Edwards and D.E. Penny, *Differential Equations and Boundary value Problems Computing and Modelling,* Pearson Education India, 2005.

2.TynMyint-U, LokenathDebnath, *Linear Partial Differential Equations for Scientists and Engineers,* 4th edition, Springer, Indian reprint, 2006.
3. S.L. Ross, *Differential Equations,* 3rd edition, John Wiley and Sons, India, 2004.

Suggested Reading

1. Martha L Abell, James P Braselton, *Differential equations with MATHEMATICA,* 3rd Edition, Elsevier Academic Press, 2004.

LIST OF PRACTICALS FOR DE-III (*MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE*)

1. Plotting of [0,1]. Legendre polynomial for n=1 to 5 in the interval Verifying graphically that all roots of $P_n(x)$ lie in the interval [0,1].
2. Automatic computation of coefficients in the series solution near ordinary points.
3. Plotting of the Bessel's function of first kind of order 0 to 3.
4. Automating the Frobeniusseries method.
5. Use of Laplace transforms to plot the solutions of
   a. Massspring systems with and without external forces and draw inferences.
   b. LCR circuits with applied voltage. Plot the graph of current and charge w.r.t. time and draw inferences.
6. Find Fourier series of different functions. Plot the graphs for *n* = 1-6. Draw inferences for the solutions as *n* tends to infinity.
7. Finding and plotting Laplace transforms of various functions and solving a differential equation using Laplace transform.
8. Finding and plotting Fourier transforms of various functions, and solving any representative partial differential equation using Fourier transform.
9. Finding and plotting the convolution of 2 functions and verify the Convolution theorem of the Fourier Transform/Laplace Transform.
10. Solve the Laplace equation describing the steady state temperature distribution in a thin rectangular slab, the problem being written as:

$$\nabla^2 u = 0, \quad 0 < x < a, 0 < y < b,$$
$$u(x,0) = f(x), \quad 0 \le x \le a,$$
$$u(x,b) = 0, \quad 0 \le x \le a,$$

$u_x(0,y) = 0,\ u_x(a,y) = 0,$

for prescribed values of a and b, and given function f(x).

VII.3 Research Methodology

Study and Practice of Modern Mathematics (3-weeks)

Studying mathematics; writing homework assignments and problem solving; writing mathematics; mathematical research; presenting mathematics; professional steps.

[1] Chapter 1-6

LaTeX and HTML (4-weeks)

Elements of LaTeX; Hands-on-training of LaTex; graphics in LaTeX; PSTricks; Beamer presentation; HTML, creating simple web pages, images and links, design of web pages.

[1] Chapter 9-11, 15

Computer Algebra Systems and Related Softwares (5-weeks)

Use of Mathematica, Maple, and Maxima as calculator, in computing functions, in making graphs; MATLAB/Octave for exploring linear algebra and to plot curve and surfaces; the statistical software R: R as a calculator, explore data and relations, testing hypotheses, generate table values and simulate data, plotting.
[1] Chapter 12-14
References:

[1] Martin J. Erickson and Donald Bindner, A Student's Guide to the Study, Practice, and Tools of Modern Mathematics, CRC Press, Boca Raton, FL, 2011.

[2] L. Lamport. LATEX: A Document Preparation System, User's Guide and ReferenceManual. Addison-Wesley, New York, second edition, 1994.

List of Practical for Research Methodology in Mathematics (2L per week per student)

Twelve practical should be done by each student. The teacher can assign practical from the following list. These are exercises from [1].

1. Exercise 4-9 (pages 73-76) (2 practical)
2. Exercise 1,3,5-9,11-13 (pages 88-90) (2 practical)
3. Exercise in page 95 (1 practical)
4. Exercise 5, 10, 11 (1 practical)
5. Exercise 1-4, 8-12 (pages 125-128) (4 practical)
6. Exercise 4, 6-10 (pages 146-147) (1 practical)
7. Exercise 1-4 (page 161) (1 practical

VIII.1: ANALYSIS-V (COMPLEX ANALYSIS)

Total marks: 150(Theory: 75, Practical: 50, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
Practical (4 periods per week per student),

(1st& 2nd Week):
Limits, Limits involving the point at infinity, continuity.
Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.
[1]: Chapter 1 (Section 11), Chapter 2 (Section 12, 13) Chapter 2 (Sections 15, 16, 17, 18, 19, 20, 21, 22)

(3rd, 4th & 5th Week):

Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions.
[1]: Chapter 2 (Sections 24, 25), Chapter 3 (Sections 29, 30, 34),Chapter 4 (Section 37, 38)

(6th Week):
Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals.
[1]: Chapter 4 (Section 39, 40, 41, 43)

(7th Week):
Antiderivatives, proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula.
[1]: Chapter 4 (Sections 44, 45, 46, 50)

(8th Week):
An extension of Cauchy integral formula, consequences of Cauchy integral formula, Liouville's theorem and the fundamental theorem of algebra.
[1]: Chapter 4 (Sections 51, 52, 53)

(9th Week):
Convergence of sequences and series, Taylor series and its examples.
[1]: Chapter 5 (Sections 55, 56, 57, 58, 59)

(10th Week):
Laurent series and its examples, absolute and uniform convergence of power series, uniqueness of series representations of power series.
[1]: Chapter 5 (Sections 60, 62, 63, 66)

(11th Week):
Isolated singular points, residues, Cauchy's residue theorem, residue at infinity.
[1]: Chapter 6 (Sections 68, 69, 70, 71)

(12th Week):
Types of isolated singular points, residues at poles and its examples, definite integrals involving sines and cosines.
[1]: Chapter 6 (Sections 72, 73, 74), Chapter 7 (Section 85).

REFERENCES:

1. James Ward Brown and Ruel V. Churchill, Complex Variables and Applications (Eighth Edition), McGraw – Hill International Edition, 2009.

SUGGESTED READING:

1. Joseph Bak and Donald J. Newman, *Complex analysis* (2nd Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.

LAB WORK TO BE PERFORMED ON A COMPUTER (MODELING OF THE FOLLOWING PROBLEMS USING MATLAB/ MATHEMATICA/ MAPLE ETC.)

1. Declaring a complex number and graphical representation.

   e.g. $Z_1 = 3 + 4i$, $Z_2 = 4 - 7i$

2. Program to discuss the algebra of complex numbers.

   e.g., if $Z_1 = 3 + 4i$, $Z_2 = 4 - 7i$, then find $Z_1 + Z_2$, $Z_1 - Z_2$, $Z_1 * Z_2$, and Z_1 / Z_2

3. To find conjugate, modulus and phase angle of an array of complex numbers.

e.g., Z = [2+ 3i 4-2i 6+11i 2-5i]

4. To compute the integral over a straight line path between the two specified end points.

e. g., $\int_C \sin Z\,dz$, where C is the straight line path from -1+ i to 2 - i.

5. To perform contour integration.

e.g., (i) $\int_C (Z^2 - 2Z + 1)dz$, where C is the Contour given by $x = y^2 + 1;\ -2 \le y \le 2$.

(ii) $\int_C (Z^3 + 2Z^2 + 1)dz$, where C is the contour given by $x^2 + y^2 = 1$, which can be

parameterized by x = cos (t), y = sin (t) for $0 \le y \le 2\pi$.

6. To plot the complex functions and analyze the graph .

e.g., (i) f(z) = Z

(ii) $f(z)=Z^3$

(iii) $f(z) = (Z^4-1)^{1/4}$

(iv) $f(z) = \overline{z},\ f(z) = iz,\ f(z) = z^2,\ f(z) = e^z$ etc.

7. To perform the Taylor series expansion of a given function f(z) around a given point z.

The number of terms that should be used in the Taylor series expansion is given for each function. Hence plot the magnitude of the function and magnitude of its Taylors series expansion.

e.g., (i) f(z) = exp(z) around z = 0, n =40.

(ii) $f(z)=\exp(z^2)$ around z = 0, n = 160.

8. To determines how many terms should be used in the Taylor series expansion of a given function f(z) around z = 0 for a specific value of z to get a percentage error of less than 5 %.

e.g., For f(z) = exp(z) around z =0, execute and determine the number of necessary terms to get a percentage error of less than 5 % for the following values of z:

(i) z = 30 + 30 i

(ii) $z = 10 + 10\sqrt{3}\ i$

9. To perform Laurents series expansion of a given function f(z) around a given point z.

e.g., (i) $f(z) = (\sin z - 1)/z^4$ around z = 0

(ii) $f(z) = \cot(z)/z^4$ around z = 0.

10. To compute the poles and corresponding residues of complex functions.

e.g., $f(z) = \dfrac{z+1}{z^3 - 2z + 2}$

12. To perform Conformal Mapping and Bilinear Transformations.

VIII.2:ALGEBRA- V (RINGS AND LINEAR ALGEBRA -II)

Total marks: 100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)

(1st,2nd , 3rd& 4th Weeks)

Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in Z[x]**.**
Divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.
[1]: Chapter 16, Chapter 17, Chapter 18.

(5th, 6th, 7th& 8th Weeks)

Dual spaces, dual basis, double dual, transpose of a linear transformationand its matrix in the dual basis, annihilators, Eigenspaces of a linearoperator, diagonalizability, invariant subspaces and Cayley-Hamiltontheorem, the minimal polynomial for a linear operator.
[2]: Chapter 2 (Section 2.6), Chapter 5 (Sections 5.1-5.2, 5.4), Chapter 7(Section 7.3).

(9th, 10th, 11th& 12th Weeks)

Inner product spaces and norms, Gram-Schmidt orthogonalisation process, orthogonal complements, Bessel's inequality, the adjoint of a linear operator, Least Squares Approximation, minimal solutions to systems of linear equations, Normal and self-adjoint operators, Orthogonal projections and Spectral theorem.
[2]: Chapter 6 (Sections 6.1-6.4, 6.6).

REFERENCES:

1. Joseph A. Gallian, Contemporary Abstract Algebra (4th Ed.), Narosa Publishing House, 1999.
2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra* (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.

SUGGESTED READING:

(Linear Algebra)

1. S Lang, Introduction to Linear Algebra (2nd edition), Springer, 2005
2. Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007
3. S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999.
4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra 2nd Ed., Prentice-Hall Of India Pvt. Limited, 1971

(Ring theory and group theory)

1. John B.Fraleigh, A first course in Abstract Algebra, 7th Edition, Pearson Education India, 2003.
2. Herstein, Topics in Algebra (2nd edition), John Wiley & Sons, 2006
3. Michael Artin, Algebra (2nd edition), Pearson Prentice Hall, 2011
4. Robinson, Derek John Scott., An introduction to abstract algebra, Hindustan book agency, 2010.

VIII.3:RESEARCH : DISSERTATION

DISCIPLINE COURSES - II

MATHEMATICS

The course structure of Discipline-II in MATHEMATICS is a blend of pure and applied papers. This study of this course would be beneficial to students belonging to variety of disciplines such as economics, physics, engineering, management sciences, computer sciences, operational research and natural sciences. The course has been designed to help one pursue a masters degree in mathematics. The first two courses on ***Calculus*** and ***Linear Algebra*** are central to both pure and applied mathematics. The next two courses with practical components are of applied nature. The course on ***Differential Equations and Mathematical Modeling*** deals with modeling of many physical, technical, or biological process in the form of differential equations and their solution procedures. The course on ***Numerical Methods*** involves the design and analysis of techniques to give approximate but accurate solutions of hard problems using iterative methods. The last two courses on ***Real Analysis*** and ***Abstract Algebra*** provides an introduction to the two branches of pure mathematics in a rigorous and definite form.

DC-II STRUCTURE

Semester	No. of Papers	Papers	Page No.
3	1	**Calculus**	**71-72**
4	1	**Linear Algebra**	**73-74**
5	1	**Differential Equations & Mathematical Modeling**	**75-77**
6	1	**Numerical Methods**	**78-80**
7	1	**Real Analysis**	**81-83**
8	1	**Abstract Algebra**	**84-85**

Out of 14 weeks in a semester, the courses are designed for 12 weeks and two weeks are given for revision/problem sessions/difficulties of students etc.

CALCULUS (SEMESTER III)

Total marks: 100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),

(1st Week)

ε-δ Definition of limit of a function, One sided limit, Limits at infinity, Horizontal asymptotes
Sections 2.3, 2.4[1]

(2nd Week)

Infinite limits, Vertical asymptotes, Linearization, Differential of a function
Sections 2.5, 3.8 [1]

(3rd Week)

Concavity, Points of inflection, Curve sketching
Sections 4.4 [1]

(4th Week)

Indeterminate forms: L'Hôpital's rule, Volumes by slicing, Volumes of solids of revolution by the disk method
Sections 4.6, 6.1(Pages 396 to 402) [1]

(5th Week)

Volumes of solids of revolution by the washer method, Volume by cylindrical shells, Length of plane curves
Sections 6.1 (Pages 403 to 405), 6.2, 6.3 [1]

(6th Week)

Area of surface of revolution, Improper integration: Type I and II, Tests of convergence and divergence
Sections 6.5 (Pages 436 to 442), 8.8 [1]

(7th Week)

Polar coordinates, Graphing in polar coordinates
Sections 10.5, 10.6 [1]

(8th Week)

Vector valued functions: Limit, Continuity, Derivatives, Integrals, Arc length, Unit tangent vector
Sections 13.1, 13.3 [1]

(9thWeek)

Curvature, Unit normal vector, Torsion, Unit binormal vector, Functions of several variables: Graph, Level curves
Sections 13.4, 13.5, 14.1 [1]

(10thWeek)

Limit, Continuity, Partial derivatives, Differentiability
Sections 14.2, 14.3 [1]

(11thWeek)

Chain Rule, Directional derivatives, Gradient
Sections 14.4, 14.5 [1]

(12thWeek)

Tangent plane and normal line, Extreme values, Saddle points
Section 14.6 (Pages 1015 to 1017), 14.7 [1]

REFERENCES:

[1] G. B. Thomas and R. L. Finney, Calculus, Pearson Education, 11/e (2012)

SUGGESTED READING:

[2] H. Anton, I. Bivens and S. Davis, Calculus, John Wiley and Sons Inc., 7/e (2011)

Note: The emphasis is on learning of methods/techniques of calculus and on the application of these methods for solving variety of problems.

LINEAR ALGEBRA (SEMESTER IV)

Total marks: 100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),

(1st Week)

Fundamental operation with vectors in Euclidean space $\mathbf{R}^n$, Linear combination of vectors, Dot product and their properties, Cauchy–Schwarz inequality, Triangle inequality, Projection vectors
Sections 1.1, 1.2 [1]

(2nd Week)

Some elementary results on vector in $\mathbf{R}^n$, Matrices: Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank
Sections 1.3 (Pages 31 to 40), 2.2 (Pages 98 to 104), 2.3 (Pages 110 to 114, Statement of Theorem 2.3) [1]

(3rdWeek)

Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices
Sections 2.3 (Pages 114 to 121, Statements of Lemma 2.7 and Theorem 2.8), 3.4 [1]

(4th Week)

Definition and examples of vector space, Some elementary properties of vector spaces, Subspace
Sections 4.1, 4.2 (Statement of Theorem 4.3) [1]

(5thWeek)

Span of a set, A spanning set for an eigenspace, Linear independence and linear dependence of vectors
Sections 4.3 (Statement of Theorem 4.5), 4.4 [1]

(6thWeek)

Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets, Application of rank: Homogenous and nonhomogenous systems of equations
Section 4.5 (Statements of Lemma 4.11 and Theorem 4.13) [1]
Section 6.6 (Pages 289 to 291) [2]

(7thWeek)

Coordinates of a vector in ordered basis, Transition matrix, Linear transformations: Definition and examples, Elementary properties

Section 6.7 (Statement of Theorem 6.15) [2]

Section 5.1 (Statements of Theorem 5.2 and Theorem 5.3) [1]

(8thWeek)

The matrix of a linear transformation, Linear operator and Similarity

Section 5.2 (Statements of Theorem 5.5 and Theorem 5.6) [1]

(9thWeek)

Application: Computer graphics- Fundamental movements in a plane, Homogenous coordinates, Composition of movements

Sections 8.8 [1]

(10th Week)

Kernel and range of a linear transformation, Dimension theorem

Sections 5.3 [1]

(11thWeek)

One to one and onto linear transformations, Invertible linear transformations, Isomorphism: Isomorphic vector spaces (to $\mathbf{R}^n$)

Sections 5.4, 5.5 (Pages 356 to 361, Statements of Theorem 5.14 and Theorem 5.15) [1]

(12th Week)

Orthogonal and orthonormal vectors, Orthogonal and orthonormal bases, Orthogonal complement, Projection theorem (Statement only), Orthogonal projection onto a subspace, Application: Least square solutions for inconsistent systems

Section 6.1 (Pages 397 to 400, Statement of Theorem 6.3), 6.2 (Pages 412 to 418, 422, Statement of Theorem 6.12), 8.12 (Pages 570 to 573, Statement of Theorem 8.12) [1]

REFERENCES:

[1] S. Andrilli and D. Hecker, Elementary Linear Algebra, Academic Press, 4/e (2012)

[2] B. Kolman and D.R. Hill, Introductory Linear Algebra with Applications, Pearson Education, 7/e (2003)

DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELING (SEMESTER V)

Total marks: 150(Theory: 75, Practical: 50, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
Practical (4 periods per week per student),

(1st Week)

First order ordinary differential equations: Basic concepts and ideas, Modeling: Exponential growth and decay, Direction field, Separable equations, Modeling: Radiocarbon dating, Mixing problem

Sections 1.1, 1.2, 1.3 (Pages 12 to 14) [1]

(2ndWeek)

Modeling: Newton's law of cooling, Exact differential equations, Integrating factors, Bernoulli equations, Modeling: Hormone level in blood, Logistic equation

Sections 1.3 (Pages 14 to 15 and Page 17), 1.4, 1.5 (Pages 29 to 31) [1]

(3rdWeek)

Orthogonal trajectories of curves, Existence and uniqueness of solutions, Second order differential equations: Homogenous linear equations of second order

Sections 1.6, 1.7, 2.1 [1]

(4thWeek)

Second order homogenous equations with constant coefficients, Differential operator, Euler-Cauchy equation

Sections 2.2, 2.3, 2.5 [1]

(5thWeek)

Existence and uniqueness theory: Wronskian, Nonhomogenous ordinary differential equations, Solution by undetermined coefficients

Sections 2.6, 2.7 [1]

(6thWeek)

Solution by variation of parameters, Higher order homogenous equations with constant coefficients, System of differential equations, Modeling: Mixing problem involving two tanks

Sections 2.10, 3.2, 4.1(Pages130 to 132) [1]

(7thWeek)

System of differential equations: Conversion of n^{th} order ODEs to a system, Basic concepts and ideas, Homogenous system with constant coefficients, Phase plane, Critical points

Sections 4.1 (Pages 134, 135), 4.2, 4.3 [1]

(8thWeek)

Criteria for critical Points and stability, Qualitative methods for nonlinear systems: Linearization of nonlinear systems, Lotka–Volterra population model

Sections 4.4, 4.5 (Pages 151 to 155) [1]

(9thWeek)

Power series method: Theory of power series methods, Legendre's equation, Legendre polynomial

Sections 5.1, 5.2, 5.3 [1]

(10thWeek)

Partial differential equations: Basic Concepts and definitions, Mathematical problems, First order equations: Classification, Construction, Geometrical interpretation, Method of characteristics

Sections 2.1, 2.2, 2.3, 2.4 [2]

(11thWeek)

General solutions of first order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations

Sections 2.6, 2.7 [2]

(12th Week)

Classification of second order partial differential equations, Reduction to canonical forms, Second order partial differential equationswith constant coefficients, General solutions

Sections 4.1, 4.2, 4.3, 4.4 [2]

PRACTICALS

1. To determine whether a given number is prime or composite.
2. To find the sum of digits of a number and decide its divisibility.
3. To compute the roots of a quadratic equation.
4. To Linear Sort a given set of numbers.
5. To compute higher degree polynomials using Horner's method.
6. To plot the direction field of first order differential equation.
7. To find the solution and plot the growth and decay model (both exponential and logistic).
8. To find the solution and plot the Lotka–Volterra model.
9. To find the solution of Cauchy problem for first order partial differential equations.
10. To plot the integral surfaces of a given first order partial differential equations with initial data.

Note: Programming is to be done in any one of Computer Algebra Systems: MATLAB/MATHEMATICA/MAPLE.

REFERENCES:

[1] Erwin Kreyszig, Advanced Engineering Mathematics, John Wiley & Sons, Inc., 9/e, (2006)

[2] TynMyint–U and LokenathDebnath; Linear Partial Differential Equations for Scientists and Engineers, Springer, Indian Reprint (2009)

FOR PRACTICALS/SUGGESTED READING:

[3] Randall J. Swift and Stephen A. Wirkus, A Course in Ordinary Differential Equations, Chapman & Hall /CRC (2007)

[4] Ioannis P. Stavroulakis and Stepan A. Tersian, Partial Differential Equations, An Introduction with Mathematica and Maple, World Scientific, 2/e (2004)

Numerical Methods (Semester VI)

Total marks: 150(Theory: 75, Practical: 50, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
Practical (4 periods per week per student),
Use of Scientific Calculators is allowed.

(1st Week)

Floating point representation and computer arithmetic, Significant digits, Errors: Round-off error, Local truncation error, Global truncation error, Order of a method, Convergence and terminal conditions, Efficient computations

Sections 1.2.3, 1.3 (Pages 16 to 25 and Page 30) [1]

(2nd Week)

Bisection method, Secant method, Regula–Falsi method

Sections 2.1, 2.2 [1]

(3rd Week)

Newton–Raphson method, Newton's method for solving nonlinear systems

Sections 2.3, 7.1.1(Pages 266 to 270) [1]

(4th Week)

Gauss elimination method (with row pivoting) and Gauss–Jordan method, Gauss Thomas method for tridiagonal systems

Sections 3.1 (Pages 110 to 115), 3.2, 3.3 [1]

(5th Week)

Iterative methods: Jacobi and Gauss-Seidel iterative methods

Sections 6.1 (Pages 223 to 231), 6.2 [1]

(6th Week)

Interpolation: Lagrange's form and Newton's form

Sections 8.1 (Pages 290 to 299 and Pages 304 to 305) [1]

(7th Week)

Finite difference operators, Gregory Newton forward and backward differences Interpolation

Sections 4.3, 4.4 (Pages 235 to 236) [2]

(8thWeek)

Piecewise polynomial interpolation: Linear interpolation, Cubic spline interpolation (only method), Numerical differentiation: First derivatives and second order derivatives, Richardson extrapolation

Sections 16.1, 16.2 (Pages 361 to 363), 16.4 [3]
Section 11.1 (Pages 426 to 430 and Pages 432 to 433) [1]

(9thWeek)

Numerical integration: Trapezoid rule, Simpson's rule (only method), Newton–Cotes open formulas

Sections 11.2 (Pages 434 to 445) [1]

(10thWeek)

Extrapolation methods: Romberg integration, Gaussian quadrature, Ordinary differential equation: Euler's method

*Section*s 11.2.4, 11.3.1 [1]
Section 20.2 (Pages 481 to 485) [3]

(11thWeek)

Modified Euler's methods: Heun method and Mid-point method, Runge-Kutta second methods: Heun method without iteration, Mid-point method and Ralston's method

Sections 20.3, 20.4 (Pages 493 to 495) [3]

(12thWeek)

Classical 4th order Runge-Kutta method, Finite difference method for linear ODE

Section 20.4.2 [3]
Section 14.2.1 [1]

PRACTICALS

1. Find the roots of the equation by bisection method (Exercises P2.1 to P2.20 [1])
2. Find the roots of the equation by secant/Regula–Falsi method (Exercises P2.1 to P2.20 [1])
3. Find the roots of the equation by Newton's method (Exercises P2.11 to 2.29 [1])
4. Find the solution of a system of nonlinear equation using Newton's method (Exercises P7.1 to P7.15 [1])
5. Find the solution of tridiagonal system using Gauss Thomas method (Exercises P3.21 to P3.25, C3.1 to C3.3, A3.7, A3.8[1])
6. Find the solution of system of equations using Jacobi/Gauss-Seidel method (Exercises P6.1 to P6.18 [1])

7. Find the cubic spline interpolating function (Exercises C8.1 to C8.5 [1])
8. Evaluate the approximate value of finite integrals using Gaussian/Romberg integration (Exercises P11.6 to P11.20 [1])
9. Solve the initial value problem using Euler's method and compare the result with the exact solutions (Exercises P12.11 to P12.20 [1])
10. Solve the boundary value problem using finite difference method (Exercises P14.1 to P14.25 [1])

Note: Programming is to be done in any one of Computer Algebra Systems: MATLAB/MATHEMATICA/MAPLE.

REFERNCES:

[1] Laurence V. Fausett, Applied Numerical Analysis, Using MATLAB, Pearson, 2/e (2012)

[2] M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, 6/e (2012)

[3] Steven C Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientists, Tata McGraw Hill, 2/e (2010)

REAL ANALYSIS (SEMESTER VII)

Total marks: 100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),

(1st Week)

Algebraic and order properties of **R**, Positive integers, Statement of well ordering principle,Least upper bound, Greatest lower bound, Completeness property of **R**

Sections 1.3, 1.4 (Page 18), 1.5 (Pages 24 to 26, Statements of Theorem 1.5.10 and Corollary 1.5.11) [1]

(2nd Week)

Archimedean property, Denseness of the sets **Q** and $\mathbf{Q}^c$in**R**, Sequences, Convergence and divergence of sequence,

Sections 1.5 (From Theorem 1.5.12 onwards), 2.1 [1]

(3rd Week)

Limit theorems, Uniqueness of limit of a sequence, Bounded sequences, Algebra of limits of sequences, Monotonic sequence

Sections 2.2 (Statements of Theorem 2.2.5, Theorem 2.2.7 and Theorem 2.2.9), 2.3 [1]

(4thWeek)

Subsequences, Nested interval theorem (without proof), Bolzano Weierstrass theorem, Cluster points

Section 2.5 (Pages 55 to 60) [1]

(5thWeek)

Cauchy sequence, Infinite series: Sequence of partial sum, Convergence and divergence, Geometric series, Algebraic theory of series

Sections 2.6, 6.1(Pages 213 to 218) [1]

(6th Week)

Integral test (without proof), Comparison tests, Ratio test (without proof), Alternating series test, Absolute convergence and conditional convergence

Sections 6.1(Pages 219 to 222, Statements of Corollary 6.1.12) , 6.2 (Statement of Theorem 6.2.5) [1]

(7th Week)

Illustrations of Taylor's series &Maclaurin's series, Taylor's theorem (without proof)

Sections 6.4 [1]

(8thWeek)

Continuity, Removable discontinuity, Algebra of continuous functions, Sequential criterion of continuity

Section 3.4 (Statements of Theorem 3.4.11 and Theorem 3.4.16) [1]

(9thWeek)

Continuity at end points of [*a*, *b*], Intermediate value theorem, Boundedness of a function, Uniform continuity

Sections 3.5 (Statement of Theorem 3.5.10), 3.6 [1]

(10thWeek)

Local maximum, Local minimum, Rolle's theorem, Mean value theorem, Monotonic function

Sections 4.3 [1]

(11thWeek)

Inverse function, Sequences and series of functions: Pointwise and uniform convergence,

Section 4.4 (Statements of Theorem 4.4.2 and Theorem 4.4.4),
7.1(Pages 258 to265) [1]

(12th Week)

Weierstrass*M*-test (without proof), Consequences of uniform convergence *Section* 7.1(Pages 265 to 267), 7.2 (Statements of Theorem 7.2.3 and Corollary 7.2.11) [1]

REFERENCE:

[1] Gerald G. Bilodeau, Paul R. Thie, G.E. Keough, An Introduction to Analysis, Jones and Bartlet India Pvt. Ltd., 2/e (2010)

SUGGESTED READING:

[2] Kenneth A. Ross, Elementary Analysis: The Theory of Calculus, Springer (2007)

ABSTRACT ALGEBRA (SEMESTER VIII)

Total marks: 100(Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),

(1st, 2nd&3rd Weeks)

Partitions and equivalence relations, Congruence modulo *n*, Algebra on circles, Roots of unity, Binary operations, Isomorphic Binary Structures, Definition & Elementary Properties of Groups, Finite Groups and Group Tables, Subgroups

Sections 0, 1 (Pages 29 to 32), 2, 3, 4, 5 [1]

(4th, 5th&6thWeeks)

Cyclic groups, Cyclic subgroups, Elementary properties of cyclic groups, Subgroups of a finite cyclic group, Groups of permutations, Dihedral group D_n, Cayley theorem, Orbits, Cycles, Even and odd permutations, Alternating groups

Sections 6 (Statement of Theorem 6.10), *Sections* 8 (Statement of Lemma 8.15), 9 (Statement of Theorem 9.15) [1]

(7th&8thWeeks)

Cosets, Lagrange's theorem, Index of a subgroup, Homomorphism and their properties, Kernel, Isomorphism, Normal subgroup

Sections 10, 13 (Statement of Theorem 13.15) [1]

(9th&10th Weeks)

Definition and examples of rings, Basic properties of ring, Field, Division ring, Ring of polynomials, Ring of quaternions

Sections 18 (Pages 181 to 184, 186 to 188), 22 (Pages 212 to 215), 24 (Pages 238 to 240) [1]

(11th&12thWeeks)

Integral domain, Characteristic of a ring, Homomorphism, Isomorphism, Ideals

Section 18 (Pages 185, 186), 19, 26 (Pages 251 to 253, 255) [1]

REFERENCES:

[1] John B.Fraleigh, A First Course in Abstract Algebra, Pearson Education, 7/e (2007)

SUGGESTED BOOKS:

[2] J. Gilbert William, Modern Algebra with Applications, Wiley India Pvt. Ltd. (2008)

APPLIED COURSES

MATHEMATICS

The four papers in this category are aimed at providing an opportunity to learn the Mathematics behind the applied coursesnamely***C++*** **Programming**, **Mathematical Finance**, **Cryptography and Networks** and **Discrete Mathematics**. C++ language is used to create computer programs which have applications in the field of systems software, client applications, entertainment software and research. Mathematical finance paper provides the understanding of mathematics behind finance and has wide applications in financial and capital markets. Security is a challenging issue of data communications that the world faces today in the age of computers.Learningeffective encryption/decryption methods to enhance data security is the basis of the paper cryptography and networks**.** Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, software development, operations research. Courses are open to all students who have studied mathematics at least till 12^{th} level.

APPLIED COURSES STRUCTURE

Semester	No/ of Papers	Papers	Page No.
3	1	C++ Programming	87-90
4	1	Mathematical Finance	91-93
5	1	Cryptography and Network Systems	94-96
6	1	Discrete Mathematics	97-99

Out of 14 weeks in a semester the courses are designed for 12 weeks and two weeks are given for revision/problem sessions/difficulties of students etc.

SEMESTER III

"C++ PROGRAMMING LANGUAGE – INTRODUCTION"

Total marks: 75 Continuous evaluation by the teacher

Total:3 periods Practical/ hands-on experience/ project work

Introduction to structured programming: data types- simple data types, floating data types, character data types, string data types, arithmetic operators and operators precedence, variables and constant declarations, expressions, input using the extraction operator >> and cin, output using the insertion operator << and cout, preprocessor directives, increment(++) and decrement(--) operations, creating a C++ program, input/ output, relational operators, logical operators and logical expressions, if and if-else statement, switch and break statements.

[1]Chapter 2(pages 37-95), Chapter3(pages 96 -129), Chapter 4(pages 134-178)

"for", "while" and "do-while" loops and continue statement, nested control statement, value returning functions, value versus reference parameters, local and global variables, one dimensional array, two dimensional array, pointer data and pointer variables,.

[1] Chapter 5 (pages 181 - 236), Chapter 6, Chapter 7(pages 287- 304()Chapter 9 (pages 357 - 390), Chapter 14 (pages 594 - 600).

Reference:

[1]D. S. Malik:C++ ProgrammingLanguage, Edition-2009, Course Technology, Cengage Learning, India Edition

Suggested Readings:

[2]E. Balaguruswami: Object oriented programming with C++, fifth edition, Tata McGraw Hill Education Pvt. Ltd.

[3]Marshall Cline, Greg Lomow, Mike Girou: C++ FAQs, Second Edition, Pearson Education.

Note: Practical programs of the following (and similar) typeare suggestive.

1. Calculate the Sum of the series 1/1 + 1/2+ 1/3……..+1/N for any positive integer N.

2. Write a user defined function to find the absolute value of an integer and use it to evaluate the function $(-1)^n/|n|$, for n = -2,-1,0,1,2.
3. Calculate the factorial of any natural number.
4. Read floating numbers and compute two averages: the average of negative numbers and the average of positive numbers.

5. Write a program that prompts the user to input a positive integer. It should then output a message indicating whether the number is a prime number.
6. Write a program that prompts the user to input the value of a, b and c involved in the equation ax^2 + bx + c = 0 and outputs the type of the roots of the equation. Also the program should outputs all the roots of the equation.
7. write a program that generates random integer between 0 and 99. Given that first two Fibonacci numbers are 0 and 1, generate all Fibonacci numbers less than or equal to generated number.
8. Write a program that does the following:
    a. Prompts the user to input five decimal numbers.
    b. Prints the five decimal numbers.
    c. Converts each decimal number to the nearest integer.
    d. Adds these five integers.
    e. Prints the sum and average of them.
9. Write a program that uses ***while***loops to perform the following steps:
    a. Prompt the user to input two integers :firstNum and secondNum (firstNumshoul be less than secondNum).
    b. Output all odd and even numbers between firstNum and secondNum.
    c. Output the sum of all even numbers between firstNum and secondNum.
    d. Output the sum of the square of the odd numbers firstNum and secondNum.
    e. Output all uppercase letters corresponding to the numbers between firstNum and secondNum, if any.
10. Write a program that prompts the user to input five decimal numbers. The program should then add the five decimal numbers, convert the sum to the nearest integer, and print the result.
11. Write a program that prompts the user to enter the lengths of three sides of a triangle and then outputs a message indicating whether the triangle is a right triangleor a scalene triangle.
12. Write a value returning function ***smaller*** to determine the smallest number from a set of numbers. Use this function to determine the smallest number from a set of 10 numbers.

13. Write a function that takes as a parameter an integer (as a **long** value) and returns the number of odd, even, and zero digits. Also write a program to test your function.
14. Enter 100 integers into an array and short them in an ascending/ descending order and print the largest/ smallest integers.
15. Enter 10 integers into an array and then search for a particular integer in the array.
16. Multiplication/ Addition of two matrices using two dimensional arrays.
17. Using arrays, read the vectors of the following type: A = (1 2 3 4 5 6 7 8) , B = (0 2 3 4 0 1 5 6) and compute the product and addition of these vectors.
18. Read from a text file and write to a text file.
19. Write a program to create the following grid using for loops:

```
1  2  3  4  5
2  3  4  5  6
3  4  5  6  7
4  5  6  7  8
5  6  7  8  9
```

20. Write a function, *reverseDigit*, that takes an integer as a parameter and returns the number with its digits reversed. For example, the value of function reverseDigit(12345) is 54321 and the value of reverseDigit(-532) is -235.

Week Wise Distribution

[Ist week]

Simple data types, Floating data types, Character data types, String data types. Arithmetic operators and operators precedence

[IInd Week]

Variables and constant declarations. Expressions

[IIIrd Week]

Input using the extraction operator >> and cin.Output using the insertion operator << and cout.

[IVth Week]

Preprocessor directives, Increment(++) and decrement operations(--). Creating a C++ program

[Vth Week]

Input/ Output. Relational operators, Logical operators and logical expressions

[VIth Week]

if and if-else statement.

[VIIth Week]

switch and break statements. “for” statement.

[VIIIth Week]

“while” and “do-while” loops and continue statement

[IXth Week]

Nested control statement, Value returning functions.

[Xth Week]

Void functions, Value versus reference parameters.

[XIth Week]

One dimensional array

[XIIth Week]

Two dimensional array, Pointer data and pointer variables.

SEMESTER IV

MATHEMATICAL FINANCE

Total marks: 75 Continuous evaluation by the teacher

Total:3 periods Practical/ hands-on experience/ project work

Week-1

Interest rates, types of rates, measuring interest rates, zero rates, bond pricing

[1] Chapter 4 (4.1-4.4)

Week-2

Forward rate, duration, convexity

[1] Chapter 4 (4.6, 4.8-4.9)

Week-3

Exchange Traded Markets and OTC markets, Derivatives- Forward contracts, futures contract, options,

Types of traders, hedging, speculation, arbitrage

[1] Chapter 1 (1.1-1.9)

Week-4

No Arbitrage principle, short selling, forward price for an investment asset

[1] Chapter 5 (5.2-5.4)

Week-5

Types of Options, Option positions, Underlying assets, Factors affecting option prices

[1] Chapter 8 (8.1-8.3), Chapter 9 (9.1)

Week-6

Bounds on option prices, put-call parity, early exercise, effect of dividends

[1] Chapter 9 (9.2-9.7)

Week-7

Binomial option pricing model, Risk neutral Valuation (for European and American options on assets following binomial tree model)

[1] Chapter 11(11.1-11.5)

Week-8

Lognormal property of stock prices, distribution of rate of return, expected return, volatility, estimating volatility from historical data

[1] Chapter 13 (13.1-13.4)

Week-10

Extension of risk neutral valuation to assets following GBM (without proof), Black Scholes formula for European options

[1] Chapter 13 (13.7-13.8)

Week-11

Hedging parameters (the Greeks: delta, gamma, theta, rho and Vega)

[1] Chapter 17 (17.1-17.9)

Week-12

Trading strategies Involving options

[1] Chapter 10 (except box spreads, calendar spreads and diagonal spreads)

Week-12

Swaps, mechanics of interest rate swaps, comparative advantage argument, valuation of interest rate swaps, currency swaps, valuation of currency swaps

[1] Chapter 7 (7.1-7.4, 7.7-7.9)

Reference

1. J.C. Hull and S. Basu, *Options, Futures and Other Derivatives* (7th Edition), Pearson Education, Delhi, 2010

For further reading

2. David G Luenberger, *Investment Science*, Oxford University Press, Delhi, 1998

Practical/Lab work using Excel

1. Computing simple, nominal and effective rates. Conversion and comparison.
2. Computing price and yield of a bond.
3. Bond duration and convexity.
4. Comparing spot and forward rates.
5. Estimating volatility from historical data of stock prices.
6. Simulating a binomial price path.
7. Computing price of simple European call and put options when the underlying follows binomial model (using Monte Carlo simulation).
8. Simulating lognormal price path.
9. Computing price of simple European call and put options when the underlying follows lognormal model (using Monte Carlo simulation).
10. Implementing the Black-Scholes formulae in a spreadsheet.
11. Computing Greeks for European call and put options.
12. Valuation of a swap.

References for Practicals:

1. Simon Benninga, *Financial Modeling*, 3rd Edition, MIT Press, Cambridge, Massachusetts, 2008
2. Alastair Day, *Mastering Financial Mathematics in Microsoft Excel: A Practical Guide for Business Calculations*, 2nd Edition, Prentice Hall, 2010

SEMESTER V

CRYPTOGRAPHY AND NETWORK SECURITY

Total marks: 75 Continuous evaluation by the teacher

Total:3 periods Practical/ hands-on experience/ project work

Definition of a cryptosystem, Symmetric cipher model, Classical encryption techniques- Substitution and transposition ciphers, caesar cipher, Playfair cipher.

[1] 2.1-2.3

Block cipher Principles, Shannon theory of diffusion and confusion, Data encryption standard (DES).

[1] 3.1, 3.2, 3.3.

Polynomial and modular arithmetic, Introduction to finite field of the form GF(p) and GF(2^n), Fermat theorem and Euler's theorem(statement only), Chinese Remainder theorem, Discrete logarithm.

[1] 4.2, 4.3, 4.5, 4.6, 4.7, 8.2, 8.4, 8.5

Advanced Encryption Standard(AES), Stream ciphers . Introduction to public key cryptography, RSA algorithm and security of RSA, Introduction to elliptic curve cryptography.

[1] 5.2-5.5(tables 5.5, 5.6 excluded),7.4, 9.1, 9.2, 10.3, 10.4

Information/Computer Security: Basic security objectives, security attacks, security services, Network security model,

[1]1.1, 1.3, 1.4, 1.6

Cryptographic Hash functions, Secure Hash algorithm, SHA-3.

[1] 11.1, 11.5, 11.6

Digital signature, Elgamal signature, Digital signature standards, Digital signature algorithm

[1] 13.1, 13.2, 13.4

E-mail security: Pretty Good Privacy (PGP)

[1] 18.1 Page 592-596(Confidentiality excluded)

REFERENCE:

[1]William Stallings, "Cryptography and Network Security", Principles and Practise, Fifth Edition, Pearson Education, 2012.

SUGGESTED READING:

[1] Douglas R. Stinson, "Cryptography theory and practice", CRC Press, Third edition, 2005.

Week Wise Distribution

[Ist week]

Definition of a cryptosystem, Symmetric cipher model, Classical encryption techniques- Substitution and transposition ciphers, caesar cipher, Playfair cipher.

[IInd Week]

Block cipher Principles, Shannon theory of diffusion and confusion.

[IIIrd Week]

Data encryption standard (DES). Polynomial and modular arithmetic, Introduction to finite field of the form GF(p) and GF(2^n).

[IVth Week]

Fermat theorem and Euler's theorem(statement only), Chinese Remainder theorem, Discrete logarithm.

[Vth Week]

Advanced Encryption Standard(AES), Stream ciphers .

[VIth Week]

Introduction to public key cryptography, RSA algorithm and security of RSA.

[VIIth Week]

Introduction to elliptic curve cryptography.

[VIIIth Week]

Basic security objectives, security attacks, security services, Network security model.

[IXth Week]

Cryptographic Hash functions, Secure Hash algorithm, SHA-3.

[Xth Week]

Digital signature, Elgamal signature.

[XIth Week]

Digital signature standards, Digital signature algorithm.

[XIIth Week]

Pretty Good Privacy (PGP).

SEMESTER VI

DISCRETE MATHEMATICS

Total marks: 75 Continuous evaluation by the teacher

Total:3 periods Practical/ hands-on experience/ project work

Week 1:

Definition, examples and properties of posets, maps between posets.

Week 2:

Algebraic lattice, lattice as a poset, duality principal, sublattice ,Hasse diagram.

Week 3:

Products and homomorphisms of lattices, Distributive lattice, complemented lattice.

References for first 3 weeks:

[1] Chapter 1 (Section 1, Section 2 - upto Theorem 2.9)

Week 4:

Boolean Algebra, Boolean polynomial, CN form, DN form.

Week 5:

Simplification of Boolean polynomials, Karnaugh diagram.

Week 6:

Switching Circuits and its applications.

References for Week 4, 5 and 6:

[1] Chapter 1 (Section 3 – Upto example 3.9), (Section 4- upto Definition 4.8, Finding CN form and DN form as in example 4.16 and 4.17, Section 6 (page 48 to 50), section 7, section 8- example 8.1)

Week 7:

Graphs, subgraph, complete graph, bipartite graph, degree sequence,Euler's theorem for sum of degrees of all vertices.

Week 8:

Eulerian circuit, Seven bridge problem, Hamiltonian cycle, Adjacency matrix.

Week 9:

Dijkstra's shortest path algorithm (improved version).

References for week 7, 8 and 9:

[2] Chapter 9 (9.2), Chapter 10 (10.1, 10.2 – 10.2.1, 10.2.2 and application to Gray codes, 10.3, 10.4)

Week 10:

Chinese postman problem, Digraphs.

Week 11:

Definitions and examples of tree and spanning tree , Kruskal's algorithm to find the minimum spanning tree.

Week 12:

Planar graphs, coloring of a graph and chromatic number.

References for Week 10, 11 and 12:

[2] Chapter 11 (11.1, 11.2- upto11.2.4)

Chapter 12 (12.1, 12.2, 12.3)

Chapter 13 (13.1- upto 13.1.6, 13.2)

References:

[1] Applied Abstract Algebra (2nd Edition) Rudolf Lidl, Gunter Pilz, Springer, 1997.

[2] Discrete Mathematics with Graph Theory (3rd Edition) Edgar G. Goodaire, Michael M. Parmenter, Pearson, 2005.

Suggested Reading:

1. Discrete Mathematics and its applications with combinatorics and graph theory by Kenneth H Rosen (7th Edition), Tata McGrawHill Education private Limited, 2011.