Your Roll Number: ....

Department of Mathematics, University of Delhi
M.Sc. Mathematics Examinations, December 2025
Part II Semester III
MMath18- 301(i) : Algebraic Topology (UPC 223502301)

Time: 3 hours   Maximum Marks: 70

Instructions: • Answer **five** questions. • **Question 1** is compulsory. Attempt any **four** questions from **Q2** to **Q7**. • Each question carries 14 marks.

(1) (a) Let A be retract of X. Show that a continuous map $f : A \to Y$ has continuous extension over X. [3 Marks]

(b) Is the punctured sphere $\mathbb{S}^n - \{*\}$ contractible ? Justify [3 Marks]

(c) Let $p : \widetilde{X} \to X$ be an 7-sheeted covering map. If $\widetilde{X}$ is simply connected then prove that $\pi_1(X, x_0) \cong \mathbb{Z}_7$. [4 Marks]

(d) Determine the group of deck transformations of the covering map $p : \mathbb{R} \to \mathbb{S}^1$ defined by the exponential map. [4 Marks]

(2) (a) Show that a continuous map $f : \mathbb{S}^n \to Y$ is null homotopic if and only if f can be continuously extended to the disc $\mathbb{D}^{n+1}$. [7 Marks]

(b) Let A be a subspace of a space X. Give an example that A is a deformation retract of X but not a strong deformation retract. [7 Marks]

(3) (a) Determine the fundamental group of $\mathbb{S}^2 \vee \mathbb{S}^1$. [7 Marks]

(b) Let f, g and h be paths in X such that $f(1) = g(0)$ and $g(1) = h(0)$. Show that $(f * g) * h \simeq f * (g * h)$ rel $\{0, 1\}$. [7 Marks]

(4) (a) Show that the quotient map $\mathbb{S}^n \to \mathbb{R}P^n$, which identifies pair of antipodal points, is a covering map. [7 Marks]

(b) Let X be a path connected space. Show that the fundamental group of X is independendent of the choice of base point. [7 Marks]

(5) (a) State the Covering Homotopy theorem. Let $p : \widetilde{X} \to X$ be a covering map and $\tilde{x} \in p^{-1}(x_0)$. Show that the path class $[f]$ of a loop f in X at x_0 belongs to $\mathrm{im}(p_\#)$ iff f lifts to a loop in $\widetilde{X}$ at $\tilde{x}$. [7 Marks]

(b) Let $p : \widetilde{X} \to X$ be a covering map. If X is locally path connected and $\widetilde{C}$ a path component of $\widetilde{X}$ then show that $p(\widetilde{C})$ is a path component of X and $p|_{\widetilde{C}} : \widetilde{C} \to p(\widetilde{C})$ is a covering map. [7 Marks]

(6) (a) Let $p : \widetilde{X} \to X$ be a regular covering map and $\tilde{x} \in \widetilde{X}$. Show that the group of deck transformations $\Delta(p)$ is isomorphic to [7 Marks]

$\pi_1(X, p(\tilde{x}))/p_{\#}\pi_1(\widetilde{X}, \tilde{x})$. (Assume that all spaces are path connected and locally path connected)

(b) Let $f : \mathbb{S}^2 \to \mathbb{R}^2$ be a continuous map. Prove that there exists a point $x \in \mathbb{S}^2$ such that $f(-x) = f(x)$. [7 Marks]

(7) (a) Let $p : \widetilde{X} \to X$ be a covering map, and $f : I \to X$ be a path with origin x_0. If $\tilde{x}_0 \in p^{-1}(x_0)$ then show that there exists unique path $f : I \to \widetilde{X}$ with $p\tilde{f} = f$ and $\tilde{f}(0) = \tilde{x}_0$. [7 Marks]

(b) Show that the Comb Space C is contractible. Deduce that any two continuous maps of a space X to C are homotopic. [4+3 Marks]

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Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.Sc. Mathematics Examinations, December 12, 2025
Part II Semester III
MMATH18-301(ii): COMMUTATIVE ALGEBRA
(Unique Paper Code: 223502302)

Time: 3 Hours — Maximum Marks: 70

Instructions: • Question 1 is compulsory. • Attempt any four questions from question 2 to question 7. • All questions carry equal marks. • Throughout the paper all the rings are assumed to be commutative ring with nonzero identity.

(1) (a) Let $A \subset B$ be an integral extension of rings and $x \in A$ is unit in B. Show that x is unit in A. [2]

(b) Let R be an integral domain such that $dim(R_{\mathfrak{p}}) = 1$ for every prime ideal $\mathfrak{p}$ of R. Show that $dim(R) = 1$. [3]

(c) Define associated prime ideals, isolated prime ideals and embedded prime ideals belonging to a decomposable ideal $\mathfrak{a}$ of a ring R. Also give an example of each. [3]

(d) Let R be a ring such that every element satisfies $x^n = x$ for some $n > 1$. Show that every prime ideal in R is maximal. [3]

(e) Let v be a discrete valuation on a field K. Show that the set $A = \{a \in K \mid v(x) \geq 0\} \cup \{0\}$ is a subring of K. [3]

(2) (a) Let $0 \to M' \xrightarrow{f} M \xrightarrow{g} M'' \to 0$ be an exact sequence of R-modules. Show that if M' and M'' are finitely generated, then so is M. [5]

(b) Show that the radical of an ideal $\mathfrak{a}$ is an ideal. Further show that if $\mathfrak{a}$ is a proper ideal, then the radical of $\mathfrak{a}$ is the intersection of all prime ideals of R containing $\mathfrak{a}$. [9]

(3) (a) Let $\mathfrak{a}$ be an ideal of a ring R and $S = 1 + \mathfrak{a}$. Show that $S^{-1}\mathfrak{a}$ is contained in the Jacobson radical of $S^{-1}R$. [4]

(b) Let M be an R-module and S be a multiplicative closed set in R. Show that $S^{-1}R \otimes_R M \cong S^{-1}M$. [5]

(c) Let $\mathfrak{a}$ be a decomposable ideal in R. Show that the union of all associated prime ideals belonging to $\mathfrak{a}$ is equals to the set $\{x \in R \mid (\mathfrak{a} : x) \neq \mathfrak{a}\}$. [5]

(4) (a) Let $A \subset B$ be rings, $\mathfrak{a}$ be an ideal of A, and C be the integral closure of A in B. Show that the integral closure of $\mathfrak{a}$ in B is the radical of extension of $\mathfrak{a}$ in C. [5]

(b) Let A be a subring of a field K. Show that the integral closure of A in K is the intersection of all the valuation rings of K containing A. [5]

(c) Let A be a subring of B such that $B \setminus A$ is closed under multiplication. Show that A is integrally closed in B. [4]

(5) (a) Let $A \subset B$ be rings and $x \in B$. Show that if there exists a faithful $A[x]$-module M which is finitely generated A-module, then x is integral over A. [4]

(b) Let $A \subset B$ be rings, B integral over A, and $\mathfrak{q} \subseteq \mathfrak{q}'$ be prime ideals of B such that $\mathfrak{q} \cap A = \mathfrak{q}' \cap A = \mathfrak{p}$. Show that $\mathfrak{q} = \mathfrak{q}'$. [5]

(c) State and prove the Going up theorem. [5]

(6) (a) Let R be a ring in which the zero ideal is a product of finitely many maximal ideals. Show that R is Noetherian if and only if R is Artinian. [5]

(b) Let $\mathfrak{a}$ be a proper ideal in a Noetherian ring R. Show that the prime ideals associated to $\mathfrak{a}$ are precisely the prime ideals which occurs in the family of ideals $\{(\mathfrak{a} : x) \mid x \in R\}$. [5]

(c) Let R be an Artinian ring. Show that the nilradical is nilpotent in R. [4]

(7) (a) Define a Dedekind domain. Show that a domain R is a Dedekind domain if and only if every nonzero fractional ideal of R is invertible. [1+6]

(b) Let R be a Noetherian one dimensional local domain in which there exists $x \in R$ such that every nonzero ideal is of the form $< x^n >$ for some $n \geq 0$. Show that R is a discrete valuation ring. [7]

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Department of Mathematics, University of Delhi

M.A./M.Sc. Mathematics, Part-II, Semester-III
Final Examination, December-2025

Paper: MMATH18-301(iii) Representation of Finite Groups
UPC-223502303

Maximum Marks: 70

Time: 3 Hrs

Note: • Question **1** is compulsory and attempt any **four** questions from the rest. All questions carry equal marks. G will denote a finite group and F a scalar field throughout the question paper.

1. (a) Show that all representations which are equivalent to a faithful representation are faithful. (2)

   (b) Suppose $G = D_{2n}$ and $F = \mathbb{R}$ or $\mathbb{C}$. Show that there is a representation $\rho : G \to GL(1, F)$ such that $a\rho = (1)$ and $b\rho = (-1)$. (3)

   (c) Give an example of an irreducible 2-dimensional FG-module when $G = D_8$. (3)

   (d) Find all the linear characters of S_n $(n \geq 2)$. (3)

   (e) Show that for any character χ of G and any element $g \in G$, $\chi(g)$ is an integer if $\chi(g)$ is a rational number. (3)

2. (a) Prove that the representations of the form $g \mapsto [g]_\beta$, $g \in G$, for some basis β of an FG-module V, are equivalent to each other and also any two equivalent representations of G arise from some FG-moudule in this way. Illustrate this through an example when $G = C_3$. (5+5)

   (b) Given $G = S_n$ and V a vector space over F, show that V becomes an FG-module with the following definition,

$$vg = \begin{cases} v, & \text{if } g \text{ is an even permutation} \\ -v, & \text{if } g \text{ is an odd permutation} \end{cases}$$

   (4)

3. (a) Let V be an FG-module over a field F. Let $V = U_1 + \cdots + U_r$ be a sum of irreducible FG-submodule of V. Prove that V can be written as a direct sum of some of the FG-submodules U_i above. (6)

   (b) State and prove Schur's lemma. (2+6)

4. (a) Prove that there is no faithful irreducible $\mathbb{C}G$-submodule when G is abelian but not cyclic. What is the conclusion when $G = C_2 \times C_2$. (5+2)

   (b) Let $\mathbb{C}G = U_1 \oplus \cdots \oplus U_r$, be a direct sum of irreducible $\mathbb{C}G$-submodules. Prove that every irreducible $\mathbb{C}G$-module is isomorphic to one of the $\mathbb{C}G$-submodules U_i. (7)

5. (a) Show that the dimensions of the space of $\mathbb{C}G$-homomorphisms from regular $\mathbb{C}G$-module $\mathbb{C}G$ to any other $\mathbb{C}G$-module U is equal to $\dim U$. (8)

(b) Prove that a group G is not simple if and only if $\chi(g) = \chi(1)$ for some non-trivial irreducible character χ of G and some $g \neq 1$. (6)

6. (a) Prove that the linear characters of G are precisely the lifts to G of the irreducible characters of G/G' where G' is the derived subgroup of G. (8)

(b) Determine the complete character table of the symmetric group S_4. (6)

7. (a) Prove that $\chi(1)$ divides $|G|$ when χ is an irreducible character of G. (7)

(b) State and prove Burnside's $p^a q^b$ Theorem. (7)

No. of Printed Pages 2

M.A./M.Sc. Mathematics Examinations (2025)
Part II, Semester III
MMATH18-302(i): Fourier Analysis
Unique Paper Code-223502304

Maximum Marks: 70

Time: 3 hr

Instructions: • Attempt **FIVE** questions in all • All questions carry equal marks • Question No. 1 is **COMPULSORY** • Symbols have their usual meaning.

(Question No. 1 is Compulsory)

(1) (a) Show that if $f \in L^1(\mathbb{T})$, then $\widehat{\overline{f}}(n) = \overline{\widehat{f}(-n)}$ for all $n \in \mathbb{Z}$. [2 Marks]

(b) Show that if $f \in L^1(\mathbb{R})$, $a \in \mathbb{R}$ and $f_a(x) = f(x-a)$ for all $x \in \mathbb{R}$, then $\widehat{f_a}(\xi) = e^{-ia\xi}\widehat{f}(\xi)$, $\xi \in \mathbb{R}$. [2 Marks]

(c) Find a summability kernel for the function $G(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, $x \in \mathbb{R}$. [5 Marks]

(d) Show that if V is a symmetric neighbourhood of the identity element e of a topological group G, then $\bigcup_{n=1}^{\infty} V^n$ is a closed subgroup of G. [5 Marks]

(2) (a) Show that if $f \in L^1(\mathbb{T})$, then $\lim_{|n|\to\infty} \widehat{f}(n) = 0$. [4 Marks]

(b) State and prove the Fejér theorem. [10 Marks]

(3) (a) Give an example, with justification, of a complete orthonormal sequence in $L^2(\mathbb{T})$. [4 Marks]

(b) Define modular function. Show that the modular function on a locally compact group is a continuous homomorphism. [1+5+4=10 Marks]

(4) (a) Let $\{K_n\}$ be the Fejér kernel on $\mathbb{T}$ and $V_n(t) = 2K_{2n+1}(t) - K_n(t)$, $n \in \mathbb{N}$, $t \in \mathbb{T}$. Show that $\{V_n\}$ is a summability kernel on $\mathbb{T}$. [5 Marks]

(b) Let $f \in L^1(\mathbb{R})$ and $F(x) = \int_{-\infty}^{x} f(y)dy$, $x \in \mathbb{R}$. Show that if $F \in L^1(\mathbb{R})$, then $\widehat{F}(\xi) = \frac{1}{i\xi}\widehat{f}(\xi)$ for all non-zero $\xi \in \mathbb{R}$. [9 Marks]

(5) (a) Define dual group. Find characters on the additive group of real numbers $(\mathbb{R}, +)$. [1+6=7 Marks]

(b) Show that $L^1(\mathbb{T})$ is a commutative Banach algebra under the convolution product. [7 Marks]

(6) (a) Show that if λ is a left Haar measure on a locally compact group G with $\lambda(G) < \infty$, then G is compact. [7 Marks]

(b) Show that if G is a topological group with the identity element e, F a compact subset of G and U an open subset of G with $F \subset U$, then there exists a neighbourhood V of e such that $(FV)\bigcup(VF) \subset U$. [7 Marks]

(7) (a) Show that if $f \in L^1(G)$, then $\|\widehat{f}\|_\infty \leq \|f\|_{L^1(G)}$, where G is a locally compact abelian group. [4 Marks]

(b) Prove that if $f \in C_c(\mathbb{R})$, then $\frac{1}{2\pi}\int|\widehat{f}(\xi)|^2 d\xi = \int|f(x)|^2 dx$. [10 Marks]

Your Roll Number: ....

Department of Mathematics, University of Delhi
M.Sc. Mathematics Examinations, December 2025
Part II Semester III
MMATH18- 302(ii): Matrix Analysis, UPC: 223502305

Time: 3 hours | Maximum Marks: 70

Instructions: • Answer **five** questions. • **Question 1** is compulsory. Attempt any **four** questions from **Q2** to **Q7**. • Each question carries 14 marks.

(1) **Prove or disprove the following (with justification):**

(a) The set $\{A \in M_n(\mathbb{R}) : \det A < 0\}$ is connected. (3)

(b) A vector norm on $M_n(\mathbb{C})$ equivalent to a matrix norm is a matrix norm. (3)

(c) The set of doubly stochastic matrices on $M_n(\mathbb{C})$ is compact. (3)

(d) Limit of a sequence of positive definite matrices is positive definite. (3)

(e) For $x, y \in \mathbb{R}^n$, if $x \prec y$ then $y_n^\downarrow \leq x_i$, for all i. (2)

(2) (a) State Fan's maximum principle. Use it to prove that for $A, B \in M_n(\mathbb{C})$,

(i) $\lambda(A+B) \prec \lambda(A) + \lambda(B)$, if A and B are Hermitian.

(ii) $\sigma(A+B) \prec_w \sigma(A) + \sigma(B)$. (2+3+3)

(b) Prove that $Sl_n(\mathbb{R})(n > 1)$ is connected. (6)

(3) (a) For a positive definite matrix $A \in M_n(\mathbb{C})$, prove that there exists a unique positive definite matrix $B \in M_n(\mathbb{C})$ such that $B^8 = A$. Further prove that there exists a polynomial p such that $B = p(A)$. (6+3)

(b) Let $A = (a_{ij}) \in M_n(\mathbb{C})$ be a diagonally dominant matrix with non-zero diagonal entries such that $|a_{ii}| > R_i'$ for all except possibly one value of i, where R_i' is the i-th deleted row sum. Prove that A is invertible. (5)

(4) (a) Let $A = SDS^{-1}$, where $S \in M_n(\mathbb{C})$ and $D \in M_n(\mathbb{C})$ is a diagonal matrix. Let μ be a eigenvalue of $A + E$, where $E \in M_n(\mathbb{C})$. Prove that there exists an eigenvalue λ of A such that (6)

$$|\mu - \lambda| \leq \kappa_\infty(S)|||E|||_\infty$$

(b) Let $A, B \in M_n(\mathbb{C})$ be such that $A \succeq B \succ 0$. Prove that $\det A \geq \det B$ and $trace(A) \geq trace(B)$. (8)

(5) (a) For $X \in M_n(\mathbb{C})$, define the matrix e^X (with justification). Compute e^X where $X = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. (3+4)

(b) State and prove Gelfand formula. (2+5)

(6) (a) Prove that the spectral norm is an induced norm and is unitarily invariant. (7)

(b) Let $A \in M_n(\mathbb{C})$ be Hermitian. Prove that A is positive definite if and only if every leading principal minor of A is non-negative. (7)

(7) (a) Prove that an absolute vector norm is monotone. Use this to prove that if $|||\cdot|||$ is the matrix norm on $M_n(\mathbb{C})$ induced by an absolute norm on $\mathbb{C}^n$ and $D = diag(a_1, a_2, \ldots, a_n)$, then $|||D||| = \max_i |a_i|$. (7)

(b) State and prove Birkhoff's theorem for doubly stochastic matrices. (7)

Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, December 2025
Part II Semester III
MMATH18-302(iii): Theory of Bounded Operators
Unique Paper Code 223502306

Time: **3 hours** Maximum Marks: **70**

Instructions: • All notations used are standard • **Question no.** 1 is **compulsory** • Attempt any **four** questions from the remaining six questions.

Part A

(1) Classify each statement below as true or false, giving a brief justification in each case:

(a) $\sigma_{app}(T) = \sigma(T)$ when $T : l^2 \to l^2$ be defined by $T(x_1, x_2, \ldots) = (x_1, \frac{x_2}{2}, \frac{x_3}{3}, \ldots)$.

(b) If T is a positive bounded linear operator on a complex Hilbert space H then $T^2 = T$.

(c) $\sigma_p(T) \cap i\mathbb{R} \not\subseteq \{0\}$ if T is a bounded self adjoint operator on a Hilbert space H.

(d) The range of a compact linear operator on an infinite dimensional Hilbert space H is always separable.

(e) If (T_n) is a sequence of compact operators on a Banach space X converging in the weak operator topology to an operator T, then T may not be compact.

(f) A unitary operator is a partial isometry.

(g) Any bounded linear operator T on a Hilbert space H can be written as $T = UP$ where P is a positive operator on H and U is a unitary operator on H.

(2x7)

Part B

Attempt four questions in all.

(2) Let $D, P : l^2 \longrightarrow l^2$ be given by $D(x_1, x_2, x_3, \ldots) = (x_1, \frac{1}{2}x_2, \frac{1}{3}x_3, \ldots)$ and $P(x_1, x_2, \ldots) = (x_1, x_2, \ldots, x_k, 0, 0, \ldots)$ where $k \in \mathbb{N}, k > 1$ is fixed.

(a) Check whether or not the operators D and P are bounded below. (4)

(b) Find the point spectrum of both these operators. (5)

(c) Find the spectrum of P. (5)

(3) Let T be a compact operator on an infinite dimensional Banach space X. Then establish the following:

(a) $\sigma(T)$ is countable set containing 0. (6 + 2)

(b) No point other then 0 can be a limit point of $\sigma(T)$. (2)
(c) Each non zero point of $\sigma(T)$ is an eigen value of T. (4)

(4) Let $T : l^2 \to l^2$ be defined by $T(\zeta_1, \zeta_2, \ldots) = (0, \zeta_1, \frac{\zeta_2}{2}, \frac{\zeta_3}{3}, \ldots)$.
(a) Show that T is a compact operator. (6)
(b) Show that $\sigma_p(T) = \emptyset$. (6)
(c) Hence or otherwise find $\sigma(T)$. (2)

(5) Suppose T_1, T_2, T are bounded self-adjoint linear operators on a complex Hilbert space H and suppose T commutes with T_1 and T_2.
(a) Prove that T^2 is positive and that $T_1 \leq T_2$ implies $T_1T^2 \leq T_2T^2$. (5)
(b) Show that if $T_1^2 \leq T_2^2$ and T_2 is compact, then T_1 is also compact. (5)
(c) Further, show that $(I + T^2)^{-1}$ exists. (4)

(6) Let $(P_n)_{n=1}^{\infty}$ be a monotonically decreasing sequence of projections P_n defined on a Hilbert space H.
(a) Show that $P_nP_m = P_mP_n = P_n$ for all m, n with $n > m$. (5)
(b) Show that (P_n) is strongly operator convergent to a projection P defined on H and find its range $P(H)$. (9)

(7) Let T be a bounded linear operator on a Hilbert space H.
(a) Prove that
(i) $N(T^*) = (T(H))^{\perp}$.
(ii) $N(T^*T) = N(T) = N(|T|)$.
(iii) If H is finite dimensional, then T, T^*T and $|T|$ have same rank. (6)
(b) Prove that the following are equivalent:
(i) T is a partial isometry.
(ii) T^*T is a projection.
(iii) $TT^*T = T$. (8)

Best Wishes

Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, DECEMBER 2025
Part II Semester III,UPC 223502310

MMATH18-304(i): COMPUTATIONAL FLUID DYNAMICS

Time: 3 hours | Maximum Marks: 70

Instructions: • Attempt **five** questions in all • Question number 1 is compulsory and attempt any **four** from the remaining questions 2 to 7 • Each question from 1 to 7 carries 14 marks • Scientific non-programming calculator for doing numerical calculations is allowed for this examination • Notations and acronyms have their usual meaning.

(1) (a) State Lax-Equivalence theorem. When do you say that a two level finite difference scheme is stable? [3 Marks

(b) Does power-law differencing scheme for steady one-dimensional convection-diffusion equations is efficient than central, upwind and hybrid differencing schemes? Justify. [3 Marks

(c) Derive the finite difference approximation with its order of accuracy for $(\frac{\partial^2 u}{\partial x^2})_{m,n}$ by making use of five nodes given by $(m-2,n), (m-1,n), (m,n), (m+1,n), (m+2,n)$. [4 Marks

(d) Define cell Peclet number. Explain the significance of transportiveness property in the finite volume discretisation scheme. [4 Marks

(2) (a) Find the order of accuracy, truncation error, consistency and stability of the Lax-Wendroff scheme for the equation $u_t + au_x = f(t,x)$, where a is a constant. [7 Marks

(b) Derive the Crank-Nicolson finite volume scheme for one-dimensional unsteady heat conduction equation given by: [7 Marks

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + S$$

where ρ is the density, c is the specific heat, k is the thermal conductivity, T is the temperature of the material and S is the source term.

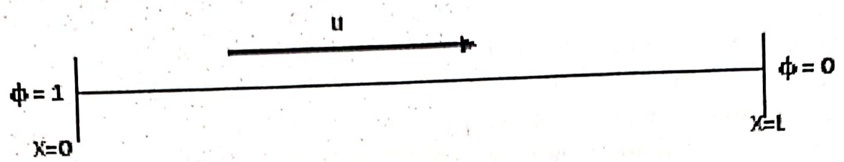
(3) (a) Use the Von-Neumann stability analysis to find stability of the Leapfrog scheme for the equation $u_{tt} = u_{xx} + u_{xx}$. [7 Marks

(b) Derive an explicit and the Peaceman-Rachford Alternating Direction Implicit (ADI) schemes along with their order of accuracy for the pde $u_t = b(u_{xx} + u_{yy})$,where b is a positive constant. [7 Marks

(4) (a) Write the general form of the discretised equation for the hybrid difference scheme for one-dimensional convection-diffusion equation. Assess this scheme on three important properties of the finite volume discretisation schemes. [7 Marks

(b) Find the order of accuracy, truncation error, consistency and stability of the BTCS scheme for the pde $u_t = bu_{xx}$,where b is a positive constant. [7 Marks

(5) (a) A property Φ is transported by means of convection and diffusion through one-dimensional domain as sketched below : Use the required [7 Marks



governing equations; the boundary conditions are $\Phi_0 = 1$ at $x = 0$ and $\Phi_L = 0$ at $x = L$. Divide the domain into 3 three control volumes and use the hybrid differencing scheme for one-dimensional convection-diffusion to calculate the finite volume solution Φ at the given nodes for $u = 2.5m/s$. Observe that $F = F_e = F_w = 2.5$, $D = D_e = D_w = 0.5$, a Peclet number $Pe_w = Pe_e = \frac{\rho u \delta x}{\Gamma} = 5$.

(b) What is the essence of a staggered grid? Elucidate it with the help of a neat diagram. Compute the convective flux / unit mass F and the diffusive conductance D at the cell faces of the u and v control volumes. [7 Marks

(6) (a) Find the solution of initial boundary value problem [7 Marks

$$u_{tt} = u_{xx}, 0 \leq x \leq 1$$

subject to the initial conditions:

$$u(x, 0) = sin(\pi x); 0 \leq x \leq 1$$
$$u_t(x, 0) = 0; 0 \leq x \leq 1$$

and the boundary condition:$u(0, t) = u(1, t) = 0, t > 0$ by using the implicit scheme with $\theta = \frac{1}{2}$.Assume $h = \frac{1}{4}, \lambda = \frac{3}{4}$ and integrate for one time step.

(b) Derive QUICK scheme for one-dimensional convection-diffusion problem. [7 Marks

(7) Consider a suitable geometry of a cylindrical fin with uniform cross-sectional area A. The base is at a temperature of $300^0C(T_B)$and the end is insulated. The fin is exposed to an ambient temperature of 60^0C. One-dimensional heat transfer in this situation is governed by [14 Marks

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0$$

where h is the conductive heat transfer coefficient, P the perimeter, k the thermal conductivity of the material and T_∞ the ambient temperature. Use a uniform grid and divide domain into five control volumes to calculate the temperature distribution along the fin. It is given that: $n^2 = hP/(kA)$, L is the length of the fin and x is the distance along the fin, $L = 5m$, $hP/(kA) = 50/m^2$ (note that kA is constant).

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Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, December 2025
Part II Semester III
MMATH18-304(ii) **UPC: 223502311**
COMPUTATIONAL METHODS FOR ODEs

Time: 3 hours Maximum Marks: 70

Instructions: • Section A is compulsory • Answer **any four** questions from Section B • Each question carries equal marks • Non-programmable scientific calculators are allowed. • Notations have their usual meaning

Section A

(1) (a) Prove that the second order classical RK scheme for the numerical solution of the first order initial value problem is absolutely stable. [2]

(b) State true and false and justify the statement: The method $y_{n+1} = y_n + hf(t_{n+1/2}, \frac{1}{2}(y_n + y_{n+1}))$ where $t_{n+1/2} = (t_n + t_{n+1})/2$ is unconditionally consistent for the numerical solution of the problem $y' = f(x, y)$. [3]

(c) Derive equivalent variational functional corresponding to the equation $y'' = y^2 + 4x,\ 0 < x < 1,\ y(0) = 4,\ y(1) = 1$. [3]

(d) Derive a first order scheme for the numerical solution of $y'' + 2xy' + y = 0,\ 0 < x < 1,\quad y(0) = 0,\ y(1) = 2$. [3]

(e) Given $\rho(\xi) = (\xi - 1)(\xi - 1/2)$ find $\sigma(\xi)$ such that the resulting method is explicit. [3]

Section B

(2) (a) Convert the problem [7]

$$y''' + 4y'' + 5y' + 2y = -4\sin(x) - 2\cos(x),\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -1$$

to a system of first order equations. Use third order Taylor series method to solve the resulting system using $h = 0.2$ in the interval $[0, 0.2]$.

(b) Solve the initial value problem $y' = t/y,\ y(0) = 1,\ t \in [0, 0.4]$ using second order Adams-Bashforth method with $h = 0.2$. Obtain the starting values using second order Taylor series method. [7]

(3) (a) Perform convergence analysis of the central difference scheme for the numerical solution of the boundary value problem $y'' + p(x)y' + q(x)y = 0, x \in [0, 1]\quad y(0) = y_0,\ y(1) = y_1$ where $q(x) > q_0 > 0$. [7]

(b) Obtain the Ritz finite element solution of the boundary value problem $y'' - \frac{3}{2}y^2 = 0,\ y(0) + y'(0) = 1,\ y(1) = 0$ using linear shape functions and two elements. [7]

(4) (a) Consider the two step method [7]

$$y_{n+1} = 2y_{n-1} - y_n + h\left[\frac{5}{2}y'_n + \frac{1}{2}y'_{n-1}\right] \quad n \geq 1$$

for the solution of the problem $y'(x) = f(x, y)$. Show that the method is of order 2 and unstable.

(b) Use the Numerov method to replace the boundary value problem $y'' = 4(y-x),\ 0 \le x \le 3, y(0) = 1,\ y(1) = 2$, by a set of linear difference equations with $h = 0.25$. Write these equations in matrix form as $My = r$. [7]

(5) (a) Consider the boundary value problem $y'' + 2y = x,\ 0 < x < 1,\quad y(0) = 4,\ y(1) = 1$ Determine the coefficients of the approximate solution function $w(x) = 1+(1-x)(a_1+a_2x)$ by Ritz method. [7]

(b) The formula $y_{j+3} = y_j + \frac{3h}{8}(y'_j + 3y'_{j+1} + 3y'_{j+2} + y'_{j+3})$ with a small steplength h is used for solving the equation $y' = -y$. Investigate the convergence properties of the method. [7]

(6) (a) Solve the initial value problem $y' = -4ty^2,\ y(0) = 1$ with $h = 0.1$ on the interval $[0, 0.2]$ using P-C method [7]

$$P:\ y_{j+1} = y_j + \frac{h}{2}(3y'_j - y'_{j-1})$$

$$C:\ y_{j+1} = y_j + \frac{h}{2}(y'_{j+1} + y'_j)$$

as $P(EC)^mE,\ m = 1$.

(b) Prove that the necessary and sufficient condition for the linear multistep method of the form [7]

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h\phi_f(y_{n+k}, y_{n+k-1}, \dots, y_n, x_n, h)$$

for the solution of the problem $y'(x) = f(x,y)$ to be consistent is

$$\rho(1) = 0,\quad \phi_f(y(x_n),\ y(x_n),\ \dots, y(x_n)\ x_n,\ 0)/\rho'(1) = f(x_n, y(x_n)),$$

where $\rho(r)$ is the first characteristic polynomial corresponding to the given method.

(7) (a) Apply the fourth order method [8]

$$u_{j+1} - 2u_j + u_{j-1} = \frac{h^2}{12}[\bar{f}_{j-1} + 10\hat{f}_j + \bar{f}_{j+1}]$$

where

$$\bar{u}'_j = (u_{j+1} - u_{j-1})/(2h),\quad \bar{u}'_{j+1} = (3u_{j+1} - 4u_j + u_{j-1})/(2h),$$

$$\bar{u}'_{j-1} = (-u_{j+1} + 4u_j - 3u_{j-1})/(2h), \hat{u}'_j = \bar{u}'_j - [h(\bar{f}_{j+1} - \bar{f}_{j-1})/20]$$

$$\hat{f}_j = f(x_j, u_j, \hat{u}'_j),\ \bar{f}_{j\pm1} = f(x_{j\pm1},\ u_{j\pm1},\ \bar{u}'_{j\pm1})$$

to the boundary value problem $u'' = f(x,u,u')$ where $f(x,u,u') = ku',\ k >> 1,\ u(0) = 1,\ u(1) = 0$ with step length h. Determine the explicit expression for u_j. For which values of $R(= kh/2)$ is the sequence $\{u_j\}_0^\infty$ bounded?

(b) Use the implicit method [6]

$$y_{j+1} = y_j + K_1,\quad K_1 = hf(t_j + h/2, y_j + K_1/2)$$

to find the solution of the initial value problem

$$y' = -2ty^2,\quad y(0) = 1,\quad 0 \le t \le 0.2,\ h = 0.2.$$

Use one iteration of Newton Raphson method to solve the algebraic equations.

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Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, December 2025
Part II Semester III

MMATH18-304(iv): METHODS OF APPLIED MATHEMATICS
(UPC 223502313)

Time: 3 hours Maximum Marks: 70

Instructions: • Question no. 1 is compulsory. • Answer any four questions from Question 2-7. • All notations have usual meaning.

(1) (a) Determine an integral equation corresponding to the IVP: $y' + y = \cos x; y(0) = 0, y'(0) = 1$. [3 Marks]

(b) Derive the resolvent kernel of Volterra integral equation in terms of iterated kernels. [3 Marks]

(c) Find the resolvent kernel for Volterra-type integral equation with the following kernel $K(x,t) = \frac{2+\cos x}{2+\cos t}$. [3 Marks]

(d) If $\alpha(x)$ is continuous in $[a,b]$, and if $\int_a^b \beta(x)h''(x)dx = 0$ for every function $h(x) \in \mathcal{D}_2(a,b)$ such that $h(a) = h(b) = 0$ and $h'(a) = h'(b) = 0$, then show that $\beta(x) = c_0 + c_1 x$ for all x in $[a,b]$, where c_0 and c_1 are constants. [3 Marks]

(e) State the Buckingham $\pi-$ theorem. [2 Marks]

(2) (a) For different values of λ, investigate the solvability of the integral equations: $\phi(x) - \lambda\int_{-1}^{1}|x-\pi|\phi(t)dt = x$. [4 Marks]

(b) Using the method of successive approximations, solve [4 Marks]

$$\phi(x) = 1 + \int_0^x (x-t)\phi(t)dt, \quad \phi_0(x) = 1.$$

(c) Determine the Euler's equation for the functional $J[y] = \int_a^b F(x, y_1, ..., y_n, y_1', ..., y_n')dx$. Also, find the extremals of the functional $J[y] = \int_a^b (2yz - 2y^2 + (y')^2 - (z')^2)dx$. [3+3 Marks]

(3) (a) Solve the following integro-differential equation: [5 Marks]

$$\phi''(x) + \phi(x) + \int_0^x \sinh(x-t)\phi(t)dt + \int_0^x \cosh(x-t)\phi'(t)dt = \cosh x; \phi(0) = -1; \phi'(0) = 1.$$

(b) Derived the Abel's generalized equation and then solve it. [5 Marks]

(c) Using Fredholm determinants, find resolvent kernel of [4 Marks]

$$k(x,t) = \sin x - \sin t, \qquad 0 \le x \le 2\pi, 0 \le t \le 2\pi.$$

(4) (a) Find the maximum of $|\int_0^1\int_0^1 K(x,t)\phi(x)\phi(t)dxdt|$ provided that $\int_0^1 \phi^2(x)dx = 1$, if $K(x,t) = xt$, $0 \le x, t \le 1$. [6 Marks]

(b) Using the Hilbert-Schmidt method, solve $\phi(x) = 2e^x + \lambda \int_0^1 K(x,t)\phi(t)dt$, where [8 Marks]

$$K(x,t) = \begin{cases} \frac{\sinh t \sinh(x-1)}{\sinh 2}, & 0 \le x \le t, \\ \frac{\sinh x \sinh(t-1)}{\sinh 2}, & t \le x \le 1. \end{cases}$$

(5) (a) Define Green's function for a two point BVP with variable coefficient. Reduce the following BVP into integral equation $y'' + \lambda y = 2x + 1, y(0) = y'(1), y'(0) = y(1)$. [3+5 Marks]

(b) Find the characteristic numbers and eigenfunctions of the homogeneous integral equation if its kernel is given by $K(x,t) = \sin x \sin t, 0 \le x, t \le 2\pi$. [6 Marks]

(6) (a) Define a functional with a suitable example. A physical phenomenon is described by the quantities P, l, m, t and ρ, representing pressure, length, mass, time, and density, respectively. If there is a physical law $f(P, l, m, t, \rho) = 0$ relating these quantities, show that there is an equivalent physical law of the form $g(l^3\rho/m, t^6P^3/m^2\rho) = 0$. [2+5 Marks]

(b) Show that regular perturbation fails on the boundary value problem: $\epsilon y'' + (1+\epsilon)y' + y = 0, 0 < t < 1, 0 < \epsilon << 1$ with $y(0) = 0$ and $y(1) = 1$. Find the exact solution and hence, find the inner and outer approximations. [7 Marks]

(7) (a) Using WKB approximation, find the solution of Schrodinger equation $ih\psi_t = -\frac{h^2}{2m}\psi_{xx} + V(x)\psi$. [8 Marks]

(b) Prove that the Nuemann series of a kernel $K(x,t)$ converges for $|\lambda| < B^{-1}$, where $B = \int_a^b \int_a^b K^2(x,t)dxdt$. [6 Marks]

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, December 2025
Part II Semester III

MMATH18-303(i): ADVANCED COMPLEX ANALYSIS
(Unique Paper Code 223502307)

Time: 3 hours Maximum Marks: 70

Instructions: • Attempt **five** questions in all. • **Question 1 is compulsory.** • All questions carry equal marks. • The symbols used have their usual meanings.

(1) (a) Show that a function $f : [a, b] \to \mathbb{R}$ is convex if and only if

$$\frac{f(u) - f(x)}{u - x} \leq \frac{f(y) - f(x)}{y - x}$$

whenever $a \leq x < u < y \leq b$. [4]

(b) For the following open subsets of $\mathbb{C}$, write them as the countable union of compact sets: (i) $\{z : |\operatorname{Im} z| < 1\}$, and (ii) $\mathbb{C} - \{0\}$. [2]

(c) Show that the family $\mathcal{F} = \{f_n : n = 1, 2, \ldots\}$ is normal, but not compact in $C(\mathbb{C}, \mathbb{C})$, where

$$f_n(z) = \frac{\bar{z}}{n}$$

for each $z \in \mathbb{C}$ and $n \in \mathbb{N}$. [4]

(d) Explicitly find an analytic one-one function which maps the first quadrant $G = \{z : \operatorname{Re} z > 0 \text{ and } \operatorname{Im} z > 0\}$ onto the unit disk. [4]

(2) (a) For a region G, let $H(G) \subset C(G, \mathbb{C})$ and $\operatorname{Har}(G) \subset C(G, \mathbb{R})$ denote the set of all analytic and harmonic functions on G respectively. Prove that $H(G)$ and $\operatorname{Har}(G)$ are complete metric spaces. [4+4]

(b) Let $a < b$ and $G = \{x + iy : a < x < b\}$ be the vertical strip. Suppose that $f : \overline{G} \to \mathbb{C}$ is continuous and f is analytic in G. Further suppose that $|f(z)| \leq 1$ for all $z \in \partial G$ and $|f(z)| < B$ for all $z \in G$. Prove that $|f(z)| \leq 1$ for all $z \in G$. [6]

(3) (a) Let f be an analytic function in $D = \{z : |z| < 1\}$ with $f(0) = 0$, $f'(0) = 1$ and $|f(z)| \leq M$ for all $z \in D$. Show that $M \geq 1$ and $B(0; 1/(6M)) \subset f(D)$. [6]

(b) State and prove Montel's theorem. [8]

(4) (a) Let G be an open subset of $\mathbb{C}$ and (Ω, d) be a complete metric space. Prove that a set $O \subseteq (C(G, \Omega), \rho)$ is open if and only if for each $f \in O$, there is a compact set K and a $\delta > 0$ such that

$$\{g \in C(G, \Omega) : d(f(z), g(z)) < \delta \text{ for all } z \in K\} \subseteq O.$$

Clearly state the results being used. [6]

(b) Let G be a simply connected region and suppose that $f : G \to \mathbb{C}$ be an analytic function that does not assume the value 0 and 1. If $g : G \to \mathbb{C}$ is an analytic function satisfying

$$f(z) = -\exp(i\pi \cosh(2g(z)))$$

for all $z \in G$, prove that $g(G)$ contains no disk of radius 1. [8]

(5) (a) Let γ be a rectifiable curve and let K be a compact set such that $K \cap \{\gamma\} = \emptyset$. If f is a continuous function on $\{\gamma\}$ and $\epsilon > 0$, prove that there is a rational function $R(z)$ having all the poles on $\{\gamma\}$ and such that

$$\left| \int_\gamma \frac{f(w)}{w - z}\, dw - R(z) \right| < \epsilon$$

for all $z \in K$. [8]

(b) Let G be a region and suppose that $u : G \to \mathbb{R}$ be a harmonic function. If there is a point a in G such that $u(z) \leq u(a)$ for all $z \in G$, show that u is a constant function. [6]

(6) (a) State three equivalent conditions for simple connectedness of a region G in the complex plane. Using each of your statements prove or disprove that the punctured plane $G = \mathbb{C} \setminus \{0\}$ is simply connected. [7]

(b) Show that

$$\sin(\pi z) = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$$

where the convergence is uniform over compact subsets of $\mathbb{C}$. [7]

(7) (a) For $|z| < 1/2$, prove that

$$\frac{1}{2}|z| \leq |\log(1+z)| \leq \frac{3}{2}|z|.$$

Use this result to deduce that if $\operatorname{Re} z_n > -1$, then the series $\sum \log(1 + z_n)$ converges absolutely if and only if the series $\sum z_n$ converges absolutely.[4+3]

(b) Let $G \neq \mathbb{C}$ be a region such that every non-vanishing analytic function on G has an analytic square root. Let $a \in G$ and consider a family $\mathcal{F}$ consisting of functions $f \in H(G)$ which are one-one, $f(a) = 0$, $f'(a) > 0$ and $f(G) \subseteq \{z : |z| < 1\}$. Assuming that $\mathcal{F}$ is non-empty, prove that $\mathcal{F}$ is normal and $\overline{\mathcal{F}} = \mathcal{F} \cup \{0\}$. [7]

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, December 2025
Part II Semester III

MMATH18-303(ii): ADVANCED MEASURE THEORY
(Unique Paper Code 223502308)

Time: 3 hours — Maximum Marks: 70

Instructions: • Attempt five questions in all. • **Question 1 is compulsory.** • All questions carry equal marks. • The symbols used have their usual meanings.

(1) (a) Let ν, μ be two signed measures on a measurable space $(X, \mathcal{B})$. Prove that $|\nu + \mu| \leq |\nu| + |\mu|$ and $|a\nu| = |a||\nu|$, for $a \in \mathbb{R}$. [4]

(b) Prove that the Lebesgue measure on $\mathbb{R}^n$ is translation invariant. [4]

(c) Let $(X, \mathcal{B}, \mu)$ be a measure space and let $f \in L^1(\mu)$. Prove that the set $\{x \in X \ : \ f(x) \neq 0\}$ is a σ-finite measurable set. [3]

(d) State the Fubini-Tonelli Theorem. [3]

(2) (a) Let μ^* be an outer measure on a nonempty set X. Prove that a countable union of μ^*-measurable sets is a μ^*-measurable set. [5]

(b) Suppose μ and ν are two finite positive measures on a measurable space $(X, \mathcal{B})$. Prove that either $\nu \perp \mu$, or there exists $\epsilon > 0$ and $B \in \mathcal{B}$ such that $\mu(B) > 0$ and B is a positive set for the signed measure $\nu - \epsilon\mu$. [5]

(c) Let $(X, \mathcal{M}, \mu)$ and $(Y, \mathcal{N}, \nu)$ be two measure spaces. Let $\mathcal{R}$ be the collection of all measurable rectangles in $X \times Y$. Let $\lambda : \mathcal{R} \to [0, \infty]$ be defined as $\lambda(M \times N) = \mu(M)\nu(N)$, for all $M \times N \in \mathcal{R}$. Prove that the collection $\mathcal{R}$ is a semiring. Prove that the set function λ is countably additive. [2+2]

(3) (a) Let $\mathcal{S}$ be non-empty collection of subsets of a non-empty set X. Let $\mu : \mathcal{S} \to [0, \infty]$ be a set function. Let μ^* be the outer measure induced by the function μ. Let E be a subset of X with $\mu^*(E) < \infty$. Prove that there is a subset A of X such that $A \in \mathcal{S}_{\sigma\delta}$ with $E \subseteq A$ and $\mu^*(A) = \mu^*(E)$. [5]

(b) Let ν be a finite signed measure and μ be a positive measure on a measurable space $(X, \mathcal{B})$. Prove that $\nu \ll \mu$ if and only if for given $\epsilon > 0$, there exists $\delta > 0$ such that $|\nu(B)| < \epsilon$ whenever $\mu(B) < \delta$ for $B \in \mathcal{B}$. [5]

(c) Let ν, μ be two signed measures on a measurable space $(X, \mathcal{B})$. Prove that $\nu \perp \mu$ if and only if both $\nu^+ \perp \mu$ and $\nu^- \perp \mu$. [4]

(4) (a) Let $\mathcal{S}$ be non-empty collection of subsets of a non-empty set X such that if $A, B \in \mathcal{S}$, then $A \setminus B \in \mathcal{S}$. Let μ be a premeasure on $\mathcal{S}$. Let $\overline{\mu}$ be the Caratheodory measure induced by μ. Prove that $\overline{\mu}$ extends μ. [5]

(b) Let $(X, \mathcal{M}, \mu)$ and $(Y, \mathcal{N}, \nu)$ be two σ-finite measure spaces. Prove that the product measure space $(X \times Y, \mathcal{M} \otimes \mathcal{N}, \mu \times \nu)$ is also σ-finite. [4]

(c) Let $(X, \mathcal{B}, \mu)$ be a finite measure space and let $1 \leq p < \infty$. Let $\Phi : L^p(\mu) \to \mathbb{R}$ be a bounded linear functional. Prove that

$$\nu(B) = \Phi(\chi_B), \ \forall \ B \in \mathcal{B},$$

defines a signed measure on $(X, \mathcal{B})$ and $\nu \ll \mu$. [4+1]

(5) (a) State and prove the Jordan Decomposition Theorem. [2+6]

(b) Let $(X, \mathcal{B}, \mu)$ be a measure space. Let $E \in \mathcal{B}$ with $\mu(E) = 1$. Compute the infimum of

$$P(f) = \left(\int_E f\, d\mu\right)\left(\int_E \frac{1}{f}\, d\mu\right),$$

when f varies over the set of functions that are measurable and positive μ-a.e. in E. For which functions is the infimum attained? [4+2]

(6) (a) Define a Radon measure. Give an example of a Radon measure on the interval $[0, 1]$ in $\mathbb{R}$. [2+1]

(b) Let μ be the Lebesgue measure and ν be the counting measure on the Borel sigma-algebra generated by open sets in $[0, 1]$. Let $D = \{(x, x) : x \in [0, 1]\}$ be the diagonal in $[0, 1] \times [0, 1]$. Compute the iterated integrals $\int_{[0,1]} \int_{[0,1]} \chi_D\, d\mu\, d\nu$ and $\int_{[0,1]} \int_{[0,1]} \chi_D\, d\nu\, d\mu$. Argue whether the function χ_D is integrable or not with respect to the product measure $\mu \times \nu$. [2+2+1]

(c) State the Riesz-Markov Representation Theorem. Prove only the uniqueness of Radon measure part of the theorem. [2+4]

(7) (a) Consider the signed measure ν on the Borel sigma-algebra $\mathcal{B}$ generated by the open sets in $[-3, 3]$ (with the usual topology on $\mathbb{R}$) defined as

$$\nu(B) = \int_B x \cos x, \ \forall\, B \in \mathcal{B}.$$

Give two Hahn decompositions of ν. [4]

(b) Let X be a locally compact Hausdorff space. Let Λ be a positive linear functional on $C_c(X)$. Show that if a measure μ on the Borel σ-algebra represents Λ, then $\mu(K) < \infty$ for all compact K in X. [5]

(c) Let $X = \{a_1, a_2, \ldots, a_n\}$. Let μ be a measure on $(X, \mathcal{P}^X)$ such that $0 < \mu(\{a_1\}) < \infty$, $\mu(\{a_i\}) = \infty$, for all $i = 2, \ldots, n$. Describe the spaces $L^1(\mu)$ and $L^\infty(\mu)$. Is $L^\infty(\mu)$ the dual of $L^1(\mu)$? Is $L^1(\mu)$ the dual of $L^\infty(\mu)$? [3+1+1]

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Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, December 2025
Part- II Semester- III
MMATH18-303(iii): General Topology
(Unique Paper Code 223502309)

Time: 3 hours | Marks: 70

Instructions: • All notations used are standard • **Question no. 1 is compulsory** • Attempt any **four** questions from the remaining six questions.

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(1) Do as directed.

(a) Is a locally compact metric space second countable? Justify your claim. [2 Marks]

(b) Prove that if X is a compact space then for any space Y, the projection map $p: X \times Y \to Y$ defined by $p(x, y) = y$ is proper. [3 Marks]

(c) Justify that every completely regular space need not be paracompact. [3 Marks]

(d) Prove that a countable regular space is normal. [3 Marks]

(e) Define cone CX and suspension ΣX of a space X. Describe $C\mathbb{S}^n$ over the n-sphere $\mathbb{S}^n$. [3 Marks]

(2) (a) Prove that a subspace of a locally compact Hausdorff space is locally compact if and only if it is locally closed. [7 Marks]

(b) Let X be a locally compact, Hausdorff space and X^* be its one-point compactification. Prove that any homeomorphism between X and the complement of a single point of a compact Hausdorff space Y extends to a homeomorphism between X^* and Y. [4 Marks]

(c) Find two spaces X and Y such that X is not homeomorphic to Y but X^* is homeomorphic to Y^*. [3 Marks]

(3) (a) Let X be a second countable space and $f : X \to Y$ be a proper surjection then prove that Y is second countable. [5 Marks]

(b) Let $\sim$ be an equivalence relation on a space X. If X is locally connected then prove that $X/\sim$ is also locally connected. [5 Marks]

(c) Let $f : X \to Y$ be a continuous surjection. Prove that f is an identification map if and only if, for any space Z, the continuity of a function $g : Y \to Z$ follows from that of $g \circ f : X \to Z$. [4 Marks]

(4) (a) Let X_α, $\alpha \in \mathcal{A}$ be a family of regular spaces then prove that $\prod_{\alpha \in \mathcal{A}} X_\alpha$ is also regular. [5 Marks]

P.T.O.

(b) Prove that a completely regular space can be embedded in a cube. [5 Marks]

(c) Prove that a completely regular space X is connected if and only if its Stone-Čech compactification $\beta(X)$ is connected. [4 Marks]

(5) (a) Let X be a normal space and A, B be disjoint closed sets in X then prove that there exists a continuous map $f : X \to [-1, 1]$ such that $f(A) = \{-1\}$ and $f(B) = \{1\}$. [9 Marks]

(b) Prove that every compact Hausdorff space is a normal space. [5 Marks]

(6) (a) Let X be a normal space, $A \subseteq X$ be closed and $f : A \to [0, 1]$ be continuous. Prove that there exists a continuous function $g : X \to [0, 1]$ such that $g_{|A} = f$. [8 Marks]

(b) Let X be a T_3-space with the property that its every open covering has a σ-locally finite open refinement. Prove that X is paracompact. [6 Marks]

(7) (a) Prove that every open covering of $\mathbb{R}_l$, $\mathbb{R}$ with lower limit topology, has a partition of unity subordinate to it. [9 Marks]

(b) Prove that a paracompact Hausdorff space is a regular space. [5 Marks]

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Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, December 2025
Part II Semester III
MMATH18-305(i): CODING THEORY
(Unique Paper Code 223503301)

Time: 2 hours

Maximum Marks: 35

Instructions: • Attempt five questions in all. • **Question 1 is compulsory.** • All questions carry equal marks. • The symbols used have their usual meanings.

(1) (a) For a binary symmetric channel with crossover probability $p < 1/2$, prove that the nearest neighbour decoding rule is same as the maximum likelihood decoding rule. [3]

(b) Let C be a linear code. Let $S(u)$ denote the syndrome of the word w for the code C. Prove that $S(u_1) = S(u_2)$ if and only if u_1 and u_2 lie in the same coset of C. [2]

(c) Argue whether the word 10010001 is a unique coset leader for a linear code with distance 7. [2]

(2) (a) Let C be a linear code over $\mathbf{F}_5$ spanned by $S = \{12400, 22021, 10030, 44401\}$. Compute a generator matrix of C and the number of elements in C. [3+1]

(b) Let C_1, C_2 be two linear codes of length n over $\mathbf{F}_q$. Prove that the linear code $C_1 + C_2$ satisfies $(C_1 + C_2)^{\perp} = C_1^{\perp} \cap C_2^{\perp}$. [3]

(3) (a) Prove that a code is u-error-detecting if and only if the distance of the code is at least $u + 1$. [4]

(b) If C is a $[n, k, d]$-linear code, prove that $d - 1 \leq n - k$. [3]

(4) Let C be a binary cyclic code of length 7 with generating polynomial $g(x) = 1 + x^2 + x^3$.

(a) Compute the dimension of C.

(b) Compute a generator matrix of C.

(c) Compute the reciprocal polynomial of C.

(d) Compute the parity-check polynomial of C.

(e) Compute a parity-check matrix of C.

(f) Show that C is a Hamming code.

(g) Compute the distance of C. [7x1]

(5) (a) Construct a Reed-Solomon code of length 12 and information rate 1/2. Verify that it is maximum distance separable (MDS).
Hint: 2 is a primitive element of the field $\mathbf{F}_{13}$. [4]

(b) Use the sphere packing bound to argue whether there a binary code with parameters $(5, 3, 3)$. [3]

(6) (a) Compute $A_2(5,4)$. Give an example of an optimal binary code of length 5 and distance 4. [3+1]

(b) Let C be a binary linear code with parity-check matrix

$$H = \begin{pmatrix} 1\,1\,0\,1\,1\,0\,0 \\ 0\,1\,1\,1\,0\,1\,0 \\ 1\,0\,1\,1\,0\,0\,1 \end{pmatrix}.$$

Compute the following: dimension of C, distance of C, and number of distinct cosets of C. [3x1]

(7) (a) Verify that $x^6 - 1 = (x+1)^2(x^2+x+1)^2$ over the field $\mathbf{F}_2$. List all the binary cyclic codes of length 6. [1+3]

(b) Let C_1, C_2 be two cyclic codes in $\mathbf{F}_q^n$ with generating polynomials $g_1(x), g_2(x)$, respectively. Show that $C_1 \cap C_2$ is a cyclic code. Compute its generating polynomial. [3]

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, December 2025
Part II Semester III

MMATH18-305(ii): STOCHASTIC CALCULUS FOR FINANCE
(Unique Paper Code 223503302)

Time: 2 hours | Maximum Marks: 35

Instructions: • Attempt **five** questions in all. • **Question 1 is compulsory**. • All questions carry equal marks. • The symbols used have their usual meanings.

(1) (a) Let X be a random variable on a probability space (X, Ω, P). Show that the probability density function f of X satisfies $\int_R f(x)dx = 1$ [2]

(b) Let $(\Omega, \mathcal{F}, P)$ be a probability space and $A \in \mathcal{F}$. Show that the indicator function (characteristic function) $\mathbf{1}_A : \Omega \longrightarrow R$ is a random variable. [2]

(c) Let $(\Omega, \mathcal{F}, P)$ be a probability space, X be a random variable and $\mathcal{G}$ be a sub-σ -algebra of $\mathcal{F}$. Show that $E[E[X|\mathcal{G}]] = E[X]$ [3]

(2) (a) Let Ω be an uncountable set. Define $\mathcal{F} = \{A \subset \Omega : A \;\; or \;\; A^c \;\; is \;\; countable\}$ and let $P(A) = \begin{cases} 0 \text{ if } A \;\; is \;\; countable \\ 1 \text{ if } A \;\; is \;\; uncountable \end{cases}$

Show that $(\Omega, \mathcal{F}, P)$ is a probability space. [5]

(b) Let X and Y be two random variables on a measurable space $(\Omega, \mathcal{F})$. Show that $X + Y$ is also a random variable on the same space. [2]

(3) (a) Let $(W(t) : t \geq 0)$ be a Brownian motion. Show that $E[e^{\alpha W(t)}] = e^{\frac{1}{2}\alpha^2 t}$ [5]

(b) Using differential form of Ito process given by $dX(t) = \Delta(t)dW(t) + \Theta(t)dt$, derive Ito multiplication rule $dX(t)dX(t) = \Delta^2(t)dt$. [2]

(4) Prove the Ito Isometry $E[I(t)^2] = E[\int_0^t \Delta^2(s)ds]$. [7]

(5) (a) Write notes on *risk free asset* and *financial derivative.* [4]

(b) Show that the cumulative distribution function F_X of a random variable X on a probability space $(\Omega, \mathcal{F}, P)$ is non-decreasing. [3]

(6) State the first and second fundamental theorems of asset pricing. Give an outline of the proof of the first theorem. [2+5]

(7) (a) State four assumptions to derive the price of an option in Black-Scholes partial differential equation. [2]

(b) Derive Black-Scholes formula for the price of European call option for a non-dividend paying stock. [5]