SEMESTER-VIII

Syllabi of Semester - VIII based on UGCF - 2022

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-VIII

DISCIPLINE SPECIFIC CORE COURSE (DSC)– 20: FIELD THEORY AND GALOIS EXTENSION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Field Theory and Galois Extension	4	3	1	0	Class XII pass with Mathematics	Group Theory, Ring Theory

Learning Objectives: The objective of the course is to introduce:

- Tools of field theory such as field extensions, splitting fields, normal extensions, separability, and separable extensions.
- Galois extensions and the Fundamental theorem of Galois theory.
- Link between group theory and the roots of polynomials, developed by Galois, to solve the problem of solvability of polynomial equations by radicals.
- Some applications, such as cyclotomic polynomials, finite fields, and simple extensions.

Learning Outcomes: This course will enable the students to:

- Identify and construct examples of fields, distinguish between algebraic and transcendental extensions, and characterize normal extensions in terms of splitting fields.
- Identify and characterize separable extensions, define Galois extensions, construct examples of automorphism groups of a field as well as prove the fundamental theorem of Galois theory.
- Use the Galois theory of equations to prove that a polynomial equation over a field is solvable by radicals if and only if its Galois group is solvable and hence deduce that a general quintic equation is not solvable by radicals.
- Define cyclotomic polynomials and find its Galois group using roots of unity, classify finite fields and prove that every finite separable extension is simple.

SYLLABUS OF DSC-20

UNIT – I: Field Extensions **(15 hours)**

Fields and prime subfields, Field extensions, Degree of field extensions, Tower theorem, Algebraic and transcendental elements, Algebraic and transcendental extensions, Monomorphism of field extensions, Ruler and compass constructions, Splitting fields, Extensions of monomorphisms, Uniqueness of splitting field.

UNIT – II: Galois Extensions and the Fundamental Theorem (15 hours)

Normal extensions, Separability and separable extensions, Monomorphisms and automorphisms of field extension, Galois extensions, Automorphism/Galois groups and fixed fields, Galois theory of polynomials, The fundamental theorem of Galois theory.

UNIT – III: Some Applications and Solvability by Radicals (15 hours)

The Discriminant, Cyclotomic polynomials, extensions and its Galois group, Solution by radicals, Existence and Uniqueness of finite fields, Simple extensions, and the primitive element theorem.

Essential Readings

1. Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.
2. Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.). Wiley.

Suggestive Readings

- Stewart, Ian (2022). Galois Theory (5th ed.). CRC Press. Chapman and Hall.
- Cox, David A. (2012). Galois Theory (2nd ed.). John Wiley & Sons.
- Cohn, P. M. (2003). Basic Algebra, Springer International Edition.

DSE Courses of B.Sc. (Hons) Mathematics, Semester -VIII

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(i): ADVANCED MECHANICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Mechanics	4	3	1	0	Class XII pass with Mathematics	Calculus, Differential Equations, Mechanics

Learning Objectives: The main objective of this course is to:

- Provide the students an enriching experience of the basic concepts of mechanics in space and its related concepts.
- Impart quality understanding to the students about Newtonian, Lagrangian and Hamiltonian mechanics along with practical applications of these concepts in real life.
- Understand the concept of fluid, their classifications, model, and approaches to study the fluid flow.

Learning Outcomes: This course will equip the students with the:

- Fundamental concepts of force systems, generalized coordinates, kinematics of a particle and a rigid body.
- Thorough and in depth understanding of the classification of dynamical systems, Lagrangian and Hamiltonian's equations.
- Formulation of mass and momentum conservation principle; solution for non-viscous flow, the motion of sphere, cylinder, and two-dimensional flow.
- Understanding of the concepts of stress and strain in viscous flow; derivation of Navier-Stokes equation of motion and related problems.

SYLLABUS OF DSE-6(i)

UNIT – I: Newtonian Mechanics (15 hours)

General force systems, Equilibrium of a system of particles, Reduction of a force systems, Equilibrium of a rigid body, Generalized coordinates and constraints, Work and potential energy, Kinematics of a particle and a rigid body. Moments and product of inertia. Kinetic energy and angular momentum, Motion of a particle and a system, Moving frame of reference, Motion of a rigid body.

UNIT – II: Lagrangian and Hamiltonian Mechanics (12 hours)

Lagrange's equations for a particle in plane, Classification of dynamical systems, Lagrange's equations for any simple dynamical system, general dynamical system and for impulsive motion; Applications of Lagrange's equations, Hamiltonian and the Canonical equations of motion, The passage from the Hamiltonian to the Lagrangian, Conservative systems.

UNIT – III: Fluid Mechanics (18 hours)

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Velocity of a fluid particle, Irrotational flow, Velocity potential, Equipotential surfaces, Streamlines and Pathlines, Mass flux density, Conservation of mass leading to equation of continuity, Boundary surface; Forces in fluid flows, Conservation of linear momentum and its mathematical formulation (Euler's equation of motion), Bernoulli's equation, Axi-symmetric flows and motion of sphere; Two-dimensional flows, Motion of cylinder, Stream function, Complex potential, Line sources and line sinks, Line doublet, Milne-Thomson circle theorem; Viscous flow, Stress components in a real fluid, Stress and strain analysis, Navier-Stokes equations of motion and its applications.

Essential Readings

1. Chorlton, F. (2005). Textbook of Fluid Dynamics. CBS Publishers, Delhi. Reprint 2018.
2. Synge, J. L. and Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.

Suggestive Readings

- Gantmacher, F. (1975). Lectures in Analytic Mechanics. MIR publisher, Moscow.
- Goldstein, H., Poole, C.P. and Safco, J.L. (2002). Classical Mechanics. (3rd ed.). Addison Wesley.
- Kundu, Piyush K. and Cohen, Ira M., Dowling, David R. (2016). Fluid Mechanics (6th ed.). Academic Press.
- Mitchell, John W. (2020). Fox and McDonald's Introduction to Fluid Mechanics. (10th ed.). John Wiley & Sons.
- Taylor, John R. (2005). Classical Mechanics. University Science Books.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(ii): CRYPTOGRAPHY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Cryptography	4	3	1	0	Class XII pass with Mathematics	Group Theory, Linear Algebra

Learning Objectives: Primary objective of this course is to:

- Learn challenges and types of attacks on the security of cryptographic protocols.
- Understand concept of confusion and diffusion, that is central to the security of symmetric key cryptography.
- Learn mathematical hard problems, which can be used to build various public key cryptosystems.
- Gain knowledge of post quantum cryptography that resist quantum attacks.

Learning Outcomes: This course will enable the students to:

- Learn classical cryptosystems Caesar cipher, Monoalphabetic cipher, Hill cipher, Vigenère cipher and their security analysis.
- Understand Feistel cipher structure to achieve confusion and diffusion in case of Data Encryption Standard (DES).
- Understand Advanced Encryption Standard (AES) structure and its operations along with key generation.
- Learn key sharing protocol – Diffie Hellman key exchange, Public-key cryptosystems – RSA, Elgamal, and Elliptic curve cryptography.
- Learn Lagrange interpolation secret sharing scheme.
- Learn hash functions and their applications, digital signatures scheme.
- Gain knowledge of code-based cryptography – McEliece cryptosystem.

SYLLABUS OF DSE-6(ii)

UNIT–I: Classical Cryptosystems and Review of Finite Fields (15 hours)

Overview of Cryptography, Symmetric key and Public-key cryptography, Security attacks, Relation between key length and security, Objectives and applications of cryptography primitives, Types of attacks from cryptanalyst view, Kerckhoff's principle; Substitution techniques - Caesar cipher, Monoalphabetic cipher, Hill cipher, Vigenère cipher, One-time pad; Euclidean Algorithm, Modular Arithmetic, Statement of Fermat's, Euler's and Chinese Remainder theorems, Discrete logarithm, Finite fields of the form GF(p) and GF(2^n), Binary and ASCII representation, Pseudo-random bit generation.

UNIT – II: Modern Block Ciphers (12 hours)

Introduction to stream and block ciphers, Diffusion and Confusion, The Feistel cipher Structure, Data Encryption Standard (DES); Advanced Encryption Standard (AES) Structure, AES transformation functions, Key expansion, AES Example.

UNIT – III: Public-key Cryptography, Hash Functions, Digital Signatures and Post Quantum Cryptography (18 hours)

Introduction to Public key cryptography, RSA cryptosystem, Diffie Hellman key exchange, Man in the middle attack, Elgamal cryptosystem, Elliptic curve arithmetic, Elliptic curve cryptography, Secret sharing; Hash functions, Applications of hash functions – MAC and digital signature, Simple Hash functions, Security requirements of Hash functions, Properties of SHA family of hash functions; Digital signatures, Elgamal and Schnorr digital signature scheme; Introduction to post quantum cryptography, Linear codes, Generating matrix, Parity check matrix, McEliece cryptosystem.

Essential Readings

1. Stallings, William (2023). Cryptography and Network Security, Principles and Practice (8th ed.). Pearson Education Limited. Global Edition.
2. Stinson, Douglas R. and Paterson, Maura, B. (2019). Cryptography: Theory and Practice (4th ed.). CRC Press.
3. Trappe, Wade and Washington, Lawrence C. (2020). Introduction to Cryptography with Coding Theory (3rd ed.). Pearson Education International.

Suggestive Readings

- Hoffstein, Jeffrey. Pipher, Jill & Silverman, Joseph H. (2014). An Introduction to Mathematical Cryptography (2nd ed.). Springer New York.
- Goldreich O. (2005). Foundations of Cryptography: Basic tools - Vol.1, Cambridge University Press.
- Goldreich O. (2009). Foundations of Cryptography: Vol.2, Basic applications, Cambridge University Press.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(iii): INDUSTRIAL MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Industrial Mathematics	4	3	0	1	Class XII pass with Mathematics	Calculus, Real Analysis, Linear Algebra, Ordinary and Partial Differential Equations

Learning Objectives: The main objective of this course is to:

- Orient the learners to understand nature and working of industrial systems and their models.
- Familiarize the learners with control and maneuvering of industrial processes through sample case-studies and encourage design-thinking and understanding.

Learning Outcomes: This course will enable the students to:

- Determine the controllability, stability, and observability of a system from the model description.
- Comprehend the signal processing landscape and analyse signals using real and spatial domain representations.
- Model/analyse an industrial system from its description and use mathematical formulations to investigate and manipulate the system for specific objectives.

SYLLABUS OF DSE-6(iii)

UNIT – I: Understanding Systems from their Mathematical Description (15 hours)

Continuous-time linear systems, Laplace transform, Transfer function and analogous systems, State-space models, Block-diagram algebra, Signal flow graph, Order of a system and reduced-order models; Discrete-time systems, Z-transform and its inverse, Feedback systems, Stability: Routh-Hurwitz criterion, Root locus method, Controllability and Observability.

UNIT – II: Mathematical Tools for Signals (15 hours)

Signal-to-noise ratio, Analog and digital messages, Channel bandwidth and rate of communication, Modulation, Randomness and redundancy; Signal energy and power, Period and aperiodic signals, Signal operations, Unit impulse function, Vector representation of signals, Orthogonality, Correlation of signals, Signal representation by orthogonal signal sets.

UNIT – III: Case Studies **(15 hours)**
Sample Cases: Continuous casting, Water filtration, Factory fires, Irrigation.

Essential Readings

1. Fulford, Glenn R., and Broadbridge, Philip (2002). Industrial Mathematics: Case Studies. Cambridge University Press.
2. Kheir, Naim A. (Ed.). (1996). Systems Modeling and Computer Simulation, CRC Press.
3. Lathi, B.P., and Ding, Zhi (2019). Modern Digital and Analog Communication Systems (5th ed.). Oxford University Press.

Suggestive Readings

- Friedman A., and Littman W. (1994). Industrial Mathematics: A Course in Solving Real-World Problems. SIAM (Society for Industrial and Applied Mathematics).
- Kreyszig, Erwin (2011). Advance Engineering Mathematics (10th ed.). John Wiley & Sons.
- MacClauer, Charles R. (2000). Industrial Mathematics: Modeling in Industry, Science, and Government. Prentice Hall, Inc.

Practical (30 hours)- Practical/Lab work using:
Mathematica/MATLAB/SciLab/C/C++/Python/R/FORTRAN or similar as per availability.

1. Use following methods to study, describe, and evaluate continuous/discrete systems:
   (a) Root locus method.
   (b) Routh-Horowitz criterion.
   (c) Transfer function using Laplace transform.
   (d) z-transform to convert continuous systems to equivalent discrete systems.
2. To apply controllability and observability analysis on a system description, using corresponding tools/libraries available.
3. To represent a signal/wave as vector data (sampling, choosing basis, and checking orthogonality).
4. To convolve and deconvolve signal/wave functions and represent the result as graphs.

Case Studies:
Besides reading the mentioned case-studies, ONE case may be chosen (in consultation with the instructor) as Semester Assignment for a brief similar study and analysis.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(iv): GEOMETRY OF CURVES AND SURFACES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Geometry of Curves and Surfaces	4	3	1	0	Class XII pass with Mathematics	Calculus

Learning Objectives: The main objective of this course is to:

- Introduces the concept of curves and surfaces in Euclidean spaces $\mathbb{R}^n$.
- Study of the curves and surfaces via the tools of calculus and introduction of concepts like first and second fundamental forms, curvatures, and differential forms.
- Complete the celebrated Gauss-Bonnet theorem that establishes a connection between curvature of a geometric object with its topology.

Learning Outcomes: This course will enable the students to:

- Understand the concept of curves and surfaces embedded in the Euclidean spaces $\mathbb{R}^n$.
- Compute the curvature and torsion for a curve in the space.
- Understand the concept of differential forms and their integration.
- Make sense of the infinitesimal distance element via the study of the Riemannian metric.
- Get prepared to venture into further study of modern differential geometry of manifolds.

SYLLABUS OF DSE-6(iv)

UNIT – I: Geometry of Curves (15 hours)

Concept of plane and space curves with examples, Parametrized plane and space-curves, Concepts of curvatures for curves, Frenet-Serret's formula for space curves, Global theorems for plane and space curves.

UNIT – II: Local Theory of Surfaces in the Space (15 hours)

Concept of surfaces in the space with examples, Fundamental forms and curvatures with examples, Orthonormal frames, Exterior differential forms in two variables and their uses.

UNIT – III: Geometry of Surfaces (15 hours)

Riemannian metric on a surface, Vector fields, Covariant derivatives, Concept of geodesic, Integration of exterior differential forms, Gauss-Bonnet theorem.

Essential Reading

1. Kobayashi, Shoshichi (2019). Differential Geometry of Curves and Surfaces. Springer Nature Singapore Pte Ltd. https://doi.org/10.1007/978-981-15-1739-6

Suggestive Readings

- Abbena Elsa, Salamon Simon, and Gray Alfred (2006). Modern Differential Geometry of Curves and Surfaces with Mathematica (3rd ed.). CRC Press.
- Carmo, Manfredo P. Do (2016). Differential Geometry of Curves and Surfaces (Revised and Updated Second Edition). Dover Publications.
- Pressley, Andrew (2010). Elementary Differential Geometry (2nd ed.). Springer-Verlag.
- Tapp, Kristopher (2016). Differential Geometry of Curves and Surfaces. Springer.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(v): INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Integral Equations and Calculus of Variations	4	3	1	0	Class XII pass with Mathematics	Ordinary, and Partial Differential Equations, Multivariate Calculus

Learning Objectives: The main objective of this course is to:

- Familiarize the learner with methods for solving Volterra and Fredholm integral equations.
- Know the determination of extremum of functional, necessary condition for an extremum, Euler's equation, and its generalization.

Learning Outcomes: This course will enable the students to:

- Compute the solutions to Volterra integral equations by method of resolvent kernel, method of successive approximations, method of Laplace transform, system of Volterra integral equations and integro-differential equation.
- Determine the solutions of Fredholm integral equations and derivation of Hilbert-Schmidt theorem.
- Understand the formulation of variational problems, the variation of a functional and its properties, extremum of functional, necessary condition for an extremum.

SYLLABUS OF DSE-6(v)

UNIT – I: Volterra Integral Equations (12 hours)

Integral equations, Introduction and relation with linear differential equations; Volterra integral equations and its solutions, Method of resolvent kernel, Method of successive approximations, Convolution type of equation, Method of Laplace transform, System of Volterra integral equations, Integro-differential equation, Abel's integral equation and its generalizations.

UNIT – II: Fredholm Integral Equations (18 hours)

Fredholm integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Integral equations with degenerate kernels, Eigenvalues and eigen functions and their properties, Hilbert-Schmidt theorem, Nonhomogeneous Fredholm integral equation with symmetric kernel, Fredholm alternative.

UNIT – III: Calculus of Variations (15 hours)

Variational problems, Variation of a functional and its properties, Extremum of functional, Necessary condition for an extremum, Euler's equation and its generalization, Variational derivative, General variation of a functional and variable end point problem, Sufficient conditions for the extremum of a functional.

Essential Readings

1. Gelfand, I. M. and Fomin, S.V. (2000). Calculus of Variations. Dover Publications, Inc.
2. Krasnov, M., Kiselev, A. and Makarenko, G. (1971). Problems and Exercises Integral Equations, Mir Publication Moscow.
3. Logan, J. David (1987). Applied Mathematics: A Contemporary Approach, John Wiley & Sons, Inc.

Suggestive Readings

- Hildebrand, F. B. (1992). Methods of Applied Mathematics (2nd ed.). Dover Publications.
- Zemyan, Stephen M. (2012). The Classical Theory of Integral Equations: A Concise Treatment. Birkhäuser.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(vi): MACHINE LEARNING: A MATHEMATICAL APPROACH

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Machine Learning: A Mathematical Approach	4	3	0	1	Class XII pass with Mathematics	Basic Knowledge of Python

Learning Objectives: The main objective of this course is to:

- Gain mathematical insights into the functioning of popular methods of Regression, Classification, Clustering and Dimension reduction.
- Understand the mathematical framework of learning and apply it to assess the performance of a number of regression, classification and density estimation algorithms
- Detect overfitting and employ regularization techniques to control it.

Learning Outcomes: This course will enable the students to:

- Learn how to build popular models of regression and classification including Linear regression, Polynomial regression, Logistic classifier, Support vector machine, Decision Tree, Random forests, Naïve Bayes classifier.
- Evaluate the performance of models on test data through analytical techniques (VC bounds and dimension) and Cross-validation to facilitate model selection and feature selection.
- Improve model performance by controlling overfitting through regularization techniques like Ridge and Lasso.

- Understand when to apply dimension reduction and combine it with other supervised learning methods.
- Understand and implement the key principles of Artificial Neural Networks in the context of regression and classification and employ them in function approximation.

SYLLABUS OF DSE-6(vi)

UNIT – I: Introduction to Machine Learning and its Applications (18 hours)

Overview of different tasks: Regression, Classification, and Clustering. Evaluation metrics– Mean Absolute error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE). Linear Regression, Cost function, Polynomial Regression, Gradient Descent Algorithm (GDA). Logistic Regression: Evaluation metrics - accuracy, precision, recall, confusion matrix, Receiver Operating Characteristic Curve (ROC curve) and Area Under ROC Curve (AUC), Vapnik-Chervonenkis (VC) dimension, VC bounds (only statement). *k*-fold validation, Concepts of training set, validation set and test set, Underfitting-Overfitting, Regularization techniques– Ridge, Lasso for Linear Regression and Logistic Regression, Bias-variance tradeoff.

UNIT – II: Popular Machine Learning Techniques (18 hours)

Cross-entropy and Gini Index, Decision Tree, Regression Tree, Random Forest and Bagging, Tree Pruning. Support Vector Machine (SVM), Kernel SVM (Gaussian) Similarity Criterion, *k*-Means clustering technique. Naive Bayes classifier- Linear Discriminant Analysis (LDA), Quadratic Discriminant Analysis (QDA). Dimensionality Reduction, Feature Selection, Principal Component Analysis (PCA).

UNIT – III: Introduction to Deep Learning (9 hours)

Artificial neural network (ANN), Activation functions – definition and examples (Sigmoid, ReLU, Tanh), neurons, layers, Cost function, Information passing, Back propagation algorithm, Optimizers, Learning rate, Statement of Universal Approximation Theorem for continuous functions, Regularization with ANN, Normalization.

Essential Readings

1. Abu-Mostafa, Y. S., Magdon-Ismail, M. & Lin, H.-T. (2012), Learning from Data, AML Book.
2. James, Gareth., Witten, D., Hastie, T., Tibshirani, R. and Taylor, J. (2023), An Introduction to Statistical Learning: with Applications in Python, Springer Nature Switzerland.
3. Ovidiu Calin, Springer. (2020). Deep Learning Architectures: A Mathematical Approach, Springer Nature Switzerland.

Suggestive Readings

- Deisenroth, M. P., Faisal, A. A., and Ong, C. S. (2020), Mathematics for Machine Learning, Cambridge University Press.
- Shalev-Shwartz, S., and Ben-David, S. (2014), Understanding Machine Learning - From Theory to Algorithms, Cambridge University Press.
- Phillips, Jeff. (2020), Mathematical Foundations for Data analysis, Springer.
- Goodfellow, I., Bengio, Y., and Courville, A. (2016), Deep Learning, MIT Press.
- Hastie, T.; Tibshirani, R., and Friedman, J. (2001), The Elements of Statistical Learning, Springer New York Inc.

Practical (30 hours)- Practical work to be performed in computer lab using Python.

Following lab exercises should be done for at least two classification problems or two regression problems or both, whenever applicable.

Following datasets can be used for classification problems

- https://scikit-learn.org/stable/datasets/toy_dataset.html
  (Toy datasets, Iris plants dataset, handwritten digits dataset, Wine recognition dataset, Breast cancer diagnostic dataset)
- https://pypi.org/project/ISLP/
  (Smarket dataset)

Following datasets can be used for regression problems

- https://scikit-learn.org/0.15/modules/classes.html#module-sklearn.datasets
  (Diabetes dataset, Boston house dataset, California housing dataset, Advertising dataset)

Following tasks needs to be performed for the below mentioned ML techniques in scikit learn (https://scikit-learn.org/stable/), whenever applicable:

- Split the dataset into two parts: training and test. Create and train model on training set and report model performance on test set.
- Test the model performance using k-fold cross validation (take k = 5 or 10) in terms of applicable metrics like – Accuracy, Precision, Recall, MAE, RMSE etc.
- Finding optimal parameters using Grid Search CV, whenever applicable; for example: in case of polynomial regression, employ Grid search CV to find the optimal value of the degree d for which the MSE is least.

<u>Practicals List</u>:

1. Create a Linear regression model. Use one variable at a time, all variable at a time, and statistically significant variables (using co-relation matrix) at a time, and observe the model performance. Preferably work with advertising dataset to predict sales in terms of the above features.
2. Fix a 10th order polynomial and sample 15 noisy data points (that is all 15 points do not lie on this polynomial). This is usually done by adding a white noise $\epsilon \sim N(0,1)$ to the polynomial $f(x)$. Using polynomial regression fit two models: one of order 10 and one of order 2. Compare the in-sample and out-sample errors for both models. Try to observe underfitting-overfitting, if any. In another scenario, take $f(x)$ to be a polynomial of order 50 and sample 15 noiseless data points (all lie on the graph of $f(x)$) and again fit a polynomial model of order 2 and 10. Compare the in-sample and out-sample errors. (refer to Exercise 4.2 and Problem 4.4 of [1]).
3. On the Smarket data, predict direction based on features Lag1 and Lag2. Split the Smarket dataset into training and testing parts in the ratio 80-20. Fit logistic regression on the training data and evaluate its accuracy on the test data via confusion matrix, ROC, and AUC. Plot decision boundary for the logistic regression in the 2D feature space spanned by Lag1 and Lag2 (you might need to rescale the variables).
4. Create decision tree models for classification and regression. Observe the effect of various parameters like - splitting criterion (Gini index, Cross entropy), max depth (for tree pruning). Examine overfitting-underfitting in the associated tree model. Display a decision tree.
5. Create Random Forest models for classification and regression. Observe the effect of number of estimators in the context of overfitting.

6. Create SVM models for classification and regression. Observe the effect of the parameter - kernel.
7. Create LDA and QDA models and assess them preferably on the digits dataset.
8. Create *k*-means cluster model for clustering. Observe the effect of parameter *k* (number of clusters). Plot *k* versus error to find out best *k* (Elbow criterion). Plot clusters in case of 2-dimensional data.
9. Demonstrate Principal Component Analysis (PCA) on a dataset with large number of features.
10. Create an ANN model for both classification and regression. Observe the effect of parameters- hidden layer sizes, activation functions (ReLU, Logistic/Sigmoid, Tanh), optimizers (Adam, Sgd), batch size, learning rate, early stopping, validation fraction, maximum number of iterations. Plot iteration number versus accuracy on training and validation dataset. The mnist dataset may be used to explore real strength of ANN. (https://www.kaggle.com/datasets/oddrationale/mnist-in-csv?resource=download in csv format).

B.A. (Prog.) Semester-VIII with Mathematics as Major
Category-II

DISCIPLINE SPECIFIC CORE COURSE (DSC-8): TOPICS IN MULTIVARIATE CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Topics in Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	Calculus

Learning Objectives: The primary objective of this course is to introduce the:
- Extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- Applications of multivariable calculus tools to physics, economics, and optimization.
- Geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- Techniques of integration to functions of two and three independent variables.

Learning Outcomes: This course will enable the students to:
- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems.

SYLLABUS OF DSC-8

UNIT-I: Calculus of Functions of Several Variables (18 hours)
Basic Concepts, Limits and Continuity, Tangent Planes, Partial Derivatives, Total Differential, Differentiability, Chain Rules, Directional Derivatives and the Gradient, Extrema of Functions of Two Variables, Method of Lagrange multipliers with one constraint.

UNIT-II: Double and Triple Integrals (15 hours)
Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

UNIT-III: Green's, Stokes' and Gauss Divergence Theorem (12 hours)

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

Essential Reading

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.

Suggestive Reading

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). First Indian Reprint.

DSE Courses of B.A. (Prog.) Semester-VIII
Category-II

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(i): APPLIED ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Applied Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra, Abstract Algebra

Learning Objectives: The primary objective of this course is to:
- Introduce the applications of linear algebra in the field of science and arts.
- Develop the analytical and numerical skills to apply the algebraic concepts in real-life situations.
- Understand the identification numbers and different check digit schemes that can be used to reduce the errors during their transmission.

Learning Outcomes: This course will enable the students to:
- Understand the system of linear equations, matrices and transformations in the fields of economics, science, engineering and computer science.
- Apply the combinatorics and graph theory in scheduling and reliability theory.
- Learn about identification numbers and using check digits to check for errors after the identification number has been transmitted.

SYLLABUS OF DSE-4(i)

UNIT-I: Applications of Linear Algebra (15 hours)
Applications of linear systems: Leontief input-output model in economics, Traffic flow, and diet problem; Applications to computer graphics, difference equations and Markov chains; Applications to linear models: Least-squares problems, and least-squares lines.

UNIT-II: Latin Squares and Graph Models (12 hours)
Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments; Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings, Interval graph modeling and Influence model, Pitcher pouring puzzle.

UNIT-III: Various Check Digit Schemes (18 hours)
Developing identification numbers, Types of identification numbers, Transmission errors, Check digits, Integer division, Modular arithmetic, US postal money orders, Airline ticket identification numbers, The Universal Product Code check digit scheme, ISBN check digit

scheme, Creating Identification numbers, IBM scheme, Symmetry, Symmetry and Rigid motions, Verhoeff check digit scheme.

Essential Readings

1. David C. Lay, Steven R. Lay and Judi J. McDonald (2016). Linear Algebra and Its Applications (5th ed.). Pearson.
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.
3. Kirtland, Joseph (2001). Identification Numbers and Check Digit Schemes. Mathematical Association of America.

Suggestive Readings

- Andirilli, Stephen and Hecker, David (2016). Elementary Linear Algebra (5th ed.). Academic Press, Elsevier.
- Lidl, Rudolf and Pilz, Günter (1998). Applied Abstract Algebra (2nd ed.). Springer. Indian Reprint 2014.
- Strang, Gilbert (2016). Introduction to Linear Algebra (5th ed.). Wellesley-Cambridge.

DISCIPLINE SPECIFIC ELECTIVE COURSE-4(ii): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

Learning Outcomes: The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type $Rr + Ss + Tt = V$.
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.

SYLLABUS OF DSE-4(ii)

UNIT–I: First-order Partial Differential Equations (18 hours)

Review of basic concepts: Origins of first-order PDEs, Lagrange's method for solving linear equations of first order; Integral surfaces passing through a given curve, and surfaces orthogonal to a given system of surfaces; Nonlinear PDEs of the first order, and compatible systems of first-order PDEs; Charpit's method for solving nonlinear PDEs, special types of first-order PDEs, and solutions satisfying given conditions; Jacobi's method for solving nonlinear PDE with three independent variables.

UNIT – II: Second-order Partial Differential Equations (15 hours)

Origins of second-order PDEs, and solving linear PDEs with constant coefficients using methods of finding the complementary function and particular integral; Monge's method of integrating nonlinear second-order PDE of type $Rr + Ss + Tt = V$ with variable coefficients.

UNIT – III: Applications of Partial Differential Equations (12 hours)

Solution of one-dimensional diffusion equation and wave equation by method of separation of variables, d'Alembert's solution of the Cauchy problem for the one-dimensional wave equation; Solutions of homogeneous one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends; Traffic flow model.

Essential Readings

1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2 Piaggio, H.T.H. (2004). Differential Equations. CBS Publishers & Distributors, Delhi.
3 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

Suggestive Readings

- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- Arrigo, Daniel (2023). An Introduction to Partial Differential Equations (2nd ed.). Springer.
- Kapoor, N. M. (2023). A Text Book of Differential Equations. Pitambar Publishing Company.

DISCIPLINE SPECIFIC ELECTIVE COURSE-4(iii): MATHEMATICAL STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Statistics	4	3	1	0	Class XII pass with Mathematics	Probability and Statistics, Multivariate Calculus

Learning Objectives: The main objective of this course is to introduce the:

- Joint behavior of several random variables theoretically and through illustrative practical examples.
- Theory underlying modern statistics to give the student a solid grounding in (mathematical) statistics and the principles of statistical inference.
- Application of the theory to the statistical modeling of data from real applications, including model identification, estimation, and interpretation.
- Theory and analysis of multivariate data which covers two-factor analysis of variance, multiple linear regression including models for contingency tables.

Learning Outcomes: The course will enable the students to:

- Understand joint distributions of random variables including the multivariate normal distribution.
- Estimate model parameters from the statistical inference based on confidence intervals and hypothesis testing.
- Understand the theory of multiple regression models and contingency tables.
- Apply principles and theory to the statistical modeling and analysis of practical problems in a variety of application areas, and to interpret results and draw conclusions in context.

SYLLABUS OF DSE-4(iii)

UNIT–I: Joint Probability Distributions (15 hours)

Joint probability mass function for two discrete random variables, Marginal probability mass function, Joint probability density function for two continuous random variables, Marginal probability density function, Independent random variables; Expected values, covariance, and correlation; Linear combination of random variables, Moment generating functions of linear combination of random variables; Conditional distributions and conditional expectation, The laws of total expectation and variance; Bivariate normal distribution.

UNIT-II: Sampling Distributions and Estimation (12 hours)

Distribution of important statistics such as the sample totals, sample means, and sample proportions; Joint sampling distribution of sample mean and sample variance, t-statistic and F-statistic distributions based on normal random samples; Concepts and criteria for point estimation, The method of moments estimators and maximum likelihood estimation; Interval estimation and basic properties of confidence intervals, One-sample t confidence interval, Confidence intervals for a population proportion and population variance.

UNIT-III: Tests of Hypotheses, ANOVA and Multiple Regression Analysis (18 hours)

Statistical hypotheses and test procedures, One-sample tests about: population mean, population proportion, and population variance; P-values for tests; Two-sample z-confidence interval and t-confidence interval tests; Single-factor ANOVA, Two-factor ANOVA without replication; Multiple linear regression model and estimating parameters; Chi-squared goodness-of-fit tests, Two-way Contingency tables.

Essential Reading

1. Devore, Jay L., Berk, Kenneth N. & Carlton Matthew A. (2021). Modern Mathematical Statistics with Applications. Third edition, Springer.

Suggestive Readings

- Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences. Ninth edition, Cengage Learning India Private Limited, Delhi. Fourth impression 2022.
- Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2019). Introduction to Mathematical Statistics. Eighth edition, Pearson. Indian Reprint 2020.
- Mood, A.M., Graybill, F.A., & Boes, D.C. (1974). Introduction the Theory of Statistics (3rd ed.). Tata McGraw Hill Pub. Co. Ltd. Reprinted 2017.
- Wackerly, Dennis D., Mendenhall III, William & Scheaffer, Richard L. (2008). Mathematical Statistics with Applications. 7th edition, Cengage Learning.

DISCIPLINE SPECIFIC ELECTIVE COURSE-4(iv): OPTIMIZATION TECHNIQUES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization Techniques	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

Learning Objectives: The primary objective of this course is to introduce:

- Nonlinear optimization problems
- Transshipment and dynamic programming problems
- Integer Programming, fractional programming problems
- Convex and generalized convex functions and their properties

Learning Outcomes: This course will enable the students to:

- Nonlinear programming problems and their applications
- Method to solve fractional programming problems with linear constraints
- Methods to solve dynamic programming problems using recursive computations

SYLLABUS OF DSE-4(iv)

UNIT-I: Transshipment and Dynamic Programming Problems (15 hours)

Transshipment problem, Shortest-route problem; Dynamic programming, Recursive forward and backward computation, Knapsack/fly-away/cargo-loading problems solution through dynamic programming.

UNIT-II: Integer Programming Problems **(15 hours)**
Integer programming problem, Gomory's cutting plane method for integer problems, Mixed integer problems, Branch and bound method.

UNIT-III: Nonlinear Programming Problems **(15 hours)**
Convex functions, Convex programming problems; Generalized convex functions; Linear fractional programming problem, Charnes and Cooper transformation, Simplex algorithm to solve linear fractional programming problem.

Essential Readings

1. Chandra, Suresh, Jayadeva and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Reading

- Swarup, K., Gupta, P.K., and Mohan, M. (1984). Operations Research. Sultan Chand.

DISCIPLINE SPECIFIC ELECTIVE COURSE-4(v): RINGS AND FIELDS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Rings and Fields	4	3	1	0	Class XII pass with Mathematics	Abstract Algebra

Learning Objectives: The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

Learning Outcomes: This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

SYLLABUS OF DSE-4(v)

UNIT-I: Ideals in the quotient rings and Euclidean rings **(15 hours)**

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

UNIT-II: Polynomial Rings and Field Extensions **(15 hours)**

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in *n* variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

UNIT-III: Algebraic Extensions **(15 hours)**

Characterization of field extensions, Finite extensions, Properties of algebraic extensions;

Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field;

Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

Suggestive Readings

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

B.Sc. (Physical Sciences/Mathematical Sciences) Semester-VIII
Category-III

DISCIPLINE SPECIFIC CORE COURSE – (DSC-8): TOPICS IN MULTIVARIATE CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Topics in Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	Calculus

Learning Objectives: The primary objective of this course is to introduce the:
- Extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- Applications of multivariable calculus tools to physics, economics, and optimization.
- Geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- Techniques of integration to functions of two and three independent variables.

Learning Outcomes: This course will enable the students to:
- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems.

SYLLABUS OF DSC-8

UNIT-I: Calculus of Functions of Several Variables (18 hours)
Basic Concepts, Limits and Continuity, Tangent Planes, Partial Derivatives, Total Differential, Differentiability, Chain Rules, Directional Derivatives and the Gradient, Extrema of Functions of Two Variables, Method of Lagrange multipliers with one constraint.

UNIT-II: Double and Triple Integrals (15 hours)
Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

UNIT-III: Green's, Stokes' and Gauss Divergence Theorem (12 hours)

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

Essential Reading

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.

Suggestive Reading

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). First Indian Reprint.

DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Semester-VIII
Category-III

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(i): APPLIED ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Applied Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra, Abstract Algebra

Learning Objectives: The primary objective of this course is to:
- Introduce the applications of linear algebra in the field of science and arts.
- Develop the analytical and numerical skills to apply the algebraic concepts in real-life situations.
- Understand the identification numbers and different check digit schemes that can be used to reduce the errors during their transmission.

Learning Outcomes: This course will enable the students to:
- Understand the system of linear equations, matrices and transformations in the fields of economics, science, engineering and computer science.
- Apply the combinatorics and graph theory in scheduling and reliability theory.
- Learn about identification numbers and using check digits to check for errors after the identification number has been transmitted.

SYLLABUS OF DSE-6(i)

UNIT-I: Applications of Linear Algebra (15 hours)
Applications of linear systems: Leontief input-output model in economics, Traffic flow, and diet problem; Applications to computer graphics, difference equations and Markov chains; Applications to linear models: Least-squares problems, and least-squares lines.

UNIT-II: Latin Squares and Graph Models (12 hours)
Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments; Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings, Interval graph modeling and Influence model, Pitcher pouring puzzle.

UNIT-III: Various Check Digit Schemes (18 hours)
Developing identification numbers, Types of identification numbers, Transmission errors, Check digits, Integer division, Modular arithmetic, US postal money orders, Airline ticket identification numbers, The Universal Product Code check digit scheme, ISBN check digit scheme, Creating Identification numbers, IBM scheme, Symmetry, Symmetry and Rigid motions, Verhoeff check digit scheme.

Essential Readings

1. David C. Lay, Steven R. Lay and Judi J. McDonald (2016). Linear Algebra and Its Applications (5th ed.). Pearson.
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.
3. Kirtland, Joseph (2001). Identification Numbers and Check Digit Schemes. Mathematical Association of America.

Suggestive Readings

- Andirilli, Stephen and Hecker, David (2016). Elementary Linear Algebra (5th ed.). Academic Press, Elsevier.
- Lidl, Rudolf and Pilz, Günter (1998). Applied Abstract Algebra (2nd ed.). Springer. Indian Reprint 2014.
- Strang, Gilbert (2016). Introduction to Linear Algebra (5th ed.). Wellesley-Cambridge.

DISCIPLINE SPECIFIC ELECTIVE COURSE-6(ii): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

Learning Outcomes: The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type $Rr + Ss + Tt = V$.
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.

SYLLABUS OF DSE-6(ii)

UNIT–I: First-order Partial Differential Equations (18 hours)

Review of basic concepts: Origins of first-order PDEs, Lagrange's method for solving linear equations of first order; Integral surfaces passing through a given curve, and surfaces orthogonal to a given system of surfaces; Nonlinear PDEs of the first order, and compatible systems of first-order PDEs; Charpit's method for solving nonlinear PDEs, special types of first-order PDEs, and solutions satisfying given conditions; Jacobi's method for solving nonlinear PDE with three independent variables.

UNIT – II: Second-order Partial Differential Equations (15 hours)

Origins of second-order PDEs, and solving linear PDEs with constant coefficients using methods of finding the complementary function and particular integral; Monge's method of integrating nonlinear second-order PDE of type $Rr + Ss + Tt = V$ with variable coefficients.

UNIT – III: Applications of Partial Differential Equations (12 hours)

Solution of one-dimensional diffusion equation and wave equation by method of separation of variables, d'Alembert's solution of the Cauchy problem for the one-dimensional wave equation; Solutions of homogeneous one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends; Traffic flow model.

Essential Readings

1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2 Piaggio, H.T.H. (2004). Differential Equations. CBS Publishers & Distributors, Delhi.
3 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

Suggestive Readings

- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- Arrigo, Daniel (2023). An Introduction to Partial Differential Equations (2nd ed.). Springer.
- Kapoor, N. M. (2023). A Text Book of Differential Equations. Pitambar Publishing Company.

DISCIPLINE SPECIFIC ELECTIVE COURSE-6(iii): MATHEMATICAL STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Statistics	4	3	1	0	Class XII pass with Mathematics	Probability and Statistics, Multivariate Calculus

Learning Objectives: The main objective of this course is to introduce the:

- Joint behavior of several random variables theoretically and through illustrative practical examples.
- Theory underlying modern statistics to give the student a solid grounding in (mathematical) statistics and the principles of statistical inference.
- Application of the theory to the statistical modeling of data from real applications, including model identification, estimation, and interpretation.
- Theory and analysis of multivariate data which covers two-factor analysis of variance, multiple linear regression including models for contingency tables.

Learning Outcomes: The course will enable the students to:

- Understand joint distributions of random variables including the multivariate normal distribution.
- Estimate model parameters from the statistical inference based on confidence intervals and hypothesis testing.
- Understand the theory of multiple regression models and contingency tables.
- Apply principles and theory to the statistical modeling and analysis of practical problems in a variety of application areas, and to interpret results and draw conclusions in context.

SYLLABUS OF DSE-6(iii)

UNIT–I: Joint Probability Distributions (15 hours)

Joint probability mass function for two discrete random variables, Marginal probability mass function, Joint probability density function for two continuous random variables, Marginal probability density function, Independent random variables; Expected values, covariance, and correlation; Linear combination of random variables, Moment generating functions of linear combination of random variables; Conditional distributions and conditional expectation, The laws of total expectation and variance; Bivariate normal distribution.

UNIT-II: Sampling Distributions and Estimation (12 hours)

Distribution of important statistics such as the sample totals, sample means, and sample proportions; Joint sampling distribution of sample mean and sample variance, *t*-statistic and *F*-statistic distributions based on normal random samples; Concepts and criteria for point estimation, The method of moments estimators and maximum likelihood estimation; Interval estimation and basic properties of confidence intervals, One-sample *t* confidence interval, Confidence intervals for a population proportion and population variance.

UNIT-III: Tests of Hypotheses, ANOVA and Multiple Regression Analysis (18 hours)

Statistical hypotheses and test procedures, One-sample tests about: population mean, population proportion, and population variance; *P*-values for tests; Two-sample *z*-confidence interval and *t*-confidence interval tests; Single-factor ANOVA, Two-factor ANOVA without replication; Multiple linear regression model and estimating parameters; Chi-squared goodness-of-fit tests, Two-way Contingency tables.

Essential Reading

1. Devore, Jay L., Berk, Kenneth N. & Carlton Matthew A. (2021). Modern Mathematical Statistics with Applications. Third edition, Springer.

Suggestive Readings

- Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences. Ninth edition, Cengage Learning India Private Limited, Delhi. Fourth impression 2022.
- Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2019). Introduction to Mathematical Statistics. Eighth edition, Pearson. Indian Reprint 2020.
- Mood, A.M., Graybill, F.A., & Boes, D.C. (1974). Introduction the Theory of Statistics (3rd ed.). Tata McGraw Hill Pub. Co. Ltd. Reprinted 2017.
- Wackerly, Dennis D., Mendenhall III, William & Scheaffer, Richard L. (2008). Mathematical Statistics with Applications. 7th edition, Cengage Learning.

DISCIPLINE SPECIFIC ELECTIVE COURSE-6(iv): OPTIMIZATION TECHNIQUES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization Techniques	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

Learning Objectives: The primary objective of this course is to introduce:

- Nonlinear optimization problems, Transshipment, and dynamic programming problems.
- Integer Programming, and fractional programming problems.
- Convex and generalized convex functions with their properties.

Learning Outcomes: This course will enable the students to:

- Nonlinear programming problems and their applications.
- Method to solve fractional programming problems with linear constraints.
- Methods to solve dynamic programming problems using recursive computations.

SYLLABUS OF DSE-6(iv)

UNIT-I: Transshipment and Dynamic Programming Problems (15 hours)

Transshipment problem, Shortest-route problem; Dynamic programming, Recursive forward and backward computation, Knapsack/fly-away/cargo-loading problems solution through dynamic programming.

UNIT-II: Integer Programming Problems (15 hours)

Integer programming problem, Gomory's cutting plane method for integer problems, Mixed integer problems, Branch and bound method.

UNIT-III: Nonlinear Programming Problems **(15 hours)**

Convex functions, Convex programming problems; Generalized convex functions; Linear fractional programming problem, Charnes and Cooper transformation, Simplex algorithm to solve linear fractional programming problem.

Essential Readings

1. Chandra, Suresh, Jayadeva and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Second Reprint 2016.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Reading

- Swarup, K., Gupta, P.K., and Mohan, M. (1984). Operations Research. Sultan Chand.

DISCIPLINE SPECIFIC ELECTIVE COURSE-6(v): RINGS AND FIELDS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Rings and Fields	4	3	1	0	Class XII pass with Mathematics	Abstract Algebra

Learning Objectives: The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

Learning Outcomes: This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

SYLLABUS OF DSE-6(v)

UNIT-I: Ideals in the quotient rings and Euclidean rings (15 hours)

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

UNIT-II: Polynomial Rings and Field Extensions (15 hours)

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in *n* variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

UNIT-III: Algebraic Extensions (15 hours)

Characterization of field extensions, Finite extensions, Properties of algebraic extensions;

Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field;

Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

Suggestive Readings

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

COMMON POOL OF GENERIC ELECTIVES (GE) Semester-VIII COURSES OFFERED BY DEPARTMENT OF MATHEMATICS

Category-IV

GENERIC ELECTIVES (GE-8(i)): RINGS AND FIELDS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Rings and Fields	4	3	1	0	Class XII pass with Mathematics	Abstract Algebra

Learning Objectives: The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

Learning Outcomes: This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

SYLLABUS OF GE-8(i)

UNIT-I: Ideals in the quotient rings and Euclidean rings (15 hours)

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

UNIT-II: Polynomial Rings and Field Extensions (15 hours)

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in n variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

UNIT-III: Algebraic Extensions **(15 hours)**

Characterization of field extensions, Finite extensions, Properties of algebraic extensions; Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field; Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

Suggestive Readings

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

GENERIC ELECTIVES (GE-8(ii)): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

Learning Outcomes: The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type $Rr + Ss + Tt = V$.
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.

SYLLABUS OF GE-8(ii)

UNIT–I: First-order Partial Differential Equations (18 hours)

Review of basic concepts: Origins of first-order PDEs, Lagrange's method for solving linear equations of first order; Integral surfaces passing through a given curve, and surfaces orthogonal to a given system of surfaces; Nonlinear PDEs of the first order, and compatible systems of first-order PDEs; Charpit's method for solving nonlinear PDEs, special types of first-order PDEs, and solutions satisfying given conditions; Jacobi's method for solving nonlinear PDE with three independent variables.

UNIT – II: Second-order Partial Differential Equations (15 hours)

Origins of second-order PDEs, and solving linear PDEs with constant coefficients using methods of finding the complementary function and particular integral; Monge's method of integrating nonlinear second-order PDE of type $Rr + Ss + Tt = V$ with variable coefficients.

UNIT – III: Applications of Partial Differential Equations (12 hours)

Solution of one-dimensional diffusion equation and wave equation by method of separation of variables, d'Alembert's solution of the Cauchy problem for the one-dimensional wave equation; Solutions of homogeneous one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends; Traffic flow model.

Essential Readings

1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2 Piaggio, H.T.H. (2004). Differential Equations. CBS Publishers & Distributors, Delhi.
3 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

Suggestive Readings

- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- Arrigo, Daniel (2023). An Introduction to Partial Differential Equations (2nd ed.). Springer.
- Kapoor, N. M. (2023). A Text Book of Differential Equations. Pitambar Publishing Company.

GENERIC ELECTIVES (GE-8(iii)): ELEMENTS OF COMPLEX ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Complex Analysis	4	3	1	0	Class XII pass with Mathematics	Metric Spaces, Multivariate Calculus

Learning Objectives: The primary objective of this course is to:

- Acquaint with the basic ideas of complex analysis.
- Learn complex-valued functions with visualization through relevant examples.
- Emphasize on Cauchy's theorems, series expansions and calculation of residues.

Learning Outcomes: The accomplishment of the course will enable the students to:

- Grasp the significance of differentiability of complex-valued functions leading to the understanding of Cauchy-Riemann equations.
- Study some elementary functions and evaluate the contour integrals.
- Learn the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues, and apply Cauchy Residue theorem to evaluate integrals.

SYLLABUS OF GE-8(iii)

Unit-I: Analytic Functions (15 hours)

Basic properties of complex numbers and their exponential form; Limits, continuity, and partial derivatives of functions of two variables. Limits, continuity, and partial derivatives of functions of a complex variable; Cauchy-Riemann Equations, Sufficient conditions for differentiability; Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions.

Unit-II: Complex Integrals (15 hours)

Derivatives of functions, Definite integrals of functions, Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals, Antiderivatives; Statement of Cauchy-Goursat theorem; Cauchy integral formula and its extension, Cauchy’s inequality, Liouville's theorem and the fundamental theorem of algebra.

Unit-III: Series and Residues (15 hours)

Convergence of sequences and series of complex numbers; Taylor, and Laurent series with examples; Isolated singular points, Residues, Cauchy's residue theorem; Types of isolated singular points, Residues at poles and its examples.

Essential Reading

1. Brown, James Ward & Churchill, Ruel V. (2014). Complex Variables and Applications (9th ed.). McGraw-Hill Education. Indian Reprint.

Suggestive Readings

- Bak, Joseph & Newman, Donald J. (2010). Complex Analysis (3rd ed.). Undergraduate Texts in Mathematics, Springer.
- Mathews, John H., & Howell, Rusell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.

GENERIC ELECTIVES (GE-8(iv)): OPTIMIZATION TECHNIQUES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization Techniques	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

Learning Objectives: The primary objective of this course is to introduce:
- Nonlinear optimization problems
- Transshipment and dynamic programming problems
- Integer Programming, fractional programming problems
- Convex and generalized convex functions and their properties

Learning Outcomes: This course will enable the students to:
- Nonlinear programming problems and their applications
- Method to solve fractional programming problems with linear constraints
- Methods to solve dynamic programming problems using recursive computations

SYLLABUS OF GE-8(iv)

UNIT-I: Transshipment and Dynamic Programming Problems (15 hours)
Transshipment problem, Shortest-route problem; Dynamic programming, Recursive forward and backward computation, Knapsack/fly-away/cargo-loading problems solution through dynamic programming.

UNIT-II: Integer Programming Problems (15 hours)
Integer programming problem, Gomory's cutting plane method for integer problems, Mixed integer problems, Branch and bound method.

UNIT-III: Nonlinear Programming Problems (15 hours)
Convex functions, Convex programming problems; Generalized convex functions; Linear fractional programming problem, Charnes and Cooper transformation, Simplex algorithm to solve linear fractional programming problem.

Essential Readings
1. Chandra, Suresh, Jayadeva, and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Reading
- Swarup, K., Gupta, P.K., and Mohan, M. (1984). Operations Research. Sultan Chand.