Teaching Plan: B.Sc. (Hons.) Mathematics, Semester-7

DSC-19: Linear Analysis

Weeks 1 and 2: Normed spaces, Banach spaces, Properties of normed spaces, Finite dimensional normed spaces and subspaces, Compactness and finite dimension.

- [1]: Chapter 2 (Overview of Section 2.1).
- [1]: Chapter 2 (Section 2.2 [2.2-1 to 2.2-6], Section 2.3 [2.3-1], Section 2.4, and Section 2.5 [2.5-1 to 2.5-3]).

Weeks 3 and 4: Matrix norms; Linear operators, Bounded linear operators; Linear functionals, Linear operators and functionals on finite dimensional spaces.

- [2]: Chapter 5 (Section 5.6 [up to Example 5.6.5, page 345]).
- [1]: Chapter 2 (Section 2.6, Section 2.7 [2.7-1 to 2.7-5, and 2.7-8 to 2.7-10], Section 2.8 [2.8-1 to 2.8-5], and Section 2.9).

Week 5 Normed spaces of operators, Dual space.

[1]: Chapter 2 (Section 2.10).

Weeks 6 and 7: Overview of inner product spaces and its properties, Hilbert spaces, Orthogonal complements and direct sums, Orthonormal sets and sequences, Bessel inequality. [1]: Chapter 3 (Section 3.1 [3.1-1 to 3.1-4, and 3.1-6 to 3.1-8), Section 3.2 [3.2-1, 3.2-2, and 3.2-4], Section 3.3 [3.3-1 (only statement), and 3.3-2 to 3.3-7], and Section 3.4 [up to 3.4-6]).

Week 8: Total orthonormal sets and sequences; Riesz representations theorem. [1]: Chapter 3 (Section 3.6 [up to 3.6.4], and Section 3.8).

Weeks 9 and 10. Hilbert-adjoint operator, Self-adjoint, Unitary and normal operators. [1]: Chapter 3 (Section 3.9, and Section 3.10).

Weeks 11 and 12: Hahn Banach theorems for real and complex vector spaces, Hahn Banach theorem for normed spaces.

[1]: Chapter 4 (Section 4.1, Section 4.2, Section 4.3 [4.3-1(only statement), and 4.3-2 to 4.3-4]).

Week 13: Reflexive spaces; Uniform boundedness theorem.

[1]: Chapter 4 (Section 4.6, and Section 4.7 [4.7-1, 4.7-2 (only statement), and 4.7-3]).

Weeks 14 and 15: Open mapping theorem, Closed graph theorem.

[1]: Chapter 4 (Section 4.12 [4.12-1, 4.12-2, and 4.12-3 (only statement)], and Section 4.13).

- 1. Kreyszig, Erwin (1989). Introductory Functional Analysis with Applications (1st ed.). John Wiley & Sons. Wiley-India Student Edition. Indian Reprint 2007.
- 2. Horn, Roger A. and Johnson, Charles R. (2013). Matrix Analysis (2nd ed.). Cambridge University Press.

DSE-5(i): Advanced Differential Equations

Weeks 1 and 2: Well posed problems, Picard's existence theorem, uniqueness and continuity theorems for initial value problems of first order.

[1]: Chapters 1, and 2

[2]: Chapter 10 (Section 10.2).

Week 3: Existence and uniqueness theorems for systems and higher order IVPs.

[2]: Chapter 10 (Section 10.4).

[1]: Chapter 5 (Section 5.1).

Week 4: Sturm separation and comparison theorems; Homogeneous linear systems.

[1]: Chapter 3 (Section 3.5); Chapter 5 (Section 5.2).

Week 5: Nonhomogeneous linear systems, Linear systems with constant coefficients.

[1]: Chapter 5 (Sections 5.3, and 5.4)

Week 6: Stability of autonomous systems of differential equations, Critical points of an autonomous system and their classification, Stability of linear systems with constant coefficients. [1]: Chapter 8 (Sections 8.1, and 8.2).

Week 7: Linear plane autonomous systems, Perturbed systems; Two-point boundary-value problem.

[1]: Chapter 8 (Sections 8.3, and 8.4).

[1]: Chapter 6 (Section 6.1).

Week 8: Green's functions and their construction; Sturm-Liouville systems, Eigenvalues and Eigenfunctions.

[1]: Chapter 6 (Sections 6.2, and 6.3).

[1]: Chapter 7 (Section 7.1).

Weeks 9 and 10: Laplace's equation, Boundary value problems, Maximum and minimum principles, Uniqueness of solution and their continuous dependence on boundary data, Solution of the Dirichlet and Neumann problem for a half plane by Fourier transform method.

[3]: Chapter 4 (Sections 1, and 4)

[Amaranath]: Chapter 2 (Section 2.4 [2.4.1, 2.4.2, 2.4.4, and 2.4.5]).

Week 11: Theory of Green's function for Laplace's equation in three dimensions and application in the solution of the Dirichlet and Neumann problems for semi-infinite spaces; [3]: Chapter 4 (Section 8 [up to the equation 12, page 171]).

Weeks 12 and 13: Wave equation, Helmholtz's first and second theorems, Theory of Green's function for wave equation and its applications.

[3]: Chapter 5 (Sections 1, 6 [up to the equation 5, page 240], and 7).

Weeks 14 and 15: Diffusion equation, Solution of initial boundary value problems for the diffusion equation, Green's function for the diffusion equation and its applications.

[3]: Chapter 6 (Sections 1, 3, and 6).

Essential Readings

- 1. Myint-U, Tyn (1978). Ordinary Differential Equations. Elsevier, North-Holland, Inc.
- 2. Ross S. L. (2007). Differential Equations (3rd ed.) John Wiley & Sons. India.
- 3. Sneddon Ian N. (2006). Elements of Partial Differential Equations. Dover Publications.

Suggestive Reading

Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.

DSE-5(ii): Dynamical Systems

Weeks 1 and 2: Dynamical systems: Discrete and continuous, Population Models, Newton's Method; Discrete dynamical system: Definition, examples and orbits, Periodic and eventually periodic points, Stable and unstable sets, Phase portrait, Graphical analysis of one-dimensional maps. [2]: Chapter 2, and Chapter 3.

Weeks 3 and 4: Hyperbolicity, A glimpse of bifurcations, Analysis of families of logistic maps. [2]: Chapter 4.

Weeks 5 to 7: Symbolic dynamics, Sequence space, Shift map, Itinerary map, Subshifts of finite type, Conjugacy and chaos, Sensitive dependence on initial conditions, Topological transitivity, Devaney chaos, Expansive homeomorphisms, Expansivity of interval and circle maps; Structural stability.

[2]: Chapters 6 to 9, and Chapter 13 [Section 13.1].

[1]: Chapter 2 (Section 2.2 [pp. 36-40, and pp. 53-56]).

Weeks 8 and 9: Sharkovsky's theorem and examples, Schwarzian derivative; Period 3 case. [2]: Chapter 10, Chapter 11, and Chapter 13 [Section 13.2]).

Weeks 10 to 12: Full shifts, Shift spaces, Languages, Higher block shifts and higher power shifts, Sliding block codes;

[3]: Chapter 1 (Sections 1.1 to 1.5).

Weeks 13 and 14: Finite type constraints, Graphs and their shifts, Graph representations of shifts of finite type, Markov chain;

[3]: Chapter 2 (Sections 2.1 to 2.3).

Week 15: Shadowing property and subshifts of finite type.

[1]: Chapter 2 (Section 2.3 [pp.78-81, 84, and pp. 86-87]).

- 1. Aoki, N. and Hiraide, K. (1994). Topological Theory of Dynamical Systems: Recent Advances. Elsevier Science, North-Holland.
- 2. Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press, Taylor & Francis Group.
- 3. Lind, Douglas and Marcus, Brian (2021). An Introduction to Symbolic Dynamics and Coding (2nd ed.). Cambridge University Press.

DSE-5(iii): Fundamentals of Topology

Weeks 1 and 2: Review of the properties of metric spaces; Space of bounded sequences; Space l_p , Space of all sequences of numbers; Convergence and completeness of these spaces; Completion of a metric space.

[2]: Chapter 1 (Examples 1.2.2 (vi), (vii), Examples 1.2.4 (ii) of Section 1.2; Examples 1.3.3 (v), (vii), Theorem 1.3.4 of Section 1.3; Propositions 1.4.10, 1.4.11 of Section 1.4; and Section 1.5).

Weeks 3 and 4: Local base and base, First and second axiom of countability, Separable and Lindelöf spaces.

[2]: Chapter 2 (Section 2.3).

Week 5: Nowhere dense subsets; Category I and II sets; Baire Category Theorem; F_{σ} set; oscillation of a real-valued function over an interval.

[2]: Chapter 2 (Section 2.4 [except Theorems 2.4.6 and 2.4.7, Examples 2.4.8 and 2.4.15]).

Week 6: Extension Theorems; Extension of the concepts of pointwise and uniform convergence of sequence and series of functions for metric spaces, Tietze's Extension Theorem.

[2]: Chapter 3 (Section 3.2, and Section 3.6 [till Examples 3.6.13 (only statements), Proposition 3.6.14, and Theorem 3.6.15 with proofs]).

Week 7: Locally connected and Arcwise connected spaces.

[2] Chapter 4 (Sections 4.2, and 4.3).

Week 8: ϵ -net for a metric space; Totally bounded space; Relation among total boundedness, completeness and compactness.

[2]: Chapter 5 (Section 5.1 [Definition 5.1.7 onwards]).

Week 9: Topology; Basis and subbasis for a topology.

[1] Chapter 2 (Sections 12, and 13).

Week 10: Product and subspace topology.

[1] Chapter 2 (Sections 15, and 16 [up to Theorem 16.3]).

Week 11: Closed sets; Closure, Interior and limit points of a set; Hausdorff spaces.

[1] Chapter 2 (Section 17).

Week 12: Continuous functions, Homeomorphism.

[1] Chapter 2 (Section 18 [up to Example 7, page 107]).

Week 13: The box and the product topology for an indexed family of topological spaces.

[1] Chapter 2 (Section 19).

Week 14: Connectedness.

[1] Chapter 3 (Section 23 [up to Theorem 23.6]).

Week 15: Compactness.

[1]: Chapter 3 (Section 26 [up to Theorem 26.6, and Statement of Theorem 26.7]).

[1]: Chapter 5 (Statement of Theorem 37.3 (Tychonoff Theorem)).

- 1. Munkres James R. (2002). Topology (2nd ed.). Prentice Hall of India Pvt. Ltd.
- 2. Shirali Satish and Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

DSE-5(iv): Information Theory and Coding

Weeks 1 to 3: A measure of uncertainty, H function as a measure of uncertainty, Sources and binary sources, Measure of information for two-dimensional discrete finite probability schemes. [3]: Chapter 3 (Sections 3.1 to 3.4, 3.6, and 3.7).

Weeks 4 and 5: Entropy, Joint entropy and conditional entropy, Relative entropy and mutual information, Chain rules for entropy, Conditional relative entropy and conditional mutual information. [1]: Chapter 2 (Sections 2.1 to 2.5).

Week 6: A measure of mutual information, Interpretation of Shannon's fundamental inequalities, Redundancy, Efficiency and channel capacity.

[3]: Chapter 3 (Sections 3.12, 3.13, and 3.14).

Week 7: Jensen's inequality and its characterizations, The log sum inequality and its applications. [1]: Chapter 2 (Sections 2.6, and 2.7)

Weeks 8 to 10: Introduction to error detecting and correcting codes, Maximum likelihood decoding, Hamming distance, Nearest neighbour/minimum distance decoding, Distance of a code, Main coding theory problems, Equivalence of codes, Sphere-packing bound, Perfect codes, Balanced block designs, Finite fields, The ISBN code.

[2]: Chapter 1, Chapter 2, and Chapter 3.

Weeks 11 and 12: Introduction to vector space over finite fields, Linear codes, Bases for linear codes. [2]: Chapter 4, and Chapter 5.

Weeks 13 to 15: Encoding and decoding with a linear code, Dual code, Generator and parity check matrices, Nearest neighbour decoding for linear codes, Syndrome decoding. Binary Hamming codes, q-ary Hamming codes.

[2]: Chapter 6, Chapter 7, and Chapter 8

- 1. Cover, Thomas M. and Thomas, Joy A. (2006). Elements of Information Theory (2nd ed.). Wiley India. Indian Reprint 2017.
- 2. Hill, Raymond. (1996). A First Course in Coding Theory. Oxford University Press.
- 3. Reza, Fazlollah M. (1961). An Introduction to Information Theory. Dover Publications Inc, New York. Reprint July 2022.

DSE-5(v): Optimization

Week 1: Problem statement of a nonlinear optimization problem, Example of production-inventory, Location facilities, Stochastic resource allocation, Convex sets.

[1]: Chapter 1 (Sections 1.1, and 1.2 [Examples A, and F]), and Chapter 2 (Section 2.1.1).

Week 2: Convex functions, Epigraph and hypograph of a function.

- [1]: Chapter 3 (Sections 3.1.1, 3.1.2, 3.1.3 [only statement with example], 3.2.1, and 3.2.2).
- [2]: Chapter 7 (Examples [7.2.1(i) to 7.2.1(v)]).

Week 3: Differentiable convex function, Twice differentiable convex function.

[1]: Chapter 3 (Sections 3.3.1, 3.3.3 [only statement], 3.3.4, 3.3.5, 3.3.6, 3.3.7 [only statement], and 3.3.8 [only statement]).

Weeks 4 and 5: Minima of convex function, Quasiconvex functions, Pseudoconvex functions.

[1]: Chapter 3 (Sections 3.4.1, 3.4.2, 3.5.1, 3.5.2, 3.5.4 [only statement with example], 3.5.10, and 3.5.11 [only part of statement relating quasiconvexity and pseudoconvexity]).

Week 6: Unconstrained problems: Necessary optimality conditions, Sufficient optimality conditions. [1]: Chapter 4 (Sections 4.1.1, 4.1.2, 4.1.3, 4.1.4 [only statement], 4.1.5, and 4.1.7).

Week 7: Problems having inequality constraints: Fritz John optimality conditions, Karush-Kuhn-Tucker (KKT) necessary optimality conditions.

[1]: Chapter 4 (Sections 4.2.8 [only statement], 4.2.9, 4.2.10, 4.2.11, and 4.2.13).

Week 8: Fritz John necessary conditions for inequality and equality constraints with examples.

[1]: Chapter 4 (Sections 4.3.2 [only statement], 4.3.3, 4.3.4, and 4.3.5).

Week 9: KKT necessary and sufficient conditions for inequality and equality constraints.

- [1]: Chapter 4 (Sections 4.3.7, and 4.3.8).
- [2]: Chapter 8 (Examples 8.5.1, and 8.5.2).

Weeks 10 and 11: Lagrangian dual problem, Weak duality theorem, Duality gap, Strong duality theorem.

- [1]: Chapter 6 (Sections 6.1 [without geometrical interpretation], and 6.1.1).
- [1]: Chapter 6 (Sections 6.2 [6.2.1, 6.2.2, 6.2.3 (only statement), and 6.2.4 (only statement)]).

Week 12: Wolfe's method for quadratic programming problem.

[2]: Chapter 7 (Section 7.7).

Weeks 13 and 14: Descent property, Order of convergence, Global convergence, Steepest descent method, Newton's method.

[2]: Chapter 9 (Sections 9.2.2, 9.2.4, 9.2.5, 9.4, and 9.5).

Week 15: Linear fractional programming problem and simplex algorithm.

[2]: Chapter 12 (Sections 12.4, and 12.5 [simplex algorithm]).

- 1. Bazaraa, Mokhtar S., Sherali, Hanif D. & Shetty, C. M. (2006). Nonlinear Programming: Theory and Algorithms (3rd ed.). John Wiley & Sons. Wiley India (2017).
- 2. Chandra, Suresh, Jayadeva and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.

DSE-5(vi): Research Methodology

Weeks 1 to 2 (For Practical mentioned in the syllabus): Use of technology which includes LaTeX, PSTricks and Beamer.

- [1]: Chapters 9, 10, and 11. {Introduce LaTeX in theory class, and continue in practical class with
- [6]: Chapters 2, 3, and 4. practical list 1-10 as per syllabus from references mentioned here}
- [8]: Chapter 8 (Note: Refer to Chapter 13 from 2nd edition).
- [10]: https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf

Weeks 3 to 5: How to learn mathematics; How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process.

- [1]: Chapters 1, and 2.
- [8]: Chapter 2, and Chapter 3 [Reading for students]). (Note. Refer to Chapters 3, and 4 from 2nd edition).
- [9]: Chapter 0 (pages 19 to 48 by Paul R. Halmos).

Weeks 6 to 8: What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), Publishing research.

- [1]: Chapter 3.
- [8]: Chapters 6, and 10. (Note. Refer to Chapters 6 and 8 from 2nd edition).
- [7]: Chapter 8 (Section 8.3).

Demonstration by few research articles from reputed journals.

Weeks 9 to 11: How to present mathematics: Preparing a mathematical talk, Oral presentation, Poster presentation.

- [1]: Chapter 4.
- [8]: Chapters 12 to 14. (Note. Refer to Chapters 10 to 12 from 2nd edition).
- [7]: Chapter 8.
- [5]: https://www.d.umn.edu/~jgallian/goodPPtalk.pdf

Weeks 12 and 13: Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics

- [1]: Chapter 7.
- [3]: https://en.wikipedia.org/wiki/Journal_ranking#Measures; https://en.wikipedia.org/wiki/Journal_Citation_Reports https://en.wikipedia.org/wiki/Scopus; https://liu.cwp.libguides.com/c.php?g=45770&p=4417804
- [4]: https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874

Weeks 14 and 15: Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

[*]: https://mathscinet.ams.org/, https://zbmath.org/; https://mjl.clarivate.com/; https://www.scopus.com/; https://ugccare.unipune.ac.in/; https://www.turnitin.com/products/ouriginal/

[7]: Chapter 7.

- 1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
- 2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
- 3. Declaration on Research Assessment.
 - https://en.wikipedia.org/wiki/San Francisco Declaration on Research Assessment
- 4. Evaluating Journals using journal metrics;
 - (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
- 5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~igallian/goodPPtalk.pdf). MATH HORIZONS.
- 6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
- 7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Pub. Pte. Ltd.
- 8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, 3rd edn., SIAM, 2000.
- 9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
- 10. Tantau, Till, Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
- 11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

Teaching Plan: B.A. (Prog.) with Mathematics, Semester-7

DSC-7: Numerical Methods

Weeks 1 to 2: Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.

- [2]: Chapter 1 (Sections 1.3.1, and 1.3.2).
- [3]: Chapter 1 (Section 1.3).

Week 3 and 4: Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

- [2]: Chapter 2 (Sections 2.1 to 2.3).
- [3]: Chapter 2 (Sections 2.2, and 2.3).

Weeks 5 to 7: Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss-Seidel method.

- [2]: Chapter 3 (Section 3.1), Chapter 6 (Sections 6.1, and 6.2), Chapter 8 (Section 8.1).
- [3]: Chapter 3 (Sections 3.2, and 3.4).

Weeks 8 to 10: Interpolation: Lagrange form, and Newton form, Finite difference operators.

[3]: Chapter 4 (Sections 4.2, and 4.3).

Weeks 11 and 12: Numerical differentiation: First and second order derivatives.

[2]: Chapter 11 (Sections 11.1 [11.1.1, and 11.1.2]).

Weeks 13 to 15: Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equations: Euler's method.

- [2]: Chapter 11 (Section 11.2 [11.2.1, and 11.2.2]).
- [1]: Chapter 22 (Sections 22.1, and 22.2).

- 1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
- 2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
- 3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.

DSE-3(i): Advanced Linear Algebra

Weeks 1 and 2: The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators.

[1]: Chapter 2 (Section 2.5 [Definitions, and statement of theorems with examples], and Section 2.6 [complete section with Theorem 2.25 (statement), annihilator definition and exercise 13,14, page 127]).

Weeks 3 and 4: Eigenvalues, Eigenvectors, Eigenspaces and characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces.

[1]: Chapter 5 (Definitions, and statement of theorems with examples from Sections 5.1, and 5.2).

Weeks 5 and 6: Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

- [1]: Chapter 5 (Definitions, and statement of theorems with examples from Sections 5.4 up to Example 7 and corollary, p. 316).
- [1]: Chapter 7 ([From Section 7.1, illustrate computing the Jordan canonical form of the matrix given in the Example 2, p. 485], and Section 7.3 [Definition, and Examples 1 to 3]).

Weeks 7 to 9: Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality.

[1]: Chapter 6 (Sections 6.1, and 6.2 [with statement of Theorem 6.4 used for Gram-Schmidt process through examples, Exercise 16 p. 353 for Bessel's inequality]).

Week 10: Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

[1]: Chapter 6 (Definitions, and statement of theorems with examples from Sections 6.3).

Weeks 11 to 13: Normal, self-adjoint, unitary and orthogonal operators and their properties.

[1]: Chapter 6 (Section 6.4 [with statements of Theorems 6.14, and 6.16], and Section 6.5 up to Example 2, p. 379]).

Weeks 14 and 15: Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

- [1]: Chapter 6 (Definition, and statement of theorems with proofs of corollaries from Sections 6.6)
- [1]: Chapter 6 (illustrate finding a singular value decomposition of matrix given in Example 3, p. 407).

Note. The priority should be given to those questions from the exercises which are similar to the Examples given in the respective Sections.

Essential Reading

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

DSE-3(ii): Elements of Metric Spaces

Week 1: Inequalities, Definition and examples of metric spaces.

[1]: Chapter 1 (Sections 1.1 [1.1.1 to 1.1.4 (for p = q = 2)], and 1.2 [1.2.1, 1.2.2 ((i), (ii), (iv), (v), and (xiv)), 1.2.3, and 1.2.4 (i)].

Week 2: Sequences in metric spaces.

[1]: Chapter 1 (Section 1.3 [1.3.1, 1.3.2, 1.3.3 ((i), (iii), and (iv))]).

Week 3: Cauchy sequences, Complete metric space.

[1]: Chapter 1 (Section 1.4 [1.4.1, 1.4.2 (i), 1.4.3, 1.4.4 (i), 1.4.5, 1.4.6, 1.4.7, and 1.4.14 ((i), and (ii))]).

Week 4: Open and closed balls, Neighborhood, Open set, Interior of a set.

[1]: Chapter 2 (Section 2.1 [2.1.1, 2.1.2 ((i), (ii), and (iii)), 2.1.3 to 2.1.6 ((i), and (iii)), 2.1.12 to 2.1.16]).

Week 5: Limit point of a set, Closed set, Closure of a set.

[1]: Chapter 2 (Section 2.1 [2.1.17 to 2.1.32 with Propositions 2.1.19 and 2.1.20 without proofs]).

Week 6: Subspaces.

[1]: Chapter 2 (Section 2.2 [2.2.1, 2.2.2 [statement only], 2.2.3, 2.2.5, and 2.2.6]).

Weeks 7 and 8: Continuous mappings, Sequential criterion, and other characterizations of continuity. [1]: Chapter 3 (Section 3.1 [3.1.1, 3.1.3, 3.1.8 (statement only), and 3.1.9 to 3.1.12]).

Week 9: Uniform continuity.

[1]: Chapter 3 (Section 3.4 [3.4.1 to 3.4.5]).

Weeks 10 and 11: Homeomorphism, Isometry and equivalent metrics.

[1]: Chapter 3 (Section 3.5 [3.5.1 to 3.5.7((i), and (ii))]).

Weeks 12 and 13: Connected subsets of \mathbb{R} , Connectedness and continuous mappings.

[1]: Chapter 4 (Section 4.1 [4.1.1, 4.1.2, 4.1.3, 4.1.4 (statement only), and 4.1.5 to 4.1.8]).

Week 14: Compactness and boundedness, Characterizations of compactness.

[1]: Chapter 5 (Section 5.1 [5.1.1, 5.1.2, 5.1.5 (statement only), and first paragraph of the Remark 5.1.6], and Section 5.2 [5.2.1, 5.2.2, 5.2.3 (all three with statements only), 5.2.4, and 5.2.5]).

Week 15: Continuous functions on compact spaces.

[1]: Chapter 5 (Section 5.3 [5.3.1 to 5.3.8]).

Note: Examples can be discussed in tutorials.

Essential Reading

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

DSE-3(iii): Mathematical Data Science

Weeks 1 and 2: Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas.

[2]: Chapter 2, Chapter 3, and relevant material for different presentation styles from Chapter 9.

[1]: Chapter 1 (up to page 28).

Weeks 3 and 4: Anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.

[1]: Relevant material from Chapters 4, 5, and 6.

[1]: Chapter 1 (pages 29-44, and 58-60).

Week 5: Model driven data in Rⁿ, Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling.

[3]: Chapter 1 (pages 12-13), and Chapter 2 (Section 2.2, 2.3 [2.3.1 to 2.3.3], and 2.4).

Weeks 6 and 7: Norms in Vector Spaces—Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets—Jaccard, and edit distances; Modeling text with distances.

[3]: Chapter 3 (Section 3.3), and Chapter 4 (Sections 4.1 to 4.4).

Weeks 8 and 9: Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.

[3]: Chapter 5, and Chapter 6 (Sections 6.1 to 6.3).

Weeks 10 and 11: Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best *k*-rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis.

[3]: Chapter 7 (Sections 7.1 to 7.7).

Weeks 12 and 13: Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez's algorithm for k-center clustering, Lloyd's algorithm for k-means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering.

[3]: Chapter 8.

Weeks 14 and 15: Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and *k*-nearest neighbors (*k*-NN) classifiers.

[3]: Chapter 9 (Sections 9.1 to 9.5).

- 1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
- 2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
- 3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer. (https://mathfordata.github.io/).

DSE-3(iv): Integral Transforms

Weeks 1 and 2: Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series, Convergence of Fourier series.

[1]: Chapter 6 (Sections 6.1 to 6.5).

Weeks 3 and 4: Examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval. [1]: Chapter 6 (Sections 6.6 to 6.9).

Weeks 5 and 6: The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

[1]: Chapter 6 (Sections 6.10, 6.11, and 6.13).

Weeks 7 to 9: Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine transforms, Convolution properties of Fourier transform.

[1]: Chapter 12 (Sections 12.1 to 12.6, and from Exercises 12.18 [questions 8, and 9]).

Week 10 and 11: Laplace transforms, Properties of Laplace transforms, Convolution theorem of the Laplace transform, Convolution properties of the Laplace transform, Laplace transforms of the Heaviside and Dirac delta functions.

- [1]: Chapter 12 (Sections 12.8 to 12.10, and from Exercises 12.18 [question 27]).
- [1]: Chapter 11 (Section 11.2 for definition of the Dirac delta function).
- [1]: Chapter 12 (Section 12.11 up to Example 12.11.4).

Weeks 12 and 13: Finite Fourier transforms and applications, Applications of Fourier transforms to ordinary differential equations and partial differential equations.

- [1]: Chapter 12 (Section 12.15).
- [2]: Chapter 2 (Section 2.10, and Section 2.12 [Examples 2.12.1 to 2.12.4, and 2.12.7]).

Weeks 14 and 15: Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

[2]: Chapter 4 (Section 4.2 up to Example 4.2.6 [p. 203], and Section 4.3 up to Example 4.3.6 [p. 231]).

- 1. Tyn Myint-U & Lokenath Debnath (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhauser. Indian Reprint.
- 2. Lokenath Debnath & Dambaru Bhatta (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press Taylor & Francis Group.

DSE-3(v): Research Methodology

Weeks 1 to 2 (For Practical mentioned in the syllabus): Use of technology which includes LaTeX, PSTricks and Beamer.

- [1]: Chapters 9, 10, and 11. {Introduce LaTeX in theory class, and continue in practical class with
- [6]: Chapters 2, 3, and 4. practical list 1-10 as per syllabus from references mentioned here}
- [8]: Chapter 8 (Note: Refer to Chapter 13 from 2nd edition).
- [10]: https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf

Weeks 3 to 5: How to learn mathematics; How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process.

- [1]: Chapters 1, and 2.
- [8]: Chapter 2, and Chapter 3 [Reading for students]). (Note. Refer to Chapters 3, and 4 from 2nd edition).
- [9]: Chapter 0 (pages 19 to 48 by Paul R. Halmos).

Weeks 6 to 8: What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), Publishing research.

- [1]: Chapter 3.
- [8]: Chapters 6, and 10. (Note. Refer to Chapters 6 and 8 from 2nd edition).
- [7]: Chapter 8 (Section 8.3).

Demonstration by few research articles from reputed journals.

Weeks 9 to 11: How to present mathematics: Preparing a mathematical talk, Oral presentation, Poster presentation.

- [1]: Chapter 4.
- [8]: Chapters 12 to 14. (Note. Refer to Chapters 10 to 12 from 2nd edition).
- [7]: Chapter 8.
- [5]: https://www.d.umn.edu/~jgallian/goodPPtalk.pdf

Weeks 12 and 13: Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics

- [1]: Chapter 7.
- [3]: https://en.wikipedia.org/wiki/Journal_ranking#Measures; https://en.wikipedia.org/wiki/Journal_Citation_Reports https://en.wikipedia.org/wiki/Scopus; https://liu.cwp.libguides.com/c.php?g=45770&p=4417804
- [4]: https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874

Weeks 14 and 15: Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

[*]: https://mathscinet.ams.org/, https://zbmath.org/; https://mjl.clarivate.com/; https://www.scopus.com/; https://ugccare.unipune.ac.in/; https://www.turnitin.com/products/ouriginal/

[7]: Chapter 7.

- 1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
- 2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
- 3. Declaration on Research Assessment.
 - https://en.wikipedia.org/wiki/San Francisco Declaration on Research Assessment
- 4. Evaluating Journals using journal metrics;
 - (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
- 5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
- 6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
- 7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
- 8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, 3rd edn., SIAM, 2000.
- 9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
- 10. Tantau, Till, Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
- 11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India:Extraordinary, Part-iii-Sec.4)

Teaching Plan: B.Sc. (Physical Sciences/Mathematical Scieces), Semester-7

DSC-7: Numerical Methods

Weeks 1 to 2: Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.

- [2]: Chapter 1 (Sections 1.3.1, and 1.3.2).
- [3]: Chapter 1 (Section 1.3).

Week 3 and 4: Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

- [2]: Chapter 2 (Sections 2.1 to 2.3).
- [3]: Chapter 2 (Sections 2.2, and 2.3).

Weeks 5 to 7: Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss-Seidel method.

- [2]: Chapter 3 (Section 3.1), Chapter 6 (Sections 6.1, and 6.2), Chapter 8 (Section 8.1).
- [3]: Chapter 3 (Sections 3.2, and 3.4).

Weeks 8 to 10: Interpolation: Lagrange form, and Newton form, Finite difference operators.

[3]: Chapter 4 (Sections 4.2, and 4.3).

Weeks 11 and 12: Numerical differentiation: First and second order derivatives.

[2]: Chapter 11 (Sections 11.1 [11.1.1, and 11.1.2]).

Weeks 13 to 15: Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equations: Euler's method.

- [2]: Chapter 11 (Section 11.2 [11.2.1, and 11.2.2]).
- [1]: Chapter 22 (Sections 22.1, and 22.2).

- 1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
- 2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
- 3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.

DSE-5(i): Advanced Linear Algebra

Weeks 1 and 2: The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators.

[1]: Chapter 2 (Section 2.5 [Definitions, and statement of theorems with examples], and Section 2.6 [complete section with Theorem 2.25 (statement), annihilator definition and exercise 13,14, page 127]).

Weeks 3 and 4: Eigenvalues, Eigenvectors, Eigenspaces and characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces.

[1]: Chapter 5 (Definitions, and statement of theorems with examples from Sections 5.1, and 5.2).

Weeks 5 and 6: Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

- [1]: Chapter 5 (Definitions, and statement of theorems with examples from Sections 5.4 up to Example 7 and corollary, p. 316).
- [1]: Chapter 7 ([From Section 7.1, illustrate computing the Jordan canonical form of the matrix given in the Example 2, p. 485], and Section 7.3 [Definition, and Examples 1 to 3]).

Weeks 7 to 9: Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality.

[1]: Chapter 6 (Sections 6.1, and 6.2 [with statement of Theorem 6.4 used for Gram-Schmidt process through examples, Exercise 16 p. 353 for Bessel's inequality]).

Week 10: Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

[1]: Chapter 6 (Definitions, and statement of theorems with examples from Sections 6.3).

Weeks 11 to 13: Normal, self-adjoint, unitary and orthogonal operators and their properties.

[1]: Chapter 6 (Section 6.4 [with statements of Theorems 6.14, and 6.16], and Section 6.5 up to Example 2, p. 379]).

Weeks 14 and 15: Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

- [1]: Chapter 6 (Definition, and statement of theorems with proofs of corollaries from Sections 6.6)
- [1]: Chapter 6 (illustrate finding a singular value decomposition of matrix given in Example 3, p. 407).

Note. The priority should be given to those questions from the exercises which are similar to the Examples given in the respective Sections.

Essential Reading

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

DSE-5(ii): Elements of Metric Spaces

Week 1: Inequalities, Definition and examples of metric spaces.

[1]: Chapter 1 (Sections 1.1 [1.1.1 to 1.1.4 (for p = q = 2)], and 1.2 [1.2.1, 1.2.2 ((i), (ii), (iv), (v), and (xiv)), 1.2.3, and 1.2.4 (i)].

Week 2: Sequences in metric spaces.

[1]: Chapter 1 (Section 1.3 [1.3.1, 1.3.2, 1.3.3 ((i), (iii), and (iv))]).

Week 3: Cauchy sequences, Complete metric space.

[1]: Chapter 1 (Section 1.4 [1.4.1, 1.4.2 (i), 1.4.3, 1.4.4 (i), 1.4.5, 1.4.6, 1.4.7, and 1.4.14 ((i), and (ii))]).

Week 4: Open and closed balls, Neighborhood, Open set, Interior of a set.

[1]: Chapter 2 (Section 2.1 [2.1.1, 2.1.2 ((i), (ii), and (iii)), 2.1.3 to 2.1.6 ((i), and (iii)), 2.1.12 to 2.1.16]).

Week 5: Limit point of a set, Closed set, Closure of a set.

[1]: Chapter 2 (Section 2.1 [2.1.17 to 2.1.32 with Propositions 2.1.19 and 2.1.20 without proofs]).

Week 6: Subspaces.

[1]: Chapter 2 (Section 2.2 [2.2.1, 2.2.2 [statement only], 2.2.3, 2.2.5, and 2.2.6]).

Weeks 7 and 8: Continuous mappings, Sequential criterion, and other characterizations of continuity. [1]: Chapter 3 (Section 3.1 [3.1.1, 3.1.3, 3.1.8 (statement only), and 3.1.9 to 3.1.12]).

Week 9: Uniform continuity.

[1]: Chapter 3 (Section 3.4 [3.4.1 to 3.4.5]).

Weeks 10 and 11: Homeomorphism, Isometry and equivalent metrics.

[1]: Chapter 3 (Section 3.5 [3.5.1 to 3.5.7((i), and (ii))]).

Weeks 12 and 13: Connected subsets of \mathbb{R} , Connectedness and continuous mappings.

[1]: Chapter 4 (Section 4.1 [4.1.1, 4.1.2, 4.1.3, 4.1.4 (statement only), and 4.1.5 to 4.1.8]).

Week 14: Compactness and boundedness, Characterizations of compactness.

[1]: Chapter 5 (Section 5.1 [5.1.1, 5.1.2, 5.1.5 (statement only), and first paragraph of the Remark 5.1.6], and Section 5.2 [5.2.1, 5.2.2, 5.2.3 (all three with statements only), 5.2.4, and 5.2.5]).

Week 15: Continuous functions on compact spaces.

[1]: Chapter 5 (Section 5.3 [5.3.1 to 5.3.8]).

Note: Examples can be discussed in tutorials.

Essential Reading

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

DSE-5(iii): Mathematical Data Science

Weeks 1 and 2: Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas.

[2]: Chapter 2, Chapter 3, and relevant material for different presentation styles from Chapter 9.

[1]: Chapter 1 (up to page 28).

Weeks 3 and 4: Anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.

[1]: Relevant material from Chapters 4, 5, and 6.

[1]: Chapter 1 (pages 29-44, and 58-60).

Week 5: Model driven data in Rⁿ, Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling.

[3]: Chapter 1 (pages 12-13), and Chapter 2 (Section 2.2, 2.3 [2.3.1 to 2.3.3], and 2.4).

Weeks 6 and 7: Norms in Vector Spaces—Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets—Jaccard, and edit distances; Modeling text with distances.

[3]: Chapter 3 (Section 3.3), and Chapter 4 (Sections 4.1 to 4.4).

Weeks 8 and 9: Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.

[3]: Chapter 5, and Chapter 6 (Sections 6.1 to 6.3).

Weeks 10 and 11: Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best *k*-rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis.

[3]: Chapter 7 (Sections 7.1 to 7.7).

Weeks 12 and 13: Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez's algorithm for k-center clustering, Lloyd's algorithm for k-means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering.

[3]: Chapter 8.

Weeks 14 and 15: Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and *k*-nearest neighbors (*k*-NN) classifiers.

[3]: Chapter 9 (Sections 9.1 to 9.5).

- 1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
- 2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
- 3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer. (https://mathfordata.github.io/).

DSE-5(iv): Integral Transforms

Weeks 1 and 2: Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series, Convergence of Fourier series.

[1]: Chapter 6 (Sections 6.1 to 6.5).

Weeks 3 and 4: Examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval. [1]: Chapter 6 (Sections 6.6 to 6.9).

Weeks 5 and 6: The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

[1]: Chapter 6 (Sections 6.10, 6.11, and 6.13).

Weeks 7 to 9: Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine transforms, Convolution properties of Fourier transform.

[1]: Chapter 12 (Sections 12.1 to 12.6, and from Exercises 12.18 [questions 8, and 9]).

Week 10 and 11: Laplace transforms, Properties of Laplace transforms, Convolution theorem of the Laplace transform, Convolution properties of the Laplace transform, Laplace transforms of the Heaviside and Dirac delta functions.

- [1]: Chapter 12 (Sections 12.8 to 12.10, and from Exercises 12.18 [question 27]).
- [1]: Chapter 11 (Section 11.2 for definition of the Dirac delta function).
- [1]: Chapter 12 (Section 12.11 up to Example 12.11.4).

Weeks 12 and 13: Finite Fourier transforms and applications, Applications of Fourier transforms to ordinary differential equations and partial differential equations.

- [1]: Chapter 12 (Section 12.15).
- [2]: Chapter 2 (Section 2.10, and Section 2.12 [Examples 2.12.1 to 2.12.4, and 2.12.7]).

Weeks 14 and 15: Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

[2]: Chapter 4 (Section 4.2 up to Example 4.2.6 [p. 203], and Section 4.3 up to Example 4.3.6 [p. 231]).

- 1. Tyn Myint-U & Lokenath Debnath (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhauser. Indian Reprint.
- 2. Lokenath Debnath & Dambaru Bhatta (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press Taylor & Francis Group.

DSE-5(v): Research Methodology

Weeks 1 to 2 (For Practical mentioned in the syllabus): Use of technology which includes LaTeX, PSTricks and Beamer.

- [1]: Chapters 9, 10, and 11. {Introduce LaTeX in theory class, and continue in practical class with
- [6]: Chapters 2, 3, and 4. practical list 1-10 as per syllabus from references mentioned here}
- [8]: Chapter 8 (Note: Refer to Chapter 13 from 2nd edition).
- [10]: https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf

Weeks 3 to 5: How to learn mathematics; How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process.

- [1]: Chapters 1, and 2.
- [8]: Chapter 2, and Chapter 3 [Reading for students]). (Note. Refer to Chapters 3, and 4 from 2nd edition).
- [9]: Chapter 0 (pages 19 to 48 by Paul R. Halmos).

Weeks 6 to 8: What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), Publishing research.

- [1]: Chapter 3.
- [8]: Chapters 6, and 10. (Note. Refer to Chapters 6 and 8 from 2nd edition).
- [7]: Chapter 8 (Section 8.3).

Demonstration by few research articles from reputed journals.

Weeks 9 to 11: How to present mathematics: Preparing a mathematical talk, Oral presentation, Poster presentation.

- [1]: Chapter 4.
- [8]: Chapters 12 to 14. (Note. Refer to Chapters 10 to 12 from 2nd edition).
- [7]: Chapter 8.
- [5]: https://www.d.umn.edu/~jgallian/goodPPtalk.pdf

Weeks 12 and 13: Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics

- [1]: Chapter 7.
- [3]: https://en.wikipedia.org/wiki/Journal_ranking#Measures; https://en.wikipedia.org/wiki/Journal_Citation_Reports https://en.wikipedia.org/wiki/Scopus; https://liu.cwp.libguides.com/c.php?g=45770&p=4417804
- [4]: https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874

Weeks 14 and 15: Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

[*]: https://mathscinet.ams.org/, https://zbmath.org/; https://mjl.clarivate.com/; https://www.scopus.com/; https://ugccare.unipune.ac.in/; https://www.turnitin.com/products/ouriginal/

[7]: Chapter 7.

- 1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
- 2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
- 3. Declaration on Research Assessment.
 - https://en.wikipedia.org/wiki/San Francisco Declaration on Research Assessment
- 4. Evaluating Journals using journal metrics;
 - (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
- 5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
- 6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
- 7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
- 8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
- 9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
- 10. Tantau, Till, Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
- 11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India:Extraordinary, Part-iii-Sec.4)

Teaching Plan for Generic Electives (GE), Semester-7

GE-7(i): Applied Algebra

Weeks 1 to 3: Applications of linear systems: Leontief input-output model in economics, Traffic flow, and diet problem; Applications to computer graphics.

- [1]: Chapter 1 (Section 1.6 [Examples 1, and 2], and Section 1.10 [up to Example 1, p. 82]).
- [1]: Chapter 2 (Section 2.6 [Examples 1, and 2], and Section 2.7 [up to Example 6, p. 142]).

Weeks 4 and 5: Difference equations and Markov chains, Applications to linear models: Least-squares problems, least-square lines.

- [1]: Chapter 4 (Section 4.8 [up to Example 6, p. 252], and Section 4.9).
- [1]: Chapter 6 (Section 6.5 [up to Example 3, p. 366, Theorems 13 and 14 without proofs], and Section 6.6 [up to Example 1, p. 372]).

Weeks 6 to 9: Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments, Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings, Interval graph modeling and Influence model, Pitcher pouring puzzle. [Lidl, Rudolf and Pilz, Günter]: Chapter 7 (Section 35 [35.1, 35.2, and 35.6]).

[2]: Chapter 1 (Section 1.1, Examples 1 to 6), and Chapter 3 (Section 3.2, Example 3, Page 106).

Weeks 10 and 11: Developing identification numbers, Types of identification numbers, Transmission errors, Check digits, Integer division, Modular arithmetic.

[3]: Chapter 1, and Chapter 2 (Sections 2.2, and 2.3).

Weeks 12 and 13: US postal money orders, Airline ticket identification numbers, The Universal Product Code check digit scheme, ISBN check digit scheme, Creating identification numbers, IBM scheme. [3] Chapter 2 (Sections 2.4 to 2.7), and Chapter 3 (Sections 3.2, and 3.5).

Weeks 14 and 15: Symmetry, Symmetry and Rigid motions, Verhoeff check digit scheme. [3] Chapter 4, and Chapter 5 (Section 5.4).

Essential Readings

- 1. David C. Lay, Steven R. Lay and Judi J. McDonald (2016). Linear Algebra and Its Applications (5th ed.). Pearson.
- 2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.
- 3. Kirtland, Joseph (2001). Identification Numbers and Check Digit Schemes. Mathematical Association of America.

Suggestive Reading

 Lidl, Rudolf and Pilz, Günter (1998). Applied Abstract Algebra (2nd ed.). Springer. Indian Reprint 2014.

GE-7(ii): Elements of Metric Spaces

Week 1: Inequalities, Definition and examples of metric spaces.

[1]: Chapter 1 (Sections 1.1 [1.1.1 to 1.1.4 (for p = q = 2)], and 1.2 [1.2.1, 1.2.2 ((i), (ii), (iv), (v), and (xiv)), 1.2.3, and 1.2.4 (i)].

Week 2: Sequences in metric spaces.

[1]: Chapter 1 (Section 1.3 [1.3.1, 1.3.2, 1.3.3 ((i), (iii), and (iv))]).

Week 3: Cauchy sequences, Complete metric space.

[1]: Chapter 1 (Section 1.4 [1.4.1, 1.4.2 (i), 1.4.3, 1.4.4 (i), 1.4.5, 1.4.6, 1.4.7, and 1.4.14 ((i), and (ii))]).

Week 4: Open and closed balls, Neighborhood, Open set, Interior of a set.

[1]: Chapter 2 (Section 2.1 [2.1.1, 2.1.2 ((i), (ii), and (iii)), 2.1.3 to 2.1.6 ((i), and (iii)), 2.1.12 to 2.1.16]).

Week 5: Limit point of a set, Closed set, Closure of a set.

[1]: Chapter 2 (Section 2.1 [2.1.17 to 2.1.32 with Propositions 2.1.19 and 2.1.20 without proofs]).

Week 6: Subspaces.

[1]: Chapter 2 (Section 2.2 [2.2.1, 2.2.2 [statement only], 2.2.3, 2.2.5, and 2.2.6]).

Weeks 7 and 8: Continuous mappings, Sequential criterion, and other characterizations of continuity. [1]: Chapter 3 (Section 3.1 [3.1.1, 3.1.3, 3.1.8 (statement only), and 3.1.9 to 3.1.12]).

Week 9: Uniform continuity.

[1]: Chapter 3 (Section 3.4 [3.4.1 to 3.4.5]).

Weeks 10 and 11: Homeomorphism, Isometry and equivalent metrics.

[1]: Chapter 3 (Section 3.5 [3.5.1 to 3.5.7((i), and (ii))]).

Weeks 12 and 13: Connected subsets of \mathbb{R} , Connectedness and continuous mappings.

[1]: Chapter 4 (Section 4.1 [4.1.1, 4.1.2, 4.1.3, 4.1.4 (statement only), and 4.1.5 to 4.1.8]).

Week 14: Compactness and boundedness, Characterizations of compactness.

[1]: Chapter 5 (Section 5.1 [5.1.1, 5.1.2, 5.1.5 (statement only), and first paragraph of the Remark 5.1.6], and Section 5.2 [5.2.1, 5.2.2, 5.2.3 (all three with statements only), 5.2.4, and 5.2.5]).

Week 15: Continuous functions on compact spaces.

[1]: Chapter 5 (Section 5.3 [5.3.1 to 5.3.8]).

Note: Examples can be discussed in tutorials.

Essential Reading

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

GE-7(iii): Introduction to Graph Theory

Week 1: Graphs and their representation, Pseudographs, Subgraphs, Degree sequence, Euler's theorem. [1]: Chapter 9 (Sections 9.1, and 9.2).

Week 2: Isomorphism of Graphs, Paths and Circuits, Connected graphs, Eulerian circuits.

[1]: Chapter 9 (Section 9.3).

[1]: Chapter 10 (Section 10.1 [Theorems 10.1.4, and 10.1.5 without proofs]).

Week 3: Hamiltonian paths cycles, Adjacency Matrix.

[1]: Chapter 10 (Sections 10.2 [Theorems 10.2.4, and 10.2.6 without proofs, exclude 10.2.3], and 10.3).

Week 4: Weighted graphs, Travelling salesman problem, Shortest path problem, Dijkstra's algorithm (without proof), Dijkstra's algorithm improved (without proof).

[1]: Chapter 10 (Section 10.4 up to 10.4.3 [applications only]).

Week 5: The Chinese postman problem; Digraphs, Bellman-Ford algorithm.

[1]: Chapter 11 (Sections 11.1, and 11.2).

Week 6: Tournaments, Directed network, Scheduling problem.

[1]: Chapter 11 (Sections 11.4, and 11.5).

Weeks 7 and 8: Trees and their properties, Spanning Trees.

[1]: Chapter 12 (Sections 12.1 [with exercise 26 on forest], and 12.2 [Theorem 12.2.3 without proof]).

Week 9 and 10: Minimum Spanning Tree Algorithms: Kruskal's algorithm, Prim's algorithm (without proofs), Acyclic digraphs and Bellman's algorithm.

[1]: Chapter 12 (Sections 12.3, and 12.4 [Proposition 12.4.5, and corollary 12.4.6 without proofs]).

Week 11: Planar graphs, Euler's formula, Kuratowski theorem.

[1]: Chapter 13 (Section 13.1).

Week 12: Graph coloring, Applications of graph coloring.

[1]: Chapter 13 (Section 13.2 [Theorem 13.2.4 without proof]).

Week 13: Circuit testing and facilities design.

[1]: Chapter 13 (Section 13.3).

Week 14 and 15: Flows and cuts, Max flow-min cut theorem, Matchings, Hall's theorem.

[1]: Chapter 14 (Sections 14.1, 14.2, and 14.4).

Essential Reading

1. Goodaire, Edgar G., & Parmenter, Michael M. (2011). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.

GE-7(iv): Topics in Multivariate Calculus

Week 1: Definition and examples of function of two variables, Limits and continuity of functions of two variables.

[1]: Chapter 11 (Sections 11.1 [Example 1], and 11.2).

Week 2: Partial differentiation and partial derivatives as slope and rate (definitions only), Higher order partial derivatives.

[1]: Chapter 11 (Section 11.3).

Week 3: Tangent planes, Total differential, Differentiability, Chain rule for one independent variable.

[1]: Chapter 11 (Sections 11.4 [except Examples 2 and 3], and 11.5 [up to Example 5]).

Weeks 4 and 5: Extensions of the chain rule to two independent parameters, Directional derivatives, Gradient, Maximal and normal property of the gradient.

[1]: Chapter 11 (Section 11.5 [Examples 6 to 8], and 11.6 [up to Example 7]).

Week 6: Extrema of functions of two variables, Lagrange multipliers method for optimization problems with one constraint.

[1] Chapter 11 (Section 11.7 [up to Example 6], and Section 11.8 [Examples 1 and 2]).

Weeks 7 and 8: Double integration over rectangular regions, and nonrectangular regions, Double integrals in polar coordinates

[1]: Chapter 12 (Sections 12.1 to 12.3).

Week 9: Triple integral over a parallelepiped, and solid regions, Volume by triple integrals.

[1]: Chapter 12 (Section 12.5).

Week 10: Triple integration in cylindrical and spherical coordinates.

[1]: Chapter 12 (Section 12.7 [except Examples 3, and 6]).

Week 11: Jacobians: Change of variables in double and triple integrals.

[1]: Chapter 12 (Section 12.8 [except Example 5]).

Week 12: Line integrals, Applications of line integrals to mass and work, Fundamental theorem for line integrals, Conservative vector fields.

[1]: Chapter 13 (Sections 13.2, and 13.3).

Week 13: Green's theorem, Area as a line integral.

[1]: Chapter 13 (Section 13.4 [up to Example 3]).

Week 14: Surface integrals.

[1]: Chapter 13 (Section 13.5 [up to Example 3]).

Week 15: Stokes' theorem (statement only) and Gauss divergence theorem (statement only).

[1]: Chapter 13 (Sections 13.6 [up to Example 3], and 13.7 [up to Example 3]).

Note. For weeks 9 to 11, please refer to sections 12.4, 12.5, and 12.6 while using the Indian print.

Essential Reading

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.