

Courses based on Under Graduate Curriculum Framework (UGCF) -2022 DEPARTMENT OF MATHEMATICS COMMON POOL OF GENERIC ELECTIVES (GE) COURSES OFFERED [For all courses (other than B.Sc. (Hons.) Mathematics) under UGCF 2022]

Level/Category	Semester	GE (Generic Electives) COURSES
100 (First Category)	Ι	 GE-1(i): Fundamentals of Calculus (3L + 1T) or GE-1(ii): Theory of Equations and Symmetries (3L + 1T)
	Π	GE-2(i): Analytic Geometry (3L + 1T) or GE-2(ii): Introduction to Linear Algebra (3L + 1T)

200	III	GE-3(i) : Differential Equations $(3L + 1T)$
(Second Category)		or
		GE-3(ii) : Lattices and Number Theory (3L + 1T)
_	IV	GE-4(i) : Elements of Real Analysis (3L + 1T)
		or
		GE-4(ii) : Linear Programming (3L + 1T)
	V	GE-5(i) : Numerical Methods (3L + 1P)
		or
		GE-5(ii) : Mathematical Python (3L + 1P)
		or
		GE-5(iii) : Elementary Mathematical Analysis (3L + 1T)
_	VI	GE-6(i) : Introduction to Mathematical Modeling (3L + 1P)
		or
		GE-6(ii) : Discrete Dynamical Systems (3L + 1P)
		or
		GE-6(iii) : Abstract Algebra (3L + 1T)

300	VII	Choose maximum <i>two</i>
(Third Category)		GE-7(i): Applied Algebra $(3L + 1T)$
		GE-7(ii) : Elements of Metric Spaces (3L + 1T)
		GE-7(iii) : Introduction to Graph Theory (3L + 1T)
		GE-7(iv) : Topics in Multivariate Calculus (3L + 1T)
	VIII	Choose maximum <i>two</i>
		GE-8(i) : Rings and Fields $(3L + 1T)$
		GE-8(ii) : Elements of Partial Differential Equations (3L + 1T)
		GE-8(iii) : Elements of Complex Analysis (3L + 1T)
		GE-8(iv) : Optimization Techniques (3L + 1T)

3L = 3 Hours Lecture; 1T = 1 Hour Tutorial; 1P = 2 Hours Practical.

UNIVERSITY OF DELHI

CNC-II/093/1(22)/2022-23/216 Dated: 10.10.2022

NOTIFICATION

Sub: Amendment to Ordinance V

[E.C Resolution No. 18-1/ (18-1-4) dated 18.08.2022]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-I of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 to be implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICS

COMMON POOL OF GENERIC ELECTIVES (GE) COURSES OFFERED BY DEPARTMENT OF MATHEMATICS CATEGORY-IV

GE-1(i)

GENERIC ELECTIVES: FUNDAMENTALS OF CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit di Lecture		of the course Practical/ Practice	Eligibility criteria	Pre- requisite of the course
						(if any)
Fundamentals of Calculus	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives

The Learning Objectives of this course is as follows:

• Understand the quantitative change in the behaviour of the variables and apply them on the problems related to the environment.

Learning Outcomes

Upon completion of this course, students will be able to:

- Understand continuity and differentiability in terms of limits.
- Describe asymptotic behavior in terms of limits involving infinity.
- Understand the importance of mean value theorems and its applications.
- Learn about Maclaurin's series expansion of elementary functions.
- Use derivatives to explore the behavior of a given function, locating and classifying its extrema, and graphing the polynomial and rational functions.

SYLLABUS OF GE-1(i)

Theory

Unit – 1

Continuity and Differentiability of Functions

Limits and continuity, Types of discontinuities; Differentiability of functions; Successive differentiation: Calculation of the nth derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

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Unit – 2

Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications to monotonic functions and inequalities; Expansion of functions: Taylor's theorem, Taylor's series, Maclaurin's series expansion of e^x , $\sin x$, $\cos x$, $\log(1 + x)$ and $(1 + x)^m$; Indeterminate forms.

Unit – 3

Tracing of Curves

Concavity and inflexion points, Asymptotes (parallel to axes and oblique), Relative extrema, Tracing graphs of polynomial functions, rational functions, and polar equations. **Practical component (if any)** – **NIL**

Essential Readings

- Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). Calculus (10th ed.). Wiley India Pvt. Ltd. New Delhi. International Student Version. Indian Reprint 2016.
- Prasad, Gorakh (2016). Differential Calculus (19th ed.). Pothishala Pvt. Ltd. Allahabad.

Suggestive Reading

• Thomas Jr., George B., Weir, Maurice D., & Hass, Joel (2014). Thomas' Calculus (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

GE-1(ii)

GENERIC ELECTIVES: THEORY OF EQUATIONS AND SYMMETRIES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title	Credits	Credit distribution of the course			Eligibility	Pre-
& Code		Lecture	Tutorial	Practical/ Practice	criteria	requisite of the course (if any)
Theory of Equations and Symmetries	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives

The goal of this course is to acquaint students with certain ideas about:

- Integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial.
- Finding roots of cubic and quartic equations in special cases using elementary symmetric functions.
- Using Cardon's and Descartes' methods, respectively.

(15 hours)

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Learning outcomes

After completion of this course, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

SYLLABUS OF GE-1(ii)

Theory

Unit - 1

Polynomial Equations and Properties

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the nth roots of unity and symmetries of the solutions.

Unit - 2

Cubic and Biquadratic (Quartic) Equations

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

Unit - 3

Symmetric Functions

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric

functions such as $\sum \alpha^2 \beta$, $\sum \alpha^2 \beta^2$, $\sum \alpha^2 \beta \gamma$, $\sum \frac{1}{\alpha^2 \beta \gamma}$, $\sum \alpha^{-3}$, $\sum (\beta + \gamma - \alpha)^2$, $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$,... of

polynomial equations; Transformation of equations by symmetric functions and in general.

Practical component (if any) – NIL

Essential Readings

- Burnside, W.S., & Panton, A.W. (1979). The Theory of Equations (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
- Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785

Suggestive Reading

• Prasad, Chandrika (2017). Text Book of Algebra and Theory of Equations. Pothishala Pvt Ltd.

(12 hours)

(18 hours)

UNIVERSITY OF DELHI

CNC-II/093/1(23)/2022-23/451 Dated: 03.03.2023

NOTIFICATION

Sub: Amendment to Ordinance V [E.C Resolution No. 38-1/ (38-1-4) dated 08.12.2022]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-II of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 to be implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICS

GENERIC ELECTIVES (GE-2(i)): ANALYTIC GEOMETRY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits		r	of the course	Eligibility	Pre-requisite
title & Code		Lecture	Tutorial	Practical/ Practice	criteria	of the course
Analytic Geometry	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The course aims at:

- Identifying and sketching curves, studying three dimensional objects, their geometric properties and applications.
- Use of vector approach to three-dimensional geometry makes the study simple and elegant.

Learning Outcomes: This course will enable the students to:

- Learn concepts in two-dimensional geometry.
- Identify and sketch conics namely, ellipse, parabola and hyperbola.
- Learn about three-dimensional objects such as straight lines and planes using vectors, spheres, cones and cylinders.

SYLLABUS OF GE-2(i)

UNIT – I: Conic Sections

Techniques for sketching parabola, ellipse and hyperbola; Reflection properties of parabola, ellipse, hyperbola, and their applications to signals; Classification of quadratic equation representing lines, parabola, ellipse and hyperbola; Rotation of axes; Second degree equations.

UNIT – II: Vectors, Lines and Planes

Rectangular coordinates in 3-dimensional space, vectors viewed geometrically, vectors in coordinate systems and vectors determined by length and angle; Dot product; Projections; Cross product, scalar triple product, vector triple product and their geometrical properties; Parametric equations of lines, direction cosines and direction ratios of a line, vector and symmetric equations of lines, angle between two lines; Planes in 3-dimensional space, coplanarity of two lines, angle between two planes, distance of a point from a plane, angle between a line and a plane, distance between parallel planes; Shortest distance between two skew lines.

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(15 hours)

(18 hours)

UNIT – III: Sphere, Cone and Cylinder

Equation of a sphere, plane section of sphere, tangents and tangent plane to a sphere; Equation of a cone, enveloping cone of a sphere, Reciprocal cones and right circular cone; Equation of a cylinder, enveloping cylinder and right circular cylinder.

Recommended Readings:

- 1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pte. Ltd. Indian reprint (2016) by Wiley India Pvt. Ltd. Delhi.
- 2. Narayan, Shanti & Mittal, P. K. (2007). *Analytical Solid Geometry*. S. Chand & Company Pvt Ltd. India.

Suggestive Readings:

- Bell, Robert J.T. (1972). An Elementary Treatise on Coordinate Geometry of Three Dimensions. Macmillan & Co. Ltd. London.
- George B. Thomas, Jr., & Ross L. Finney (2012). *Calculus and Analytic Geometry* (9th ed.). Pearson Indian Education Services Pvt Ltd. India.

GE-2(ii)

GENERIC ELECTIVES (GE-2(ii)): INTRODUCTION TO LINEAR ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title	Credits	Credit d	istribution	of the course	Eligibility	Pre-requisite
& Code		Lecture	Tutorial	Practical/	criteria	of the course
				Practice		
Introduction to Linear Algebra	4	3	1	0	Class XII pass with Mathematic	NIL
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Learning Objectives: The objective of the course is:

- To introduce the concept of vectors in \mathbb{R}^n .
- Understand the nature of solution of system of linear equations.
- To view the $m \times n$ matrices as a linear function from R^n to R^m and vice versa.
- To introduce the concepts of linear independence and dependence, rank and linear transformations has been explained through matrices.

Learning Outcomes: This course will enable the students to:

- Visualize the space R^n in terms of vectors and the interrelation of vectors with matrices.
- Understand important uses of eigenvalues and eigenvectors in the diagonalization of matrices.
- Familiarize with concepts of bases, dimension and minimal spanning sets in vector spaces.
- Learn about linear transformation and its corresponding matrix.

SYLLABUS OF GE-2(ii)

UNIT – I: Vectors and Matrices

Fundamental operations and properties of vectors in \mathbb{R}^n , Linear combinations of vectors, Dot product and their properties, Cauchy-Schwarz and triangle inequality, Orthogonal and parallel vectors; Solving system of linear equations using Gaussian elimination, and Gauss-Jordan row reduction, Reduced row echelon form; Equivalent systems, Rank and row space of a matrix; Eigenvalues, eigenvectors and characteristic polynomial of a square matrix; Diagonalization.

UNIT – II: Vector Spaces

Definition, examples and some elementary properties of vector spaces; Subspaces, Span, Linear independence and dependence; Basis and dimension of a vector space; Diagonalization and bases.

UNIT – III: Linear Transformations

Definition, examples and elementary properties of linear transformations; The matrix of a linear transformation; Kernel and range of a linear transformation, The dimension theorem, one-to-one and onto linear transformations.

Essential Reading

1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebra* (5th ed.). Elsevier India.

Suggestive Reading

• Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.

(18 hours)

(15 hours)

UNIVERSITY OF DELHI

CNC-II/093/1(25)/2023-24/64 Dated: 30.05.2023

NOTIFICATION

Sub: Amendment to Ordinance V

[E.C Resolution No. 60/ (60-1-7/) dated 03.02.2023]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

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Syllabi of Semester-III of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

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GENERIC ELECTIVES-GE-3(i): DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	istribution	of the course	Eligibility	Pre-requisite
title & Code		Lecture	Tutorial	Practical/ Practice	criteria	of the course (if any)
Differential Equations	4	3	1	0	Class XII pass with Mathematics	Nil

Learning Objectives

The primary objective of this course is to introduce:

- Ordinary and partial differential equations.
- Basic theory of higher order linear differential equations, Wronskian and its properties.
- Various techniques to find the solutions of above differential equations which provide a basis to model complex real-world situations.

Learning Outcomes

This course will enable the students to:

- Solve the exact, linear, Bernoulli equations, find orthogonal trajectories and solve rate problems.
- Apply the method of undetermined coefficients and variation of parameters to solve linear differential equations.
- Solve Cauchy-Euler equations and System of linear differential equations.
- Formulate and solve various types of first and second order partial differential equations.

SYLLABUS OF GE-3(i)

Unit – 1

Ordinary Differential Equations

First order ordinary differential equations: Basic concepts and ideas, First order Exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Initial value problems, Applications of first order differential equations: Orthogonal trajectories and Rate problems; Basic theory of higher order linear differential equations, Wronskian and its properties.

Unit – 2

Explicit Methods of Solving Higher-Order Linear Differential Equations

Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients, Method of variation of parameters, Two-point boundary value problems, Cauchy-Euler equations, System of linear differential equations.

(15 hours)

Unit – 3

First and Second Order Partial Differential Equations

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

Essential Readings

- 1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

Suggestive Readings

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

GE-3(ii)

GENERIC ELECTIVES-GE-3(ii): LATTICES AND NUMBER THEORY

Course	Credits	Credit d	istribution	of the course	Eligibility	Pre-requisite
title & Code		Lecture	Tutorial	Practical/ Practice	criteria	of the course (if any)
Lattices and Number Theory	4	3	1	0	Class XII pass with Mathematics	Nil

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Learning Objectives

The primary objective of this course is to introduce:

- The concepts of ordered sets, lattices, sublattices and homomorphisms between lattices.
- Distributive lattices along with Boolean algebra and their applications in the real-world.
- Divisibility theory of congruences along with some applications.
- The number-theoretic functions and quadratic reciprocity law.

Learning Outcomes

This course will enable the students to:

- Understand the notion of ordered sets. Learn about lattices, distributive lattices, sublattices and homomorphisms between lattices.
- Become familiar with Boolean algebra, Boolean polynomials, switching circuits and their applications.
- Learn the concept of Karnaugh diagrams and Quinn–McCluskey method which gives an aid to apply truth tables in real-world problems.

- Learn about some fascinating properties of prime numbers, and some of the open problems in number theory, viz., Goldbach conjecture etc.
- Know about modular arithmetic and number-theoretic functions like Euler's Phi-function.
- Find quadratic residues and nonresidues modulo primes using Gauss's Quadratic Reciprocity Law.

SYLLABUS OF GE-3(ii)

Unit – 1

Partially Ordered Sets and Lattices

Definitions, Examples and basic properties of partially ordered sets, Order isomorphism, Hasse Diagram, Maximal and minimal elements, Dual of an ordered set, Duality principle; Statements of Well Ordering Principle and Zorn's Lemma; Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications.

Unit – 2

Divisibility and Theory of Congruences

The division algorithm: GCD, The Euclidean algorithm, Diophantine equation ax + by = cPrimes: The Fundamental Theorem of Arithmetic, Infinitude of primes, Twin primes and Goldbach conjecture.

The theory of congruences: Basic properties and applications, Linear congruences and the Chinese Remainder Theorem, Fermat's Little Theorem and Wilson's Theorem.

Unit – 3

Number-Theoretic Functions, Primitive roots and Quadratic Reciprocity Law

Number-Theoretic Functions: Sum and number of divisors, Euler's Phi-function and Euler's generalization of Fermat's Little Theorem.

Primitive roots: The order of an integer modulo *n*, and primitive roots of an integer. Quadratic Reciprocity Law: Quadratic residue and nonresidue, Euler's Criterion, The Legendre symbol and its properties and Quadratic Reciprocity Law.

Essential Readings

- 1. Davey, B A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.), Cambridge University Press, Cambridge.
- 2. Lidl, Rudolf & Pilz, Günter. (1998). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics, Springer (SIE), Indian Reprint 2004.
- 3. Burton, David M. (2012). Elementary Number Theory (7th ed.), Mc-Graw Hill Education Pvt. Ltd. Indian Reprint.

Suggestive Readings

- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.
- Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2018.
- Jones, G. A., & Jones, J. Mary. (2005). Elementary Number Theory. Springer Undergraduate Mathematics Series (SUMS). Indian Reprint.

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(21 hours)

(12 hours)

UNIVERSITY OF DELHI

CNC-II/093/1(26)/2023-24/194 Dated: 14.09.2023

NOTIFICATION

Sub: Amendment to Ordinance V

[E.C Resolution No. 14/ (14-1-7/) and 27-1-2/ dated 09.06.2023 and 25.08.2023 respectively]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-IV, V and VI of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

- 1. Department of Mathematics
- 2. Department of Statistics
- 3. Department of Operational Research
- 4. Department of Computer Science

GENERIC ELECTIVES (GE-4(i)): ELEMENTS OF REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title &	Credits	Credit	distribution		•	Pre-requisite
Code		Lecture		Practical/ Practice		of the course (if any)
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

Learning Outcomes: This course will enable the students to:

- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Learn to apply various tests such as limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF GE-4(i)

UNIT-I: Basic Properties of the Set of Real Numbers

Field and order properties of \mathbb{R} , basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of \mathbb{R} .

UNIT-II: Real Sequences

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

UNIT-III: Infinite Series of Real Numbers

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series, Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's *n*th root test, Raabe's test; Alternating series, Leibniz alternating series test, Absolute and conditional convergence.

(18 hours)

(15 hours)

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

GE-4(ii)

GENERIC ELECTIVES (GE-4(ii)): LINEAR PROGRAMMING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title &	Credits	Credit	listribution			Pre-requisite
Code		Lecture		Practical/ Practice		of the course (if any)
Linear Programming	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The solution of linear programming problem using simplex method.
- The solution of transportation and assignment problems.
- Game theory which makes possible the analysis of the decision-making process of two interdependent subjects.

Learning Outcomes: This course will enable the students to:

- Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- Write the dual of a linear programming problem.
- Solve the transportation and assignment problems.
- Learn about solution of rectangular games using graphical method and dominance.
- Formulate game to a pair of associated prima-dual linear programming problems.

SYLLABUS OF GE-4(ii)

UNIT-I: Linear Programming Problem, Simplex Method, and Duality (18 hours) Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity; Introduction to the simplex method: Optimality criterion and unboundedness, Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples.

UNIT-II: Transportation and Assignment Problems

Definition of transportation problem, finding initial basic feasible solution using Northwestcorner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problem; Hungarian method of solving assignment problem.

UNIT-III: Two-Person Zero-Sum Games

Introduction to game theory, rectangular games, Mixed strategies, Dominance principle; Formulation of game to primal and dual linear programming problems.

Essential Readings

- 1. Thie, Paul R., & Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.
- 2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Hillier, F. S., & Lieberman, G. J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.

(12 hours)

GE-V(i)

GENERIC ELECTIVES (GE-5(i)): NUMERICAL METHODS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	_	Credit	distribution			Pre-requisite
		Lecture		Practical/ Practice	criteria	of the course
Numerical Methods	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The core purpose of the course is to:

• Acquaint students with various topics in numerical solutions of nonlinear equations in one variable, interpolation and approximation, numerical differentiation and integration, direct methods for solving linear systems, numerical solution of ordinary differential equations using Computer Algebra System (CAS).

Learning Outcomes: The course will enable the students to:

- Find the consequences of finite precision and the inherent limits of numerical methods.
- Appropriate numerical methods to solve algebraic and transcendental equations.
- Solve first order initial value problems of ODE's numerically using Euler methods.

SYLLABUS OF GE-5(i)

UNIT-I: Errors and Roots of Transcendental and Polynomial Equations (12 hours) Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence, and terminal conditions; Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

UNIT-II: Algebraic Linear Systems and Interpolation (18 hours)

Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss–Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators.

UNIT-III: Numerical Differentiation, Integration and ODE (15 hours)

First and second order numerical derivatives; Trapezoidal rule, Simpson's rule for numerical integration; Ordinary differential equation: Euler's, and Runge-Kutta method.

Essential Readings

- 1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
- 2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
- 3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.

Suggestive Reading

• Bradie, Brian (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

Note: Non programmable scientific calculator may be allowed in the University examination.

Practical (30 hours): Practical/Lab work to be performed in Computer Lab: Use of computer algebra software (CAS), for example Python/SageMath/Mathematica/MATLAB/ Maple/ Maxima/Scilab etc., for developing the following numerical programs:

- 1. Bisection method
- 2. Secant method and Regula-Falsi method
- 3. Newton-Raphson method
- 4. Gauss-Jacobi method and Gauss-Seidel method
- 5. Lagrange interpolation and Newton interpolation
- 6. Trapezoidal rule and Simpson's rule
- 7. Euler's, and Runge-Kutta methods for solving first order initial-value problems of ordinary differential equations.

GE-5(ii)

GENERIC ELECTIVES (GE-5(ii)): MATHEMATICAL PYTHON

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title &						Pre-requisite
Code		Lecture	Tutorial	Practical/ Practice	criteria	of the course
Mathematical Python	4	3	0	1		Basic knowledge of python

Learning Objectives: The Learning Objectives of this course are as follows:

- To be able to model and solve mathematical problems using Python Programs.
- To experience utility of open-source resources for numerical and symbolic mathematical software systems.

Learning Outcomes: This course will enable the students to use Python:

- For numerical and symbolic computation in mathematical problems from calculus, algebra, and geometry.
- To tabulate and plot diverse graphs of functions and understand tracing of shapes, geometries, and fractals.
- To prepare smart documents with LaTeX interface.

SYLLABUS OF GE-5(ii)

Theory

UNIT – I: Drawing Shapes, Graphing and Visualization (15 hours)

Drawing diverse shapes using code and Turtle; Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots, Animations of decay, Bayes update, Random walk.

UNIT – II: Numerical and Symbolic Solutions of Mathematical Problems (18 hours) NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace, Solving dynamical systems on coupled ordinary differential equations, Functional programming fundamentals using NumPy; Symbolic computation and SymPy: Differentiation and integration of functions, Limits, Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify), Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

UNIT – III: Document Generation with Python and LaTeX (12 hours)

Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats; Pylatex and writing document files from Python with auto-computed values, Plots and visualizations.

Practical (30 hours): Software labs using IDE such as Spyder and Python Libraries.

- Installation, update, and maintenance of code, troubleshooting.
- Implementation of all methods learned in theory.
- Explore and explain API level integration and working of two problems with standard Python code.

Essential Readings

- 1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
- 2. Farrell, Peter and et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
- 3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

Suggestive Readings

- Morley, Sam (2022). Applying Math with Python (2nd ed.). Packet Publishing Ltd. ISBN: 978-1-80461-837-0
- Online resources and documentation on the libraries, such as:
 - o https://matplotlib.org
 - o https://sympy.org
 - o https://pandas.pydata.org
 - o https://numpy.org
 - o https://pypi.org
 - o https://patrickwalls.github.io/mathematicalpython/

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

GE-5(iii): Elementary Mathematical Analysis

(Passed in the syllabus of 6th semester DSC-6 of B.A. (Prog.), vide University of Delhi Notification number: CNC-II/093/1(26)/2023-24/194) dated 14.09.2023)

Course title & Code	Credits	Credit	distributior		Pre-requisite of the course	
		Lecture	Tutorial	Practical/ Practice		(if any)
Elementary Mathematical Analysis	4	3	1	0	Class XII pass with Mathematics	Elements of Real Analysis

Learning Objectives: The primary objective of this course is to introduce:

- Sequential criterion for limits and continuity of real-valued functions.
- Riemann integral of real-valued function f on [a, b] using Darboux sums.
- Pointwise and uniform convergence of sequences and series of functions. Learning

Outcomes: This course will enable the students to:

- Apply sequential continuity criterion for the proof of intermediate value theorem.
- Understand the basic tool used to calculate integrals.
- Apply uniform convergence for term-by-term integration in power series expansion.

SYLLABUS OF GE-5(iii)

UNIT-I: Continuous Functions

Sequential criterion for limits and continuity of functions, Continuity on intervals, Intermediate value theorem and applications; Uniform continuity.

UNIT-II: The Riemann Integral

Riemann integration, criterion for integrability and examples; Integrability of continuous and monotone functions, Algebraic properties of the Riemann integral, Fundamental theorem of calculus (first form).

UNIT-III: Uniform Convergence

Sequences and series of functions: Pointwise and uniform convergence, Uniform Cauchy criterion, Weierstrass M-test, Implications of uniform convergence in calculus; Power series, Radius and interval of convergence, Applications of Abel's theorem for power series.

Essential Reading

 Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

(15 hours)

(18 hours)

(12 hours)

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GENERIC ELECTIVES (GE-6(i)): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

	Credits	Credit	distribution			Pre-requisite of the course
Code		Lecture		Practical/ Practice		
Introduction to Mathematical Modeling	4	3	0	1	with	GE-3(i): Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF GE-6(i)

UNIT-I: Compartmental Models

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation (15 hours)

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling: Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

- 1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
- 2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.
- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Modeling

- of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.
- 1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
 - a. Exponential decay and growth model.
 - b. Lake pollution model (with constant/seasonal flow and pollution concentration).
 - c. Case of single cold pill and a course of cold pills.
 - d. Limited growth of population (with and without harvesting).
 - e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
 - f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
 - g. Ecosystem model of competing species
 - h. Battle model
- 2. Random number generation and then use it to simulate area under a curve and volume under a surface.
- 3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.

a.
$$y = a x^2 + b x + c$$

b.
$$y = a x^n$$

- 4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
 - a. $y = b x^n$
 - b. $y = b e^{a x}$
 - c. $y = a \ln x + b$
 - d. $y = a x^2$
 - e. $y = a x^3$.

GE-6(ii)

GENERIC ELECTIVES (GE-6(ii)): DISCRETE DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			0 0 0	Pre-
		Lecture	Tutorial	Practical/ Practice		requisite of the course
Discrete Dynamical Systems	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The fundamental concepts of discrete dynamical systems and emphasis on its study through several applications.
- The concepts of the fixed points, chaos and Lyapunov exponents for linear and nonlinear equations have been explained through examples.
- Various applications of chaos in higher dimensional models.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of difference equation, chaos and Lyapunov exponents.
- Obtain fixed points and discuss the stability of the dynamical system.
- Find Lyapunov exponents, Bifurcation, and Period-doubling for nonlinear equations.
- Analyze the behavior of different realistic systems with chaos cascade.

SYLLABUS OF GE-6(ii)

UNIT-I: Discrete-time Models

Discrete dynamical systems concepts and examples; Some linear models: Bouncing ball, investment growth, population growth, financial, economic and linear price models; Nonlinear models: Density-dependent population, contagious-disease, economic and nonlinear price models; Some linear systems models: Prey-predator, competing species, overlapping-generations, and economic systems.

UNIT-II: Linear Equations, Systems, their Solutions and Dynamics (18 hours)

Autonomous, non-autonomous linear equations and their solutions, time series graphs; Homogenous, non-homogeneous equations and their solutions with applications; Dynamics of autonomous linear equations, fixed points, stability, and oscillation; Homogeneous, nonhomogeneous linear systems and their dynamics, solution space graphs, fixed points, sinks, sources and saddles.

UNIT-III: Nonlinear Equations, their Dynamics and Chaos (15 hours)

Autonomous nonlinear equations and their dynamics: Exact solutions, fixed points, stability; Cobweb graphs and dynamics: Linearization; Periodic points and cycles: 2-cycles, *m*-cycles,

and their stability; Parameterized families; Bifurcation of fixed points and period-doubling; Characterizations and indicators of chaos.

Practical (30 hours)- Use of Excel/SageMath/MATHEMATICA/MATLAB/Scilab Software:

- 1. If Rs. 200 is deposited every 2 weeks into an account paying 6.5% annual interest compounded bi-weekly with an initial zero balance:
 - (a) How long will it take before Rs. 10,000/- is in account?
 - (b) During this time how much is deposited and how much comes from interest?
 - (c) Create a time series graph for the bi-weekly account balances for the first 40 weeks of saving scenario.

[1] Computer Projects 2.5 pp. 68

- 2. (a) How much can be borrowed at an annual interest rate of 6% paid quarterly for 5 years in order to have the payments equal Rs. 1000/- every 3 months.
 - (b) What is the unpaid balance on this loan after 4 years.
 - (c) Create a time series graph for the unpaid balances each quarter for the loan process.

[1] Computer Projects 2.5 pp. 68

- 3. Four distinct types of dynamics for any autonomous linear equation:
 - $x_{n+1} = a x_n + b$ for different values of a and b.

[1] Dynamics of autonomous linear equation, pp. 74

- 4. Find all fixed points and determine their stability by generating at least the first 100 iterates for various choices of initial values and observing the dynamics
 - a. $I_{n+1} = I_n r I_n + s I_n (1 I_n 10^{-6})$ for: (i) r = 0.5, s = 0.25, (ii) r = 0.5, s = 1.75, (iii) r = 0.5, s = 2.0.
 - b. $P_{n+1} = \frac{1}{P_n} + 0.75 P_n + c$ for: (i) c = 0; (ii) c = -1; (iii) c = -1.25; (iv) c = -1.38.
 - c. $x_{n+1} = a x_n (1 x_n^2)$ for: (i) a = 0.5; (ii) a = 1.5; (iii) a = 2.25; (iv) a = 2.3.

[1] Computer Projects 3.2 pp. 110

- 5. Determine numerically whether a stable cycle exists for the given parameter values, and if so, its period. Perform at least 200 iterations each time and if a cycle is found (approximately), use the product of derivatives to verify its stability.
 - a. $P_{n+1} = r P_n \left(1 \frac{P_n}{5000}\right)$, for: (i) r = 3.4; (ii) r = 3.5; (iii) r = 3.566; (iv) r = 3.569; (v) r = 3.845. b. $P_{n-1} = r P_n e^{-P_n/1000}$
 - b. $P_{n+1} = r P_n e^{-P_n/1000}$ for: (*i*) r = 5; (*ii*) r = 10; (*iii*) r = 14; (*iv*) r = 14.5; (*v*) r = 14.75.

[1] Computer Projects 3.5 pp. 154

6. Find through numerical experimentation the approximate intervals of stability of the (a) 2-cycle; (b) 4-cycle; (c) 8-cycle; (d) 16-cycle; (e) 32-cycle for the following

a.
$$f_r(x) = r x e^{-x}$$

b.
$$f_r(x) = r x^2 (1-x)$$

- c. $f_a(x) = x (a x^2)$
- d. $f_c(x) = \frac{2}{x} + 0.75 x c$

[1] Computer Projects 3.6 pp. 164

- 7. Through numerical simulation, show that each of the following functions undergoes a period doubling cascade:
 - a. $f_r(x) = r x e^{-x}$
 - b. $f_r(x) = r x^2 (1-x)$
 - c. $f_r(x) = r x e^{-x^2}$

d.
$$f_r(x) = \frac{rx}{(x^2+1)^2}$$

e.
$$f_a(x) = x (a - x^2)$$

[1] Computer Projects 3.7 pp. 175

8. Discuss (a) Pick two initial points close together, i.e., that perhaps differ by 0.001 or 0.00001, and perform at least 100 iterations of $x_{n+1} = f(x_n)$. Do solutions exhibit sensitive dependence on initial conditions?

(b) For several random choices of x_0 compute at least 1000 iterates x_n and draw a frequency distribution using at least 50 sub-intervals. Do dense orbits appear to exit? (c) Estimate the Lyapunov exponent L by picking several random choices of x_0 and computing $\frac{1}{N}\sum_{n=1}^{N} \ln|f'(x_n)|$ for N = 1000, 2500, 5000, etc.

Does *L* appear to be positive? i). $f(x) = 2 - x^2$ ii). $f(x) = \frac{2}{x} + \frac{3x}{4} - 2$.

[1] Computer Projects 3.8 pp. 187

- 9. Show that f(x) = r x (1 x) for r > 4 and $f(x) = 6.75 x^2 (1 x)$ have horseshoes and homoclinic orbits, and hence chaos. [1] Computer Projects 3.8 pp. 188
- 10. Find the fixed point and determine whether it is a sink, source or saddle by iterating and graphing in solution space the first few iterates for several choices of initial conditions.

a.
$$x_{n+1} = x_n - y_n + 30$$

$$y_{n+1} = x_n + y_n - 20.$$

b. $x_{n+1} = x_n + y_n$

$$y_{n+1} = x_n - y_n.$$

[1] Computer Projects 4.2 pp. 207

Essential Reading

1. Marotto, Frederick R. (2006). Introduction to Mathematical Modeling Using Discrete Dynamical Systems. Thomson, Brooks/Cole.

Suggestive Readings

- Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press Taylor & Francis Group, LLC.
- Lynch, Stephen (2017). Dynamical Systems with Applications using Mathematica[®] (2nd ed.). Birkhäuser.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.

GE-6(iii)-Vide E.C.(1270)-27.07.2024, Appendix-47, Resolution No. 5-23, page 31

GE-6(iii): Abstract Algebra

(Passed in the syllabus of 4th semester DSC-A-4 of B.A. (Prog.), vide University of Delhi Notification number: CNC-II/093/1(26)/2023-24/194) dated 14.09.2023)

Course title & Code		Credit	distribution		criteria	Pre-requisite of the course (if any)
		Lecture		Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains, and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

Learning Outcomes: This course will enable the students to:

- Appreciate ample types of groups present around us which explains our surrounding better, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals, and factor rings.

SYLLABUS OF GE- 6(iii)

UNIT-I: Introduction to Groups

(12 hours)

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

UNIT-III: Rings, Integral Domains and Fields

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

Suggestive Reading

• Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.