Courses based on Under Graduate Curriculum Framework (UGCF) -2022

DEPARTMENT OF MATHEMATICS

B.Sc. (Hons.) Mathematics

(Discipline Specific Core (DSC) and Discipline Specific Elective (DSE) Courses)

Sem.	DSC (1-20) (3L + 1T/1P = 4 credits)	DSE (1-6) (3L + 1T/1P = 4 credits)
I **(p. 1-6)**	**DSC-1**: Algebra (3L + 1T) **DSC-2**: Elementary Real Analysis (3L + 1T) **DSC-3**: Probability and Statistics (3L + 1P)	
II **(p. 7-11)**	**DSC-4:** Linear Algebra (3L + 1T) **DSC-5**: Calculus (3L + 1T) **DSC-6:** Ordinary Differential Equations (3L + 1P)	
III **(p. 13-18)**	**DSC-7:** Group Theory (3L + 1T) **DSC-8:** Riemann Integration (3L + 1T) **DSC-9:** Discrete Mathematics (3L + 1P)	**DSE-1:** Choose any *one* (i) Graph Theory (3L + 1T) **(p. 19-20)** (ii) Mathematical Python (3L + 1P) **(p. 20-21)** (iii) Number Theory (3L + 1T) **(p. 22-23)**
IV **(p. 25-29)**	**DSC-10:** Sequences and Series of Functions (3L + 1T) **DSC-11:** Multivariate Calculus (3L + 1T) **DSC-12:** Numerical Analysis (3L + 1P)	**DSE-2:** Choose any *one* (i) Biomathematics (3L + 1T) **(p. 30-31)** (ii) Mathematical Modeling (3L + 1P) **(p. 31-33)** (iii) Mechanics (3L + 1T) **(p. 33-34)**
V **(p. 35-38)**	**DSC-13:** Metric Spaces (3L + 1T) **DSC-14:** Ring Theory (3L + 1T) **DSC-15:** Partial Differential Equations (3L + 1P)	**DSE-3:** Choose any *one* (i) Mathematical Data Science (3L + 1P) **(p. 39-40)** (ii) Linear Programming and Applications (3L + 1T) **(p. 40-41)** (iii) Mathematical Statistics (3L + 1T) **(p. 42-43)**
VI **(p. 44-49)**	**DSC-16:** Advanced Group Theory (3L + 1T) **DSC-17:** Advanced Linear Algebra (3L + 1T) **DSC-18:** Complex Analysis (3L + 1P)	**DSE-4:** Choose any *one* (i) Mathematical Finance (3L + 1P) **(p. 49-51)** (ii) Integral Transforms (3L + 1T) **(p. 51-52)** (iii) Research Methodology (3L + 1P) **(p. 52-54)**
VII **(p. 55-56)**	**DSC-19:** Linear Analysis (3L + 1T)	**DSE-5:** Choose at least *one* and at most *three* (i) Advanced Differential Equations (3L + 1T) **(p. 57-58)** (ii) Dynamical Systems (3L + 1T) **(p. 58-59)** (iii) Fundamentals of Topology (3L + 1T) **(p. 60-61)** (iv) Information Theory and Coding (3L + 1T) **(p. 61-62)** (v) Optimization (3L + 1T) **(p. 62-63)** (vi) Research Methodology (3L + 1P) **(p. 64-65)**
VIII **(p. 66-67)**	**DSC-20:** Field Theory and Galois Extension (3L + 1T)	**DSE-6:** Choose at least *one* and at most *three* (i) Advanced Mechanics (3L + 1T) **(p. 68-69)** (ii) Cryptography (3L + 1T) **(p. 69-71)** (iii) Industrial Mathematics (3L + 1P) **(p. 71-72)** (iv) Geometry of Curves and Surfaces (3L + 1T) **72-73** (v) Integral Equations and Calculus of Variations (3L+1T) **74-75** (vi) Machine Learning: A Mathematical Approach (3L + 1P) **(p. 75-78)**

3L = 3 Hours Lecture; 1T = 1 Hour Tutorial; 1P = 2 Hours Practical.

UNIVERSITY OF DELHI

CNC-II/093/1(22)/2022-23/216
Dated: 10.10.2022

N O T I F I C A T I O N

Sub: Amendment to Ordinance V

[E.C Resolution No. 18-1/ (18-1-4) dated 18.08.2022]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-I of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 to be implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICS

B.SC. (H) MATHEMATICS

Category-I

DISCIPLINE SPECIFIC CORE COURSE – 1: ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Algebra	4	3	1	0	Class XII pass with Mathematics	Nil

Learning Objectives

The primary objective of this course is to introduce:

- The basic tools of theory of equations, number theory, and group theory.
- Symmetry group of a plane figure, basic concepts of cyclic groups.
- Classification of subgroups of cyclic groups.

Learning Outcomes:

This course will enable the students to:

- Determine number of positive/negative real roots of a real polynomial.

- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Employ De-Moivre's theorem in a number of applications to solve numerical problems.
- Use modular arithmetic and basic properties of congruences.
- Recognize the algebraic structure, namely groups, and classify subgroups of cyclic groups.

SYLLABUS OF DSC-1

Theory

Unit – 1 **(18 hours)**
Theory of Equations and Complex Numbers
General properties of polynomials and equations, Fundamental theorem of algebra, Relations between the roots and the coefficients, Upper bounds for the real roots; Theorems on imaginary, integral and rational roots; Newton's method for integral roots, Descartes' rule of signs; De-Moivre's theorem for integer and rational indices and their applications, The nth roots of unity, Cardan's solution of the cubic, Descartes' solution of the quartic equation.

Unit – 2 **(12 hours)**
Basic Number Theory
Division algorithm in $\mathbb{Z}$, Divisibility and the Euclidean algorithm, Fundamental theorem of arithmetic, Modular arithmetic and basic properties of congruences.

Unit – 3 **(15 hours)**
Basics of Group Theory
Groups, Basic properties, Symmetries of a square, Dihedral group, Order of a group, Order of an element, Subgroups, Center of a group, Centralizer of an element, Cyclic groups and properties, Generators of a cyclic group, Classification of subgroups of cyclic groups.

Practical component (if any) - NIL

Essential Readings

1. Andreescu, Titu & Andrica, D. (2014). Complex numbers from A to...Z. (2nd ed.). Birkhäuser.
2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785
3. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
4. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2018.

Suggestive Readings

- Burnside, W.S., & Panton, A.W. (1979), The Theory of Equations, Vol. 1. Eleventh

Edition, (Fourth Indian Reprint. S. Chand & Co. New Delhi), Dover Publications, Inc.

- Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint.
- Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE – 2: ELEMENTARY REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elementary Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives
The course will develop a deep and rigorous understanding of:
- Real line ℝ with algebraic.
- Order and completeness properties to prove the results about convergence and divergence of sequences and series of real numbers.

Learning Outcomes
This course will enable the students to:
- Understand the fundamental properties of the real numbers, including completeness and Archimedean, and density property of rational numbers in ℝ.
- Learn to define sequences in terms of functions from ℕ to a subset of ℝ and find the limit.
- Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate the limit superior and limit inferior of a bounded sequence.
- Apply limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF DSC - 2

Theory
Unit – 1 **(12 hours)**

Real Number System

Algebraic and order properties of $\mathbb{R}$, Absolute value of a real number, Bounded above and bounded below sets, Supremum and infimum of a non-empty subset of $\mathbb{R}$, The completeness property of $\mathbb{R}$, Archimedean property, Density of rational numbers in $\mathbb{R}$.

Unit – 2 **(18 hours)**

Sequences

Sequences and their limits, Convergent sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Subsequences, Bolzano-Weierstrass theorem for sequences, Limit superior and limit inferior for bounded sequence, Cauchy sequence, Cauchy's convergence criterion.

Unit – 3 **(15 hours)**

Infinite Series

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence, Tests for convergence of positive term series, Integral test, Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's nth root test, Raabe's test, Alternating series, Leibniz test, Absolute and conditional convergence.

Practical component (if any) – NIL

Essential Readings

1. Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
2. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
3. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Aliprantis C. D., & Burkinshaw, O. (1998). Principles of Real Analysis (3rd ed.). Academic Press.
- Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian reprint.
- Thomson, B. S., Bruckner, A. M., & Bruckner, J. B. (2001). Elementary Real Analysis. Prentice Hall.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE – 3: PROBABILITY AND STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Probability and Statistics	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives

The Learning Objectives of this course are as follows:

- To make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness.
- To render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

Learning Outcomes

This course will enable the students to:

- Understand some basic concepts and terminology - population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.
- Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential and lognormal.
- Understand the remarkable fact that the empirical frequencies of so many natural populati ons, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

SYLLABUS OF DSC – 3

Theory

Unit – 1 **(15 hours)**

Descriptive Statistics, Probability, and Discrete Probability Distributions

Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces and events, Probability axioms and properties, Conditional probability, Bayes' theorem and independent events; Discrete random variables and probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

Unit – 2 **(15 hours)**

Continuous Probability Distributions

Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential and lognormal distributions.

Unit – 3 **(15 hours)**

Central Limit Theorem and Regression Analysis

Sampling distribution and standard error of the sample mean, Central Limit Theorem and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

Practical (30 hours)

Software labs using Microsoft Excel or any other spreadsheet.

1) Presentation and analysis of data (univariate and bivariate) by frequeny tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
2) Fitting of binomial, Poisson and normal distributions.
3) Illustrating the Central Limit Theorem through Excel.
4) Fitting of regression line using the principle of least squares.
5) Computation of sample correlation coefficient.

Essential Reading

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2020.

Suggestive Reading

- Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). Introduction to the Theory of Statistics (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

UNIVERSITY OF DELHI

CNC-II/093/1(23)/2022-23/451
Dated: 03.03.2023

NOTIFICATION

Sub: Amendment to Ordinance V

[E.C Resolution No. 38-1/ (38-1-4) dated 08.12.2022]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-II of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 to be implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics

DISCIPLINE SPECIFIC CORE COURSE – 4: LINEAR ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Algebra	4	3	1	0	Class XII pass with Mathematics	DSC-I: Algebra

Learning Objectives: The objective of the course is to introduce:
- The concept of vectors in R^n, and their linear independence and dependence.
- Rank and nullity of linear transformations through matrices.
- Various applications of vectors in computer graphics and movements in plane.

Learning Outcomes: This course will enable the students to:

- Visualize the space R^n in terms of vectors and their interrelation with matrices.
- Familiarize with basic concepts in vector spaces, linear independence and span of vectors over a field.
- Learn about the concept of basis and dimension of a vector space.
- Basic concepts of linear transformations, dimension theorem, matrix representation of a linear transformation with application to computer graphics.

SYLLABUS OF DSC-4

UNIT – I: Matrices and System of Linear Equations (18 hours)

Fundamental operations with vectors in Euclidean space R^n, Linear combinations of vectors, Dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Solving linear systems using Gaussian elimination, Gauss-Jordan row reduction, Reduced row echelon form, Equivalent systems, Rank and row space, Eigenvalues, Eigenvectors, Eigenspace, Diagonalization, Characteristic polynomial of a matrix, Cayley-Hamilton theorem.

UNIT – II: Introduction to Vector Spaces (12 hours)

Vector spaces, Subspaces, Algebra of subspaces, Linear combination of vectors, Linear span, Linear independence, Bases and dimension, Dimension of subspaces.

UNIT – III: Linear Transformations (15 hours)

Linear transformations, Null space, Range, Rank and nullity of a linear transformation, Matrix representation of a linear transformation, Algebra of linear transformations, Invertibility and isomorphisms; Application**:** Computer Graphics-Fundamental movements in a plane, homogenous coordinates, composition of movements.

Essential Readings

1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebr*a (5th ed.). Elsevier India.
2. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2003). *Linear Algebra* (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.

Suggestive Readings

- Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.
- Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.
- Hoffman, Kenneth, & Kunze, Ray Alden (1978). *Linear Algebra* (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.

DISCIPLINE SPECIFIC CORE COURSE – 5: CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Calculus	4	3	1	0	Class XII pass with Mathematics	DSC-2: Elementary Real Analysis

Learning Objectives: The primary objective of this course is:

- To introduce the basic tools of calculus, also known as 'science of variation'.
- To provide a way of viewing and analyzing the real-world.

Learning Outcomes: This course will enable the students to understand:

- The notion of limits, continuity and uniform continuity of functions.
- Geometrical properties of continuous functions on closed and bounded intervals.
- Applications of derivative, relative extrema and mean value theorems.
- Higher order derivatives, Taylor's theorem, indeterminate forms and tracing of curves.

SYLLABUS OF DSC-5

UNIT – I: Limits and Continuity **(15 hours)**

Limits of functions (ε - δ and sequential approach), Algebra of limits, Squeeze theorem, One-sided limits, Infinite limits and limits at infinity; Continuous functions and its properties on closed and bounded intervals; Uniform continuity.

UNIT – II: Differentiability and Mean Value Theorems **(15 hours)**

Differentiability of a real-valued function, Algebra of differentiable functions, Chain rule, Relative extrema, Interior extremum theorem, Rolle's theorem, Mean-value theorem and its applications, Intermediate value theorem for derivatives.

UNIT – III: **(15 hours)**

Successive Differentiation, Taylor's Theorem and Tracing of Plane Curves

Higher order derivatives and calculation of the nth derivative, Leibnitz's theorem; Taylor's theorem, Taylor's series expansions of e^x, $\sin x$, $\cos x$. Indeterminate forms, L'Hôpital's rule; Concavity and inflexion points; Singular points, Asymptotes, Tracing graphs of rational functions and polar equations.

Essential Readings

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Bartle, Robert G., & Sherbert, Donald R. (2011). *Introduction to Real Analysis* (4th ed.). John Wiley & Sons. Wiley India edition reprint.

3. Prasad, Gorakh (2016). *Differential Calculus* (19th ed.). Pothishala Pvt. Ltd. Allahabad.
4. Ross, Kenneth A. (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian reprint.

Suggestive Readings

- Apostol, T. M. (2007). *Calculus*: *One-Variable Calculus with an Introduction to Linear Algebra* (2nd ed.). Vol. 1. Wiley India Pvt. Ltd.
- Ghorpade, Sudhir R. & Limaye, B. V. (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). Indian reprint.

DISCIPLINE SPECIFIC CORE COURSE – 6: ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Ordinary Differential Equations	**4**	**3**	**0**	**1**	**Class XII pass with Mathematics**	**NIL**

Learning Objectives: The main objective of this course is to introduce the students:

- The exciting world of differential equations.
- Their applications and mathematical modeling.

Learning Outcomes: The course will enable the students to:

- Learn the basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Solve first order non-linear differential equations, linear differential equations of higher order and system of linear differential equations using various techniques.
- Apply these techniques to solve and analyze various mathematical models.

SYLLABUS OF DSC-6

UNIT – I: First-Order Differential Equations (12 hours)

Concept of implicit, general and singular solutions for the first order ordinary differential equation; Bernoulli's equation, Exact equations, Integrating factors, Initial value problems, Reducible second order differential equations; Applications of first order differential equations to Newton's law of cooling, exponential growth and decay problems.

UNIT – II: Second and Higher-Order Differential Equations (18 hours)

General solution of homogenous equation of second order, Principle of superposition for a homogenous equation, Wronskian and its properties, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Method of variation of parameters, Method of undetermined coefficients, Two-point boundary value problems, Cauchy- Euler's equation, System of linear differential equations, Application of second order differential equation: Simple pendulum problem.

UNIT – III: Formulation and Analysis of Mathematical Models (15 hours)
Introduction to compartmental models, Lake pollution model; Density-dependent growth model, Interacting population models, Epidemic model of influenza and its analysis, Predator-prey model and its analysis, Equilibrium points, Interpretation of phase plane

Practical (30 hours)- Practical / Lab work to be performed in a Computer Lab:

Modeling of the following problems using SageMath/Mathematica/MATLAB/Maple/Maxima /Scilab etc.

1. Solutions of first, second and third order differential equations.
2. Plotting of family of solutions of differential equations of first, second and third order.
3. Solution of differential equations using method of variation of parameters.
4. Growth and decay model (exponential case only).
5. Lake pollution model (with constant/seasonal flow and pollution concentration).
6. Density-dependent growth model.
7. Predatory-prey model (basic Volterra model, with density dependence, effect of DDT, two prey one predator).
8. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). *Mathematical Modeling with Case Studies*, Using Maple and MATLAB (3rd ed.). CRC Press. Taylor & Francis Group.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). *Differential Equations and Boundary Value Problems*: *Computing and Modeling* (5th ed.). Pearson Education.
3. Ross, Shepley L. (2014). *Differential Equations* (3rd ed.). Wiley India Pvt. Ltd.

Suggestive Reading

- Simmons, George F. (2017). *Differential Equations with Applications and Historical Notes* (3rd ed.). CRC Press. Taylor & Francis Group.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

UNIVERSITY OF DELHI

CNC-II/093/1(25)/2023-24/64
Dated: 30.05.2023

N O T I F I C A T I O N

Sub: Amendment to Ordinance V

[E.C Resolution No. 60/ (60-1-7/) dated 03.02.2023]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-III of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICS

B.Sc. (Hons) MATHEMATICS

Category-I

DISCIPLINE SPECIFIC CORE COURSE -7: GROUP THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Group Theory	4	3	1	0	Class XII pass with Mathematics	Algebra

Learning Objectives

The primary objective of this course is to introduce:

- Symmetric groups, normal subgroups, factor groups, and direct products of groups.
- The notions of group homomorphism to study the isomorphism theorems with applications.
- Classification of groups with small order according to isomorphisms.

Learning Outcomes

This course will enable the students to:

- Analyse the structure of 'small' finite groups, and examine examples arising as groups of permutations of a set, symmetries of regular polygons.
- Understand the significance of the notion of cosets, Lagrange's theorem and its consequences.
- Know about group homomorphisms and isomorphisms and to relate groups using these mappings.
- Express a finite abelian group as the direct product of cyclic groups of prime power orders.
- Learn about external direct products and its applications to data security and electric circuits.

SYLLABUS OF DSC - 7

Unit – 1 **(18 hours)**

Permutation Groups, Lagrange's Theorem and Normal Subgroups

Permutation groups and group of symmetries, Cycle notation for permutations and properties, Even and odd permutations, Alternating groups; Cosets and its properties, Lagrange's theorem and consequences including Fermat's Little theorem, Number of elements in product of two finite subgroups; Normal subgroups, Factor groups, Cauchy's theorem for finite Abelian groups.

Unit – 2 **(15 hours)**

Group Homomorphisms and Automorphisms

Group homomorphisms, isomorphisms and properties, Cayley's theorem; First, Second and Third isomorphism theorems for groups; Automorphism, Inner automorphism, Automorphism groups, Automorphism groups of cyclic groups, Applications of factor groups to automorphism groups.

Unit – 3 **(12 hours)**
Direct Products of Groups and Fundamental Theorem of Finite Abelian Groups
External direct products of groups and its properties, The group of units modulo n as an external direct product, Applications to data security and electric circuits; Internal direct products; Fundamental theorem of finite abelian groups and its isomorphism classes.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.

Suggestive Readings

- Artin, Michael. (1991). Algebra (2nd ed.). Pearson Education. Indian Reprint 2015.
- Dummit, David S., & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.
- Herstein, I. N. (1975). Topics in Algebra (2nd ed.). Wiley India, Reprint 2022.
- Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE -8: RIEMANN INTEGRATION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Riemann Integration	4	3	1	0	Class XII pass with Mathematics	Elementary Real Analysis, and Calculus

Learning Objectives

The primary objective of this course is to:

- Understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration.
- Learn some of the properties of Riemann integrable functions, its generalization and the applications of the fundamental theorems of integration.
- Get an exposure to the utility of integration for practical purposes.

Learning Outcomes

This course will enable the students to:

- Learn about some of the classes and properties of Riemann integrable functions, and the applications of the Riemann sums to the volume and surface of a solid of revolution.
- Get insight of integration by substitution and integration by parts.
- Know about convergence of improper integrals including, beta and gamma functions.

SYLLABUS OF DSC - 8

Unit – 1 **(18 hours)**

The Riemann Integral

Definition of upper and lower Darboux sums, Darboux integral, Inequalities for upper and lower Darboux sums, Necessary and sufficient conditions for the Darboux integrability; Riemann's definition of integrability by Riemann sum and the equivalence of Riemann's and Darboux's definitions of integrability; Definition and examples of the Riemann-Stieltjes integral.

Unit – 2 **(15 hours)**

Properties of The Riemann Integral and Fundamental Theorems

Riemann integrability of monotone functions and continuous functions, Properties of Riemann integrable functions; Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability; Intermediate value theorem for integrals, Fundamental Theorems of Calculus (I and II).

Unit – 3 **(12 hours)**

Applications of Integrals and Improper Integrals

Methods of integration: integration by substitution and integration by parts; Volume by slicing and cylindrical shells, Length of a curve in the plane and the area of surfaces of revolution. Improper integrals of Type-I, Type-II and mixed type, Convergence of improper integrals, The beta and gamma functions and their properties.

Essential Readings

1. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer.
2. Anton, Howard, Bivens Irl and Davis Stephens (2012). Calculus (10th edn.). John Wiley & Sons, Inc.
3. Denlinger, Charles G. (2011). Elements of Real Analysis, Jones & Bartlett India Pvt. Ltd., Indian Reprint.
4. Ghorpade, Sudhir R. and Limaye, B. V. (2006). A Course in Calculus and Real Analysis. Undergraduate Texts in Mathematics, Springer (SIE). Indian Reprint.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley, Indian Edition.
- Kumar Ajit and Kumaresan S. (2014). A Basic Course in Real Analysis. CRC Press, Taylor & Francis Group, Special Indian Edition.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE– 9: DISCRETE MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Discrete Mathematics	4	3	0	1	Class XII pass with Mathematics	Algebra and Linear Algebra

Learning Objectives

The primary objective of the course is to:

- Make students embark upon a journey of enlightenment, starting from the abstract concepts in mathematics to practical applications of those concepts in real life.
- Make the students familiar with the notion of partially ordered set and a level up with the study of lattice, Boolean algebra and related concepts.
- Culminate the journey of learning with practical applications using the knowledge attained from the abstract concepts learnt in the course.

Learning Outcomes

This course will enable the students to:

- Understand the notion of partially ordered set, lattice, Boolean algebra with applications.
- Handle the practical aspect of minimization of switching circuits to a great extent with the methods discussed in this course.
- Apply the knowledge of Boolean algebras to logic, set theory and probability theory.

SYLLABUS OF DSC - 9

Unit – 1 **(15 hours)**

Cardinality and Partially Ordered Sets

The cardinality of a set; Definitions, examples and basic properties of partially ordered sets, Order-isomorphisms, Covering relations, Hasse diagrams, Dual of an ordered set, Duality principle, Bottom and top elements, Maximal and minimal elements, Zorn's lemma, Building new ordered sets, Maps between ordered sets.

Unit – 2 **(15 hours)**

Lattices

Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products, Lattice isomorphism; Definitions, examples and properties of modular and distributive lattices; The $M_3 – N_5$ theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

Unit – 3 **(15 hours)**

Boolean Algebras and Applications

Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem, Boolean polynomials, Boolean polynomial functions, Equivalence of Boolean polynomials, Disjunctive normal form and conjunctive normal form of Boolean polynomials; Minimal forms of Boolean polynomials, Quine-McCluskey method, Karnaugh diagrams, Switching circuits and applications, Applications of Boolean algebras to logic, set theory and probability theory.

Practical (30 hours):

Practical/Lab work to be performed in a computer Lab using any of the Computer Algebra System Software such as Mathematica/MATLAB /Maple/Maxima/Scilab/SageMath etc., for the following problems based on:

1) Expressing relations as ordered pairs and creating relations.
2) Finding whether or not, a given relation is:
   i. Reflexive ii. Antisymmetric iii. Transitive iv. Partial order
3) Finding the following for a given partially ordered set
   i. Covering relations.
   ii. The corresponding Hasse diagram representation.
   iii. Minimal and maximal elements.
4) Finding the following for a subset *S* of a given partially ordered set *P*
   i. Whether a given element in *P* is an upper bound (lower bound) of *S* or not.
   ii. Set of all upper bounds (lower bounds) of *S*.
   iii. The least upper bound (greatest lower bound) of *S*, if it exists.
5) Creating lattices and determining whether or not, a given partially ordered set is a lattice.
6) Finding the following for a given Boolean polynomial function:
   i. Representation of Boolean polynomial function and finding its value when the Boolean variables in it take particular values over the Boolean algebra {0,1}.
   ii. Display in table form of all possible values of Boolean polynomial function over the Boolean algebra {0,1}.
7) Finding the following:
   i. Dual of a given Boolean polynomial/expression.
   ii. Whether or not two given Boolean polynomials are equivalent.
   iii. Disjunctive normal form (Conjunctive normal form) from a given Boolean expression.
   iv. Disjunctive normal form (Conjunctive normal form) when the given Boolean polynomial function is expressed by a table of values.
8) Representing a given circuit diagram (expressed using gates) in the form of Boolean expression.
9) Minimizing a given Boolean expression to find minimal expressions.

Essential Readings

1. Davey, B. A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.). Cambridge University press, Cambridge.
2. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
3. Lidl, Rudolf & Pilz, Gunter. (2004). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint.

Suggested Readings

- Donnellan, Thomas. (1999). Lattice Theory (1st ed.). Khosla Pub. House. Indian Reprint.
- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.

B.Sc. (Hons) Mathematics, Semester-III, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE -1(i): GRAPH THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Graph Theory	4	3	1	0	Class XII pass with Mathematics	Nil

Learning Objectives

The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

Learning Outcomes

This course will enable the students to:

- Learn modelling of real-world problems by graphs.
- Know characteristics of different classes of graphs.
- Learn representation of graphs in terms of matrices.
- Learn algorithms to optimize a solution.
- Understand some properties of graphs and their applications in different practical situations.

SYLLABUS OF DSE - 1(i)

Unit – 1 **(12 hours)**

Graphs, Paths and Circuits

Definition, Examples and basic properties of graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits, Connected graphs, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling salesman problem, Shortest path, Dijkstra's algorithm.

Unit – 2 **(15 hours)**

Applications of Paths and Circuits, Trees

Applications of Path and Circuits: The Chinese Postman Problem, Digraphs, Bellman-Ford Algorithm, Tournaments, Scheduling Problem, Trees, Properties of Trees, Spanning Trees, Minimum Spanning Tree Algorithms.

Unit – 3 **(18 hours)**

Connectivity and Graph Coloring, Planar Graphs

Cut-vertices, Blocks and their Characterization, Connectivity and edge-connectivity, Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring and applications, Matchings, Hall's theorem, Independent sets and covers.

Essential Readings

1. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
2. Chartrand, Gary, & Zhang, Ping (2012). A First Course in Graph Theory. Dover Publications.

Suggestive Readings

- Bondy, J. A., and Murty, U.S.R. (2008). Graph Theory. Graduate Texts in Mathematics, Springer.
- Diestel, Reinhard (2017). Graph Theory (5th ed.). Graduate Texts in Mathematics, Springer.
- West, Douglas B. (2001). Introduction to Graph Theory (2nd ed.). Prentice Hall. Indian Reprint.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC ELECTIVE COURSE– 1(ii): MATHEMATICAL PYTHON

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Python	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of Python

Learning Objectives

The Learning Objectives of this course are as follows:

- To be able to model and solve mathematical problems using Python Programs.
- To experience utility of open-source resources for numerical and symbolic mathematical software systems.

Learning Outcomes

This course will enable the students to use Python:

- For numerical and symbolic computation in mathematical problems from calculus, algebra, and geometry.
- To tabulate and plot diverse graphs of functions and understand tracing of shapes, geometries, and fractals.
- To prepare smart documents with LaTeX interface.

SYLLABUS OF DSE - 1(ii)

Theory

Unit – 1 **(15 hours)**
Drawing Shapes, Graphing and Visualization
Drawing diverse shapes using code and Turtle; Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots, Animations of decay, Bayes update, Random walk.

Unit – 2 **(18 hours)**
Numerical and Symbolic Solutions of Mathematical Problems
NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace, Solving dynamical systems on coupled ordinary differential equations, Functional programming fundamentals using NumPy; Symbolic computation and SymPy: Differentiation and integration of functions, Limits, Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify), Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

Unit – 3 **(12 hours)**
Document Generation with Python and LaTeX
Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats; Pylatex and writing document files from Python with auto-computed values, Plots and visualizations.

Practical (30 hours): Software labs using IDE such as Spyder and Python Libraries.

- Installation, update, and maintenance of code, troubleshooting.
- Implementation of all methods learned in theory.
- Explore and explain API level integration and working of two problems with standard Python code.

Essential Readings

1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
2. Farrell, Peter and et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

Suggested Readings

- Morley, Sam (2022). Applying Math with Python (2nd ed.). Packet Publishing Ltd. ISBN: 978-1-80461-837-0
- Online resources and documentation on the libraries, such as:
  - https://matplotlib.org
  - https://sympy.org
  - https://pandas.pydata.org
  - https://numpy.org
  - https://pypi.org
  - https://patrickwalls.github.io/mathematicalpython/

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC ELECTIVE COURSE-1(iii): NUMBER THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Number Theory	4	3	1	0	Class XII pass with Mathematics	Algebra

Learning Objectives

The primary objective of this course is to introduce:

- The number theoretic techniques of computations with the flavour of abstraction.
- The Euclidean algorithm, linear Diophantine equations, congruence equations, arithmetic functions and their applications, Fermat's little, Euler's and Wilson's theorems.
- Primitive roots, quadratic residues and nonresidues, the Legendre symbol and the law of Quadratic Reciprocity.
- Introduction to cryptography, public-key cryptosystems and applications.

Learning Outcomes

This course will enable the students to:

- Use modular arithmetic in solving linear and system of linear congruence equations.
- Work with the number theoretic functions, their properties and their use.
- Learn the forms of positive integers that possess primitive roots and the Quadratic Reciprocity Law which deals with the solvability of quadratic congruences.
- Understand the public-key cryptosystems, in particular, RSA.

SYLLABUS OF DSE - 1(iii)

Unit – 1 **(12 hours)**

Linear Diophantine equation and Theory of Congruences

The Euclidean Algorithm and linear Diophantine equation; Least non-negative residues and complete set of residues modulo n; Linear congruences, The Chinese remainder theorem and system of linear congruences in two variables; Fermat's little theorem, Wilson's theorem and its converse, Application to solve quadratic congruence equation modulo odd prime p.

Unit – 2 **(21 hours)**

Number-Theoretic Functions and Primitive Roots

Number-theoretic functions for the sum and number of divisors, Multiplicative function, Möbius inversion formula and its properties; Greatest integer function with an application to the calendar; Euler's Phi-function, Euler's theorem and some properties of the Phi-function; The order of an integer modulo n and primitive roots for primes, Primitive roots of composite numbers n: when n is of the form 2^k, and when n is a product of two coprime numbers.

Unit – 3 **(12 hours)**

Quadratic Reciprocity Law and Public Key Cryptosystems

The quadratic residue and nonresidue of an odd prime and Euler's criterion, The Legendre symbol and its properties, Quadratic Reciprocity law and its application; Introduction to cryptography, Hill's cipher, Public-key cryptography and RSA.

Essential Reading

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

Suggestive Readings

- Andrews, George E. (1994). Number Theory. Dover publications, Inc. New York.
- Robbins, Neville (2007). Beginning Number Theory (2nd ed.). Narosa Publishing House Pvt. Ltd. Delhi.
- Rosen, Kenneth H. (2011). Elementary Number Theory and its Applications (6th ed.). Pearson Education. Indian Reprint 2015.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

UNIVERSITY OF DELHI

CNC-II/093/1(26)/2023-24/194

Dated: 14.09.2023

NOTIFICATION

Sub: Amendment to Ordinance V

[E.C Resolution No. 14/ (14-1-7/) and 27-1-2/ dated 09.06.2023 and 25.08.2023 respectively]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-IV, V and VI of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

1. Department of Mathematics
2. Department of Statistics
3. Department of Operational Research
4. Department of Computer Science

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-IV

DISCIPLINE SPECIFIC CORE COURSE – 10: SEQUENCES AND SERIES OF FUNCTIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Sequences and Series of Functions	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The objective of the course is to introduce:
- The sequences and series of real-valued functions as a generalization to the sequences and series of real numbers.
- The situations under which the process of convergence of a sequence and series of real-valued functions may commute with the processes of calculus while taking differentiation, or integration.
- An important class of series functions (i.e., power series), and the elementary functions-exponential, logarithmic and trigonometric.

Learning Outcomes: This course will enable the students to:
- Learn about Cauchy criterion for uniform convergence and Weierstrass *M*-test for uniform convergence of series of real-valued functions.
- Know about the constraints for the inter-changeability of differentiation, and integration with infinite sum of a series of functions.
- Handle the convergence of power series and properties of the limit function, including differentiation and integration of power series.
- Appreciate utility of polynomials in the space of continuous functions.

SYLLABUS OF DSC-10

UNIT – I: Sequences of Functions (18 hours)

Pointwise and uniform convergence of sequence of functions, The uniform norm, Cauchy criterion for uniform convergence, Continuity of the limit function of a sequence of functions, Interchange of the limit and derivative, and the interchange of the limit and integral of a sequence of functions, Bounded convergence theorem.

UNIT – II: Series of Functions (12 hours)

Pointwise and uniform convergence of series of functions, Theorems on the continuity, differentiability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass *M*-test for uniform convergence.

UNIT – III: Power Series (15 hours)

Definition of a power series, Radius of convergence, Absolute convergence (Cauchy-Hadamard theorem), Differentiation and integration of power series, Abel's theorem, Weierstrass's approximation theorem; The exponential, logarithmic and trigonometric functions: Definitions and their basic properties.

Essential Readings

1. Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). Wiley India Edition. Indian Reprint.
2. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

Suggestive Readings

- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
- Denlinger, Charles G. (2011). Elements of Real Analysis. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

DISCIPLINE SPECIFIC CORE COURSE – 11: MULTIVARIATE CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The primary objective of this course is to introduce:

- The extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- The geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- The techniques of integration to functions of two and three independent variables.
- The applications of multivariate calculus tools to physics, economics, optimization etc.

Learning Outcomes: This course will enable the students to:

- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double, and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems, and learn applications.

SYLLABUS OF DSC-11

UNIT – I: Calculus of Functions of Several Variables (18 hours)

Basic concepts, Limits and continuity, Partial derivatives, Tangent planes, Total differential, Differentiability, Chain rules, Directional derivatives and the gradient, Extrema of functions of two variables, Method of Lagrange multipliers with one constraint.

UNIT – II: Double and Triple Integrals (15 hours)

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integrals over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

UNIT – III: Green's, Stokes' and Gauss Divergence Theorem (12 hours)

Vector field, Divergence and curl, Line integrals and applications to mass and work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

Essential Reading

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. Pearson Education. Indian Reprint.

Suggestive Reading

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). Indian Reprint.

DISCIPLINE SPECIFIC CORE COURSE – 12: NUMERICAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Numerical Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The main objective of this course is to introduce:

- Various computational techniques to find approximate value for possible root(s) of algebraic and non-algebraic equations.
- Methods to solve system of linear equations and ordinary differential equations.
- The use of computer algebra system (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem-solving skills.

Learning Outcomes: This course will enable the students to:

- Learn some numerical methods to find the zeroes of nonlinear functions of a single variable, up to a certain given level of precision.
- Learn Gauss−Jacobi, Gauss−Seidel methods to solve system of linear equations.
- Get aware of using interpolation techniques, for example in finding values of a tabulated function at points which are not part of the table.
- Learn finding numerical solutions of difference equations which are obtained converting differential equations using techniques from calculus.

SYLLABUS OF DSC-12

UNIT – I: Methods for Solving Algebraic and Transcendental Equations (12 hours)

Rate and order of convergence; Bisection method, Method of false position, Fixed point iteration method, Newton's method, and Secant method, their order of convergence and convergence analysis.

UNIT – II: Techniques to Solve Linear Systems and Interpolation (15 hours)

LU decomposition and its applications; Iterative methods: Gauss−Jacobi, Gauss−Seidel methods; Lagrange and Newton interpolation, Piecewise linear interpolation.

UNIT – III: Numerical Differentiation and Integration (18 hours)

First and higher order approximation for the first derivative, Approximation for the second derivative; Numerical integration by closed Newton−Cotes formulae: Trapezoidal rule, Simpson's rule and its error analysis; Euler's method to solve ODE's, Modified Euler method, Runge−Kutta Method (fourth-order).

Essential Reading

1. Bradie, Brian. (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third impression 2011.

Suggestive Readings

- Gerald, Curtis F., & Wheatley, Patrick O. (2007). Applied Numerical Analysis (7th ed.). Pearson Education. India.
- Jain, M. K., Iyengar, S. R. K., & Jain, R. K. (2012). Numerical Methods for Scientific and Engineering Computation. (6th ed.). New Age International Publisher, India, 2016.

Note: Non programmable scientific calculator may be allowed in the University examination.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Use of computer algebra system (CAS) software: Python/SageMath/Mathematica/MATLAB/Maple/Maxima/ Scilab etc., for developing the following numerical programs:

1. Bisection method.

2. Newton-Raphson method.
3. Secant method.
4. LU decomposition method.
5. Gauss−Jacobi method.
6. Gauss−Seidel method.
7. Lagrange interpolation.
8. Newton interpolation.
9. Trapezoidal rule.
10. Simpson's rule.
11. Euler’s method.
12. Runge−Kutta Method (fourth-order).

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

B.Sc. (Hons) Mathematics, Semester-IV, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): BIOMATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Biomathematics	4	3	1	0	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to:

- Develop and analyse the models of the biological phenomenon with emphasis on population growth and predator-prey models.
- Interpret first-order autonomous systems of nonlinear differential equations using the Poincaré phase plane.
- Apply the basic concepts of probability to understand molecular evolution and genetics.

Learning Outcomes: This course will enable the students to:

- To learn and appreciate study of long-term behavior arising naturally in study of mathematical models and their impact on society at large.
- To understand spread of epidemic technically through various models and impact of recurrence phenomena.
- Learn what properties like Chaos and bifurcation means through various examples and their impact in Bio-Sciences.

SYLLABUS OF DSE-2(i)

UNIT – I: Mathematical Modeling for Biological Processes (15 hours)

Formulation a model through data, A continuous population growth model, Long-term behavior and equilibrium states, The Verhulst model for discrete population growth, Administration of drugs, Differential equation of chemical process and predator-prey model (Function response: Types I, II and III).

UNIT – II: Epidemic Model: Formulation and Analysis (15 hours)

Introduction to infectious disease, The SIS, SIR and SEIR models of the spread of an epidemic, Analyzing equilibrium states, Phase plane analysis, Stability of equilibrium points, Classifying the equilibrium state; Local stability, Limit cycles, Poincaré-Bendixson theorem.

UNIT – III: Bifurcation, Chaos and Modeling Molecular Evolution (15 hours)

Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation and period-doubling, Chaos, Stability of limit cycles, Introduction of the Poincaré plane; Modeling molecular evolution: Matrix models of base substitutions for DNA sequences, Jukes-Cantor and Kimura models, Phylogenetic distances.

Essential Readings

1. Robeva, Raina S., et al. (2008). An Invitation to Biomathematics. Academic press.
2. Jones, D. S., Plank, M. J., & Sleeman, B. D. (2009). Differential Equations and Mathematical Biology (2nd ed.). CRC Press, Taylor & Francis Group.
3. Allman, Elizabeth S., & Rhodes, John A. (2004). Mathematical Models in Biology: An Introduction. Cambridge University Press.

Suggestive Readings

- Linda J. S. Allen (2007). An Introduction to Mathematical Biology. Pearson Education.
- Murray, J. D. (2002). Mathematical Biology: An Introduction (3rd ed.). Springer.
- Shonkwiler, Ronald W., & Herod, James. (2009). Mathematical Biology: An Introduction with Maple and MATLAB (2nd ed.). Springer.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: Primary objective of this course is to introduce:

- Mathematical modeling as the representation of a system by a set of mathematical relations or equations.
- Mathematical epidemiological models susceptible-infectious-recovered (SIR) and its variant SEIR (S-Exposed-IR) for the spread of diseases.
- Monte Carlo simulation techniques, and simplex method for solving linear programming problems.

Learning Outcomes: This course will enable the students to:

- Understand the methodology of solving SIR models for disease spread.
- Learn significance of dieting model that provides important insights and guides to a biomedical issue that is of interest to the general public.
- Understand nonlinear systems and phenomena with stability analysis ranges from phase plane analysis to ecological and mechanical systems.

- Use Monte Carlo simulation technique to approximate area under a given curve, and volume under a given surface.

SYLLABUS OF DSE-2(ii)

UNIT – I: Mathematical Epidemiological and Dieting Models (15 hours)
Modeling concepts and examples, Scaling of variables, and approximations of functions; SIR and SEIR models for disease spread: Methodology, Standard and solvable SIR models, Basic reproduction number; Dieting model with analysis and approximate solutions.

UNIT – II: Modeling with Nonlinear Systems and Phenomena (15 hours)
Stability and the phase plane, Almost linear systems; Ecological models: Predators and competitors, Critical points, Oscillating populations, Survival of single species, Peaceful coexistence of two species, Interaction of logistic populations, Wildlife conservation preserve; Nonlinear mechanical systems: Hard and soft spring oscillations, Damped nonlinear vibrations.

UNIT – III: Simulation and Optimization Modeling (15 hours)
Monte Carlo simulating deterministic, and probabilistic behavior, Generating random numbers; Linear programming model: Geometric and algebraic solutions, Simplex method and its tableau format, Sensitivity analysis.

Essential Readings

1. Mickens, Ronald E. (2022). Mathematical Modelling with Differential Equations. CRC Press, Taylor & Francis Group.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2023). Differential Equations and Boundary Value Problems: Computing and Modeling (6th ed.). Pearson.
3. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). Brooks/Cole, Cengage Learning India Pvt. Ltd.

Suggestive Readings

- Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modeling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press. Taylor & Francis Group.
- Ross, Shepley L. (2014). Differential Equations (3rd ed.). Wiley India Pvt. Ltd.
- Simmons, George F. (2017). Differential Equations with Applications and Historical Notes (3rd ed.). CRC Press. Taylor & Francis Group.

Practical (30 hours)- Practical work to be performed in Computer Lab: Modeling of the following problems using: R/Python/SageMath/Mathematica/MATLAB/Maxima/Scilab etc.

1. a) Simulation of SIR model and its variants using some initial parameter values, and finding basic reproduction number for analysis.
   b) Analysis of the dieting process, which includes both body-mass loss and gain.
2. Nonlinear Systems and Phenomena.
   a) Plot phase plane portraits and solutions of first-order equations.
   b) Obtain interesting and complicated phase portraits for almost linear systems.

c) Discuss large wildlife conservation preserve model and obtain (i) The period of oscillation of the rabbit and fox populations, (ii) The maximum and minimum numbers of rabbits and foxes.
d) Discuss the Rayleigh and van der Pol models.

3. (i) Random number generation and then use it for the following:
   a) Simulate area under a given curve.
   b) Simulate volume under a given surface.

   (ii) [2] Chapter 7 (Projects 7.4 and 7.5).

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(iii): MECHANICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mechanics	4	3	1	0	Class XII pass with Mathematics	DSC-5: Calculus DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to:

- Starting Newtonian laws, learning various technical notions which explains various states of motion under given forces.
- Deals with the kinematics and kinetics of the rectilinear and planar motions of a particle including constrained oscillatory motions of particles, projectiles, and planetary orbits.
- Understand hydrostatic pressure and thrust on plane surfaces.

Learning Outcomes: This course will enable the students to:

- Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces.
- Apply the concepts of center of gravity, laws of static and kinetic friction.
- Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions.
- Evaluate the hydrostatic pressure at any given depth in a heavy homogeneous liquid at rest under gravity.

SYLLABUS OF DSE-2(iii)

UNIT – I: Statics **(15 hours)**

Fundamental laws of Newtonian mechanics, Law of parallelogram of forces, Equilibrium of a particle, Lamy's theorem, Equilibrium of a system of particles, External and internal forces, Couples, Reduction of a plane force system, Work, Principle of virtual work, Potential energy and conservative field, Mass centers, Centers of gravity, Friction.

UNIT – II: Dynamics **(18 hours)**

Kinemetics of a particle, Motion of a particle, Motion of a system, Principle of linear momentum, Motion of mass center, Principle of angular momentum, Motion relative to mass center, Principle of energy, D'Alembert's principle; Moving frames of reference, Frames of reference with uniform translational velocity, Frames of reference with constant angular velocity; Applications in plane dynamics- Motion of a projectile, Harmonic oscillators, General motion under central forces, Planetary orbits.

UNIT – III: Hydrostatics **(12 hours)**

Shearing stress, Pressure, Perfect fluid, Pressure at a point in a fluid, Transmissibility of liquid pressure, Compression, Specific gravity, Pressure of heavy fluid- Pressure at all points in a horizontal plane, Surface of equal density; Thrust on plane surfaces.

Essential Readings

1. Synge, J. L., & Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.
2. Ramsey, A. S. (2017). Hydrostatics. Cambridge University Press. Indian Reprint.

Suggestive Readings

- Roberts, A. P. (2003). Statics and Dynamics with Background Mathematics. Cambridge University Press.
- Ramsey, A. S. (1985). Statics (2nd ed.). Cambridge University Press.

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics Semester-V

DISCIPLINE SPECIFIC CORE COURSE – 13: METRIC SPACES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Metric Spaces	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

Learning Outcomes: This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities. Become aware one such formulations leading to metric spaces.
- Analyse how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.
- Know about Banach fixed point theorem, whose far-reaching consequences have resulted into an independent branch of study in analysis, known as fixed point theory.

SYLLABUS OF DSC-13

UNIT – I: Topology of Metric Spaces (18 hours)

Definition, examples, sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor's theorem, Subspaces.

UNIT – II: Continuity and Uniform Continuity in Metric Spaces (15 hours)

Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity; Homeomorphism, Isometry and equivalent metrics, Contraction mapping, Banach fixed point theorem.

UNIT – III: Connectedness and Compactness (12 hours)

Connectedness, Connected subsets of $\mathbb{R}$, Connectedness and continuous mappings, Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

Essential Reading

3. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

Suggestive Readings

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. New Delhi.
- Rudin, Walter. Principles of mathematical Analysis (3rd ed.).
- Simmons, George F. (2004). Introduction to Topology and Modern Analysis. McGraw-Hill Education. New Delhi.

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

DISCIPLINE SPECIFIC CORE COURSE – 14: RING THEORY

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Ring Theory	4	3	1	0	Class XII pass with Mathematics	DSC-7: Group Theory

Learning Objectives: The primary objective of this course is to:
- Introduce the fundamental theory of rings, and their homomorphisms.
- Develop the basic concepts of polynomial rings and irreducibility tests for polynomials over the ring of integers, and rational numbers.
- Introduce polynomial analog of a prime number.
- Describe polynomial rings, principal ideal domains, Euclidean domains and unique factorization domains, and their relationships.

Learning Outcomes: This course will enable the students to:
- Learn about the fundamental concept of rings, integral domains, and fields.
- Know about ring homomorphisms and isomorphisms theorems of rings, and construct quotient fields for integral domains.
- Appreciate the significance of unique factorization in rings and integral domains.
- Apply several criteria for determining when polynomials with integer coefficients have rational roots or are irreducible over the field of rational numbers.

SYLLABUS OF DSC-14

UNIT – I: Introduction to Rings and Ideals (18 hours)

Definition and examples of rings, Properties of rings, Subrings, Integral domains and fields, Characteristic of a ring; Ideals, operations on ideals, ideal generated by a set and properties, Factor rings, Prime ideals and maximal ideals, Principal ideal domains.

UNIT – II: Ring Homomorphisms and Polynomial Rings (15 hours)

Definition, examples and properties of ring homomorphisms; First, second and third isomorphism theorems for rings; The field of quotients; Polynomial rings over commutative rings, Division algorithm and consequences.

UNIT–III: Unique Factorization Domain and Divisibility in Integral Domains (12 hours)

Factorization of polynomials, Reducibility tests, Mod p Irreducibility test, Eisenstein's criterion, Unique factorization in $\mathbb{Z}[x]$; Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
2. Dummit, David S. & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.

Suggestive Readings

- Herstein, I. N. (2006). Topics in Algebra (2nd ed.). Wiley Student Edition. India.
- Hungerford, Thomas W. (2012). Abstract Algebra: An Introduction (3rd ed.). Cengage Learning.

DISCIPLINE SPECIFIC CORE COURSE – 15: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Partial Differential Equations	4	3	0	1	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Basic concepts of first and second order linear/nonlinear partial differential equations.
- Modeling of wave equation, heat equation, Burgers equation, traffic flow and their solutions.

Learning Outcomes: The course will enable the students to learn:

- The method of characteristics and reduction to canonical forms to solve first and second order linear/nonlinear partial differential equations.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.
- The Cauchy problem and solutions of wave equations with initial boundary-value problems, and non-homogeneous boundary conditions.

SYLLABUS OF DSC-15

UNIT – I: First Order Partial Differential Equations (15 hours)

Basic concepts, classification, construction, and geometrical interpretation; Method of characteristics and general solutions, Cauchy problem for a first-order PDE, Canonical forms of first-order linear equations; Method of separation of variables; Charpit's method for solving non-linear PDEs.

UNIT – II: Classification and Solutions of Second-Order Linear PDEs (12 hours)

Classification (hyperbolic, parabolic, and elliptic), reduction to canonical forms, and general solutions of second-order linear PDEs; Higher order linear partial differential equations with constant coefficients.

UNIT – III: Applications of Partial Differential Equations (18 hours)

Mathematical models: The vibrating string, vibrating membrane, conduction of heat in solids, the gravitational potential, conservation laws and the Burgers equation, Traffic flow; Cauchy problem and wave equations: Solutions of homogeneous wave equations with initial boundary-value problems, and non-homogeneous boundary conditions, Cauchy problem for non-homogeneous wave equations.

Essential Readings

1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.

2 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

Suggestive Readings

- Abell, Martha & Braselton, J.P. (2004) Differential Equations with Mathematica, Elsevier, Academic Press, Third Edition.
- Stavroulakis, Ioannis P & Tersian, Stepan A. (2004). Partial Differential Equations: An Introduction with Mathematica and MAPLE (2nd ed.). World Scientific.

Practical (30 hours)- Practical / Lab work to be performed in a Computer Lab:

Modeling of the following similar problems using SageMath/Python/Mathematica/ MATLAB/Maple/Maxima/Scilab:

1. General solution of first and second order partial differential equations.
2. Solution and plotting of Cauchy problem for first order PDEs.
3. Plotting the characteristics for the first order partial differential equations.
4. Solution of vibrating string problem using D'Alembert formula with initial conditions.
5. Solution of heat equation $u_t = k\, u_{xx}$ with initial conditions.
6. Solution of one-dimensional wave equation with initial conditions:
   i. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ x \in \mathbb{R}, t > 0$
   ii. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ u(0,t) = 0,\ x \in \mathbb{R}, t > 0$
   iii. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ u_x(0,t) = 0,\ x \in \mathbb{R}, t > 0$
7. Solution of traffic flow problem with given initial conditions, and plotting of the characteristic base curves and the traffic density function.

B.Sc. (Hons) Mathematics, Semester-V, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(i): MATHEMATICAL DATA SCIENCE

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title &	Credits	Credit distribution of the course	Eligibility	Pre-requisite of

Code		Lecture	Tutorial	Practical/ Practice	criteria	the course (if any)
Mathematical Data Science	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of R/Python DSC-3: Probability & Statistics

Learning Objectives: The main objective of this course is to:

- Introduce various types of data and their sources, along with steps involved in data science case-study, including problems with data and their rectification and creation methods.
- Cover dimensionality reduction techniques, clustering algorithms and classification methods.

Learning Outcomes: The course will enable the students to:

- Gain a comprehensive understanding of data science, its mathematical foundations including practical applications of regression, principal component analysis, singular value decomposition, clustering, support vector machines, and *k*-NN classifiers.
- Demonstrate data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation and regularization using R/Python.
- Use real-world datasets to practice dimensionality reduction techniques such as PCA, SVD, and multidimensional scaling using R/Python.

SYLLABUS OF DSE-3(i)

UNIT-I: Principles of Data Science (12 hours)

Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas, anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.

Unit-II: Mathematical Foundations (15 hours)

Model driven data in R^n, Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling; Norms in Vector Spaces– Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets– Jaccard, and edit distances; Modeling text with distances; Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.

Unit-III: Dimensionality Reduction, Clustering and Classification (18 hours)

Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best *k*-rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis; Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez's algorithm for *k*-center clustering, Lloyd's algorithm for *k*-means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering; Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and *k*-nearest neighbors (*k*-NN) classifiers.

Essential Readings

1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer. (https://mathfordata.github.io/).

Suggestive Readings

- Frank Emmert-Streib, et al. (2022). Mathematical Foundations of Data Science Using R. (2nd ed.). De Gruyter Oldenbourg.
- Wes McKinney. (2022). Python for Data Analysis (3rd ed.). O'Reilly.
- Wickham, Hadley, et al. (2023). R for Data Science (2nd ed.). O'Reilly.

Practical (30 hours)- Practical work to be performed in Computer Lab using R/Python:

1. To explore different types data (nominal, ordinal, interval, ratio) and identify their properties.
2. To deal with dirty and missing data, such as imputation, deletion, and data normalization.
3. Use the real-world datasets (https://data.gov.in/) to demonstrate the following:
   a) Data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation, and regularization.
   b) Dimensionality reduction techniques such as principal component analysis, singular value decomposition (SVD), and multidimensional scaling.
   c) Clustering algorithms such as *k*-means, hierarchical, and density-based clustering and evaluate the quality of the clustering results.
   d) Classification methods such as linear classifiers, support vector machines (SVM), and *k*-nearest neighbors (*k*-NN).

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(ii): LINEAR PROGRAMMING AND APPLICATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming and Applications	4	3	1	0	Class XII pass with Mathematics	DSC-4: Linear Algebra

Learning Objectives: Primary objective of this course is to introduce:

- Simplex Method for linear programming problems.
- Dual linear programming problems.
- The applications of linear Programming to transportation, assignment, and game theory.

Learning Outcomes: The course will enable the students to:

- Learn about the basic feasible solutions of linear programming problems.
- Understand the theory of the simplex method to solve linear programming problems.
- Learn about the relationships between the primal and dual problems.
- Solve transportation and assignment problems.
- Understand two-person zero sum game, games with mixed strategies and formulation of game to primal and dual linear programing problems to solve using duality.

SYLLABUS OF DSE-3(ii)

UNIT– I: Introduction to Linear Programming (12 hours)

Linear programming problem: Standard, Canonical and matrix forms, Geometric solution; Convex and polyhedral sets, Hyperplanes, Extreme points; Basic solutions, Basic feasible solutions, Correspondence between basic feasible solutions and extreme points.

UNIT– II: Optimality and Duality Theory of Linear Programming Problem (18 hours)

Simplex method: Optimal solution, Termination criteria for optimal solution of the linear programming problem, Unique and alternate optimal solutions, Unboundedness; Simplex algorithm and its tableau format; Artificial variables, Two-phase method, Big-M method. Duality Theory: Motivation and formulation of dual problem, Primal-Dual relationships, Fundamental theorem of duality; Complementary slackness.

UNIT – III: Applications (15 hours)

Transportation Problem: Definition and formulation, Northwest-corner, Least- cost, and Vogel's approximation methods of finding initial basic feasible solutions; Algorithm for solving transportation problem.

Assignment Problem: Mathematical formulation and Hungarian method of solving.

Game Theory: Two-person zero sum game, Games with mixed strategies, Formulation of game to primal and dual linear programming problems, Solution of games using duality.

Essential Readings

1. Bazaraa, Mokhtar S., Jarvis, John J., & Sherali, Hanif D. (2010). Linear Programming and Network Flows (4th ed.). John Wiley and Sons. Indian Reprint.
2. Hillier, Frederick S. & Lieberman, Gerald J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.
3. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Thie, Paul R., & Keough, G. E. (2008). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd. Indian Reprint 2014.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(iii): MATHEMATICAL STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/		

				Practice		
Mathematical Statistics	**4**	**3**	**1**	**0**	**Class XII pass with Mathematics**	**DSC-3: Probability & Statistics DSC-11: Multivariate Calculus**

Learning Objectives: The main objective of this course is to introduce:

- The joint behavior of several random variables theoretically and through illustrative practical examples.
- The theory underlying modern statistics to give the student a solid grounding in (mathematical) statistics and the principles of statistical inference.
- The application of the theory to the statistical modeling of data from real applications, including model identification, estimation, and interpretation.
- The idea of Fisher information to find the minimum possible variance for an unbiased estimator, and to show that the MLE is asymptotically unbiased and normal.

Learning Outcomes: The course will enable the students to:

- Understand joint distributions of random variables including the bivariate normal distribution.
- Estimate model parameters from the statistical inference based on point estimation and hypothesis testing.
- Apply Rao-Blackwell theorem for improving an estimator, and Cramér-Rao inequality to find lower bound on the variance of unbiased estimators of a parameter.
- Understand the theory of linear regression models and contingency tables.

SYLLABUS OF DSE - 3(iii)

UNIT–I: Joint Probability Distributions (15 hours)

Joint probability mass function for two discrete random variables, Marginal probability mass function, Joint probability density function for two continuous random variables, Marginal probability density function, Independent random variables; Expected values, covariance, and correlation; Linear combination of random variables and their moment generating functions; Conditional distributions and conditional expectation, Laws of total expectation and variance; Bivariate normal distribution.

UNIT-II: Sampling Distributions and Point Estimation (15 hours)

Distribution of important statistics such as the sample totals, sample means, and sample proportions, Central limit theorem, Law of large numbers; Chi-squared, *t*, and *F* distributions; Distributions based on normal random samples; Concepts and criteria for point estimation, The methods of moments and maximum likelihood estimation (MLE); Assessing estimators: Accuracy and precision, Unbiased estimation, Consistency and sufficiency, The Neyman factorization theorem, Rao-Blackwell theorem, Fisher Information, The Cramér-Rao inequality, Efficiency,

UNIT-III: Confidence Intervals, Tests of Hypotheses and Linear Regression Analysis (15 hours)

Interval estimation and basic properties of confidence intervals, One-sample t confidence interval, Confidence intervals for a population proportion and population variance. Statistical hypotheses and test procedures, One-sample tests about a population mean and a population proportion, P-values for tests; The simple linear regression model and its estimating parameters; Chi-squared goodness-of-fit tests, Two-way contingency tables.

Essential Reading

1. Devore, Jay L., Berk, Kenneth N. & Carlton Matthew A. (2021). Modern Mathematical Statistics with Applications. (3rd ed.). Springer.

Suggestive Readings

- Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences. Ninth edition, Cengage Learning India Private Limited, Delhi. Fourth impression 2022.
- Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2019). Introduction to Mathematical Statistics. Eighth edition, Pearson. Indian Reprint 2020.
- Mood, A.M., Graybill, F.A., & Boes, D.C. (1974). Introduction the Theory of Statistics (3rd ed.). Tata McGraw Hill Pub. Co. Ltd. Reprinted 2017.
- Wackerly, Dennis D., Mendenhall III, William & Scheaffer, Richard L. (2008). Mathematical Statistics with Applications. 7th edition, Cengage Learning.

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-VI

DISCIPLINE SPECIFIC CORE COURSE – 16: ADVANCED GROUP THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Group Theory	4	3	1	0	Class XII pass with Mathematics	DSC-7: Group Theory

Learning Objectives: The objective of the course is to introduce:
- The concept of group actions.
- Sylow's Theorem and its applications to groups of various orders.
- Composition series and Jordan-Hölder theorem.

Learning Outcomes: This course will enable the students to:
- Understand the concept of group actions and their applications.
- Understand finite groups using Sylow's theorem.
- Use Sylow's theorem to determine whether a group is simple or not.
- Understand and determine if a group is solvable or not.

SYLLABUS OF DSC-16

UNIT – I: Group Actions (18 hours)
Definition and examples of group actions, Permutation representations; Centralizers and Normalizers, Stabilizers and kernels of group actions; Groups acting on themselves by left multiplication and conjugation with consequences; Cayley's theorem, Conjugacy classes, Class equation, Conjugacy in S_n, Simplicity of A_5.

UNIT – II: Sylow Theorems and Applications (15 hours)
p-groups, Sylow p-subgroups, Sylow's theorem, Applications of Sylow's theorem, Groups of order pq and p^2q (p and q both prime); Finite simple groups, Nonsimplicity tests.

UNIT – III: Solvable Groups and Composition Series (12 hours)
Solvable groups and their properties, Commutator subgroups, Nilpotent groups, Composition series, Jordan-Hölder theorem.

Essential Readings

1. Dummit, David S., & Foote, Richard M. (2004). Abstract Algebra (3rd ed.). John Wiley & Sons. Student Edition, Wiley India 2016.
2. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
3. Beachy, John A., & Blair, William D. (2019). Abstract Algebra (4th ed.). Waveland Press.

Suggestive Readings

- Fraleigh, John B., & Brand Neal E. (2021). A First Course in Abstract Algebra (8th ed.). Pearson.
- Herstein, I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.
- Rotman, Joseph J. (1995). An Introduction to the Theory of Groups (4th ed.). Springer.

DISCIPLINE SPECIFIC CORE COURSE – 17: ADVANCED LINEAR ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Linear Algebra	4	3	1	0	Class XII pass with Mathematics	DSC-4: Linear Algebra

Learning Objectives: The objective of the course is to introduce:

- Linear functionals, dual basis and the dual (or transpose) of a linear transformation.
- Diagonalization problem and Jordan canonical form for linear operators or matrices using eigenvalues.
- Inner product, norm, Cauchy-Schwarz inequality, and orthogonality on real or complex vector spaces.
- The adjoint of a linear operator with application to least squares approximation and minimal solutions to linear system.
- Characterization of self-adjoint (or normal) operators on real (or complex) spaces in terms of orthonormal bases of eigenvectors and their corresponding eigenvalues.

Learning Outcomes: This course will enable the students to:

- Understand the notion of an inner product space in a general setting and how the notion of inner products can be used to define orthogonal vectors, including to the Gram-Schmidt process to generate an orthonormal set of vectors.
- Use eigenvectors and eigenspaces to determine the diagonalizability of a linear operator.
- Find the Jordan canonical form of matrices when they are not diagonalizable.

- Learn about normal, self-adjoint, and unitary operators and their properties, including the spectral decomposition of a linear operator.
- Find the singular value decomposition of a matrix.

SYLLABUS OF DSC-17

UNIT-I: Dual Spaces, Diagonalizable Operators and Canonical Forms (18 hours)
The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

UNIT-II: Inner Product Spaces and the Adjoint of a Linear Operator (12 hours)
Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

UNIT-III: Class of Operators and Their Properties (15 hours)
Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

Essential Reading

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

Suggestive Readings

- Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.
- Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.

DISCIPLINE SPECIFIC CORE COURSE – 18: COMPLEX ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Complex Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2 & 11: Real Analysis, Multivariate Calculus

Learning Objectives: The main objective of this course is to:

- Acquaint with the basic ideas of complex analysis.
- Learn complex-valued functions with visualization through relevant practicals.

- Emphasize on Cauchy's theorems, series expansions and calculation of residues.

Learning Outcomes: The accomplishment of the course will enable the students to:

- Grasp the significance of differentiability of complex-valued functions leading to the understanding of Cauchy-Riemann equations.
- Study some elementary functions and evaluate the contour integrals.
- Learn the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues, and apply Cauchy Residue theorem to evaluate integrals.

SYLLABUS OF DSC-18

UNIT – I: Analytic and Elementary Functions (15 hours)

Functions of a complex variable and mappings, Limits, Theorems on limits, Limits involving the point at infinity, Continuity and differentiation, Cauchy-Riemann equations and examples, Sufficient conditions for differentiability, Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions.

UNIT – II: Complex Integration (15 hours)

Derivatives of functions, Definite integrals of functions; Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals; Antiderivatives; Cauchy-Goursat theorem; Cauchy integral formula and its extension with consequences; Liouville's theorem and the fundamental theorem of algebra.

UNIT – III: Series and Residues (15 hours)

Taylor and Laurent series with examples; Absolute and uniform convergence of power series, Integration, differentiation and uniqueness of power series; Isolated singular points, Residues, Cauchy's residue theorem, Residue at infinity; Types of isolated singular points, Residues at poles and its examples, An application to evaluate definite integrals involving sines and cosines.

Essential Reading

1. Brown, James Ward, & Churchill, Ruel V. (2014). Complex Variables and Applications (9th ed.). McGraw-Hill Education. Indian Reprint.

Suggestive Readings

- Bak, Joseph & Newman, Donald J. (2010). Complex Analysis (3rd ed.). Undergraduate Texts in Mathematics, Springer.
- Mathews, John H., & Howell, Rusell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.
- Zills, Dennis G., & Shanahan, Patrick D. (2003). A First Course in Complex Analysis with Applications. Jones & Bartlett Publishers.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following similar problems using SageMath/Python/Mathematica/Maple/ MATLAB/Maxima/ Scilab etc.

1. Make a geometric plot to show that the nth roots of unity are equally spaced points that lie on the unit circle $C_1(0) = \{z : |z| = 1\}$ and form the vertices of a regular polygon with n sides, for n = 4, 5, 6, 7, 8.
2. Find all the solutions of the equation $z^3 = 8i$ and represent these geometrically.
3. Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units.
   Show the effect of rotation of this ellipse by an angle of $\frac{\pi}{6}$ radians and shifting of the centre from (0,0) to (2,1), by making a parametric plot.
4. Show that the image of the open disk $D_1(-1-i) = \{z : |z+1+i| < 1\}$ under the linear transformation $w = f(z) = (3-4i)z + 6 + 2i$ is the open disk:
   $$D_5(-1+3i) = \{w: |w+1-3i| < 5\}.$$
5. Show that the image of the right half-plane Re $z = x > 1$ under the linear transformation $w = (-1+i)z - 2 + 3i$ is the half-plane $v > u + 7$, where u = Re(w), etc. Plot the map.
6. Show that the image of the right half-plane A = {z: Re $z \geq \frac{1}{2}$} under the mapping $w = f(z) = \frac{1}{z}$ is the closed disk $\overline{D_1(1)} = \{w: |w-1| \leq 1\}$ in the w- plane.
7. Make a plot of the vertical lines $x = a$, for $a = -1, -\frac{1}{2}, \frac{1}{2}, 1$ and the horizontal lines $y = b$, for $b = -1, -\frac{1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $f(z) = \frac{1}{z}$.
8. Find a parametrization of the polygonal path $C = C_1 + C_2 + C_3$ from $-1+i$ to $3-i$, where C_1 is the line from: $-1+i$ to -1, C_2 is the line from: -1 to $1+i$ and C_3 is the line from $1+i$ to $3-i$. Make a plot of this path.
9. Plot the line segment 'L' joining the point $A = 0$ to $B = 2 + \frac{\pi}{4}i$ and give an exact calculation of $\int_L e^z dz$.
10. Evaluate $\int_C \frac{1}{z-2} dz$, where C is the upper semicircle with radius 1 centered at $z = 2$ oriented in a positive direction.
11. Show that $\int_{C_1} z dz = \int_{C_2} z dz = 4 + 2i$, where C_1 is the line segment from $-1-i$ to $3+i$ and C_2 is the portion of the parabola $x = y^2 + 2y$ joining $-1-i$ to $3+i$.
    Make plots of two contours C_1 and C_2 joining $-1-i$ to $3+i$.
12. Use the ML inequality to show that $\left|\int_C \frac{1}{z^2+1} dz\right| \leq \frac{1}{2\sqrt{5}}$, where C is the straight-line segment from 2 to $2+i$. While solving, represent the distance from the point z to the points i and $-i$, respectively, i.e., $|z-i|$ and $|z+i|$ on the complex plane $\mathbb{C}$.
13. Find and plot three different Laurent series representations for the function:
    $f(z) = \frac{3}{2+z-z^2}$, involving powers of z.
14. Locate the poles of $f(z) = \frac{1}{5z^4+26z^2+5}$ and specify their order.
15. Locate the zeros and poles of $g(z) = \frac{\pi\cot(\pi z)}{z^2}$ and determine their order. Also justify that Res (g, 0) = $-\pi^2/3$.

16. Evaluate $\int_{C_1^+(0)} \exp\left(\frac{2}{z}\right) dz$, where $C_1^+(0)$ denotes the circle $\{z: |z| = 1\}$ with positive orientation. Similarly evaluate $\int_{C_1^+(0)} \frac{1}{z^4+z^3-2z^2} dz$.

B.Sc. (Hons) Mathematics, Semester-VI, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(i): MATHEMATICAL FINANCE

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Finance	4	3	0	1	Class XII pass with Mathematics	DSC-3, 11, & 15: Probability and Statistics, Multivariate Calculus, & PDE's

Learning Objectives: The main objective of this course is to:
- Introduce the application of mathematics in the financial world.
- Understand some computational and quantitative techniques required for working in the financial markets and actuarial sciences.

Learning Outcomes: The course will enable the students to:
- Know the basics of financial markets and derivatives including options and futures.
- Learn about pricing and hedging of options.
- Learn the Itô's formula and the Black–Scholes model.
- Understand the concepts of trading strategies.

SYLLABUS OF DSE-4(i)

Unit - I: Interest Rates, Bonds and Derivatives (15 hours)
Interest rates, Types of rates, Measuring interest rates, Zero rates, Bond pricing, Forward rates, Duration, Convexity, Exchange-traded markets and Over-the-counter markets, Derivatives, Forward contracts, Futures contracts, Options, Types of traders, Hedging, Speculation, Arbitrage, No Arbitrage principle, Short selling, Forward price for an investment asset.

Unit - II: Properties of Options and the Binomial Model (15 hours)

Types of options, Option positions, Underlying assets, Factors affecting option prices, Bounds for option prices, Put-call parity (in case of non-dividend paying stock only), Early exercise, Trading strategies involving options (except box spreads, calendar spreads and diagonal spreads), Binomial option pricing model, Risk-neutral valuation (for European and American options on assets following binomial tree model).

Unit - III: The Black-Scholes Model and Hedging Parameters (15 hours)
Brownian motion (Wiener Process), Geometric Brownian Motion (GBM), The process for a stock price, Itô's lemma, Lognormal property of stock prices, Distribution of the rate of return, Expected return, Volatility, Estimating volatility from historical data, Derivation of the Black-Scholes-Merton differential equation, Extension of risk-neutral valuation to assets following GBM (without proof), Black−Scholes formulae for European options, Hedging parameters - The Greek letters: Delta, Gamma, Theta, Rho and Vega; Delta hedging, Gamma hedging.

Essential Readings

1. Hull, John C., & Basu, S. (2022). Options, Futures and Other Derivatives (11th ed.). Pearson Education, India.
2. Benninga, S. & Mofkadi, T. (2021). Financial Modeling, (5th ed.). MIT Press, Cambridge, Massachusetts, London, England.

Suggestive Readings

- Luenberger, David G. (2013). Investment Science (2nd ed.). Oxford University Press.
- Ross, Sheldon M. (2011). An elementary Introduction to Mathematical Finance (3rd ed.). Cambridge University Press.
- Day, A.L. (2015). Mastering Financial Mathematics in Microsoft Excel: A Practical Guide for Business Calculations (3rd ed.). Pearson Education Ltd.

Note: Use of non-programmable scientific calculator is allowed in theory examination.

Practical (30 hours)- Practical/Lab work using Excel/R/Python/MATLAB/MATHEMATICA

1. Computing simple, nominal, and effective rates. Conversion and comparison.
2. Computing price and yield of a bond.
3. Comparing spot and forward rates.
4. Computing bond duration and convexity.
5. Trading strategies involving options.
6. Simulating a binomial price path.
7. Computing price of European call and put options when the underlying follows binomial model (using Monte Carlo simulation).
8. Estimating volatility from historical data of stock prices.
9. Simulating lognormal price path.
10. Computing price of European call and put options when the underlying follows lognormal model (using Monte Carlo simulation).

11. Implementing the Black-Scholes formulae.
12. Computing Greeks for European call and put options.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(ii): INTEGRAL TRANSFORMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Integral Transforms	4	3	1	0	Class XII pass with Mathematics	DSC-6,15: ODE's, PDE's DSC-8, 10: Riemann Integration, Sequences & Series of Functions

Learning Objectives: Primary objective of this course is to introduce:
- The basic idea of integral transforms of functions and their applications through an introduction to Fourier series expansion of a periodic function.
- Fourier transform and Laplace transform of functions of a real variable with applications to solve ODE's and PDE's.

Learning Outcomes: The course will enable the students to:
- Understand the Fourier series associated with a periodic function, its convergence, and the Gibbs phenomenon.
- Compute Fourier and Laplace transforms of classes of functions.
- Apply techniques of Fourier and Laplace transforms to solve ordinary and partial differential equations and initial and boundary value problems.

SYLLABUS OF DSE-4(ii)

UNIT-I: Fourier Series and Integrals (18 hours)
Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series: Convergence, examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval, The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

UNIT-II: Integral Transform Methods (15 hours)
Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine

transforms, Convolution properties of Fourier transform; Laplace transforms, Properties of Laplace transforms, Convolution theorem and properties of the Laplace transform, Laplace transforms of the heaviside and Dirac delta functions.

UNIT-III: Applications of Integral Transforms (12 hours)

Finite Fourier transforms and applications, Applications of Fourier transform to ordinary and partial differential equations; Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

Essential Readings

1. Tyn Myint-U & Lokenath Debnath (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhauser. Indian Reprint.
2. Lokenath Debnath & Dambaru Bhatta (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press Taylor & Francis Group.

Suggestive Readings

- Baidyanath Patra (2018). An Introduction to Integral Transforms. CRC Press.
- Joel L. Schiff (1999). The Laplace Transform-Theory and Applications. Springer.
- Rajendra Bhatia (2003). Fourier Series (2nd ed.). Texts and Readings in Mathematics, Hindustan Book Agency, Delhi.
- Yitzhak Katznelson (2004). An Introduction to Harmonic Analysis (3rd ed.). Cambridge University Press.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(iii): RESEARCH METHODOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

Learning Outcomes: The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

SYLLABUS OF DSE - 4(iii)

UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
3. Declaration on Research Assessment. https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment
4. Evaluating Journals using journal metrics; (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.

10. Tantau, Till,Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

Practical (30 hours): Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

Syllabi of Semester-VII and VIII based on UGCF - 2022

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-VII

DISCIPLINE SPECIFIC CORE COURSE – 19: LINEAR ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Analysis	4	3	1	0	Class XII pass with Mathematics	Metric Spaces, Advanced Linear Algebra

Learning Objectives: The objective of the course is to introduce:
- Norm and normed spaces, Banach spaces and Hilbert spaces as complete normed spaces and their properties.
- Various forms of matrix norms with examples.
- Classes of bounded linear operators on normed spaces and Hilbert spaces, respectively.
- Four important theorems: Hahn-Banach, Uniform boundedness, Open mapping, and Closed graph as the cornerstones of the theory of Banach spaces.

Learning Outcomes: This course will enable the students to:
- Analyze and demonstrate examples of normed linear spaces with their properties.
- Characterize the bounded linear operators on normed spaces as continuous functions.
- Understand and apply Schwarz and Bessel's inequality, Parseval's identity.
- Illustrate linear operators, self-adjoint, unitary and normal operators on Hilbert spaces.
- Prove and apply fundamental theorems from the theory of normed and Banach spaces.

SYLLABUS OF DSC-19

UNIT – I: Normed Spaces and Banach Spaces (15 hours)

Normed spaces, Banach spaces, Properties of normed spaces, Finite dimensional normed spaces and subspaces, Compactness and finite dimension; Matrix norms; Linear operators, Bounded linear operators; Linear functionals, Linear operators and functionals on finite dimensional spaces; Normed spaces of operators, Dual space.

UNIT – II: Hilbert Spaces **(15 hours)**

Overview of inner product spaces and its properties, Hilbert spaces, Orthogonal complements and direct sums, Orthonormal sets and sequences, Bessel inequality; Total orthonormal sets and sequences; Riesz representations theorem, Hilbert-adjoint operator, Self-adjoint, Unitary and normal operators.

UNIT – III: Fundamental Theorems for Normed and Banach Spaces **(15 hours)**

Hahn Banach theorems for real and complex vector spaces, Hahn Banach theorem for normed spaces; Reflexive spaces**;** Uniform boundedness theorem, Open mapping theorem, Closed graph theorem.

Essential Readings

1. Kreyszig, Erwin (1989). Introductory Functional Analysis with Applications (1st ed.). John Wiley & Sons. Wiley-India Student Edition. Indian Reprint 2007.
2. Horn, Roger A. and Johnson, Charles R. (2013). Matrix Analysis (2nd ed.). Cambridge University Press.

Suggestive Readings

- Bollobás Béla (1999). Linear Analysis: An Introductory Course (2nd ed.). Cambridge University Press.
- Rynne, Bryan P. and Youngson, Martin A. (2008). Linear Functional Analysis (2nd ed.). Springer-Verlag London Limited.

DSE Courses of B.Sc. (Hons) Mathematics, Semester-VII

DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(i): ADVANCED DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Differential Equations	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus, Ordinary and Partial Differential Equations

Learning Objectives: The main objective of this course is to:
- Study the existence, uniqueness, and stability of solutions of IVPs, to explore the solution of system of linear equations.
- Study Green's function and its applications in boundary value problems, Eigenvalues and Eigenfunctions of Sturm Liouville systems.
- Investigate the solutions and applications of Laplace, wave, and diffusion equations.

Learning Outcomes: This course will enable the students to find the:
- Existence, uniqueness, and continuity of solutions of IVPs.
- Properties of zeros of solutions of linear second order ODE's.
- Green's function of a BVP and its applications.
- Eigenvalues and eigenfunctions of Sturm-Liouville systems.
- Solutions of Laplace, wave, and diffusion equations with their applications.

SYLLABUS OF DSE-5(i)

UNIT – I: Existence and Uniqueness for Initial-Value Problems (15 hours)
Well posed problems, Picard's existence theorem, uniqueness and continuity theorems for initial value problems of first order; Existence and uniqueness theorems for systems and higher order IVPs; Sturm separation and comparison theorems; Homogeneous linear systems, Nonhomogeneous linear systems, Linear systems with constant coefficients.

UNIT – II: Stability Theory and Boundary-Value Problems (10 hours)
Stability of autonomous system of differential equations, Critical point of an autonomous system and their classification, Stability of linear systems with constant coefficients, Linear plane autonomous systems, Perturbed systems; Two-point boundary-value problem, Green's functions and their construction; Sturm-Liouville systems, Eigenvalues and Eigenfunctions.

UNIT – III: Laplace, Wave and Diffusion Equations with Applications (20 hours)
Laplace's equation, Boundary value problems, Maximum and minimum principles, Uniqueness of solution and their continuous dependence on boundary data, Solution of the Dirichlet and Neumann problem for a half plane by Fourier transform method, Theory of Green's function for Laplace's equation in three dimension and application in solution of Dirichlet and Neumann problem for semi-infinite spaces; Wave equation, Helmholtz's first and second theorems, Theory of Green's function for wave equation and its applications; Diffusion equation, Solution of initial boundary value problems for diffusion equation, Green's function for diffusion equation and its applications.

Essential Readings

1. Myint-U, Tyn (1978). Ordinary Differential Equations. Elsevier, North-Holland, Inc.
2. Ross S. L. (2007). Differential Equations (2nd ed.) John Wiley & Sons. India.
3. Sneddon Ian N. (2006). Elements of Partial Differential Equations. Dover Publications.

Suggestive Readings

- Coddington, E. A. (2012). An Introduction to Ordinary Differential Equations. Dover Publications.
- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- McOwen, Robert C. (2003). Partial Differential Equations, Pearson Education.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(ii): DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Dynamical Systems	4	3	1	0	Class XII pass with Mathematics	Calculus, Differential Equations, Linear Algebra, Metric spaces

Learning Objectives: Primary objective of this course is to introduce:
- The fundamental concepts of dynamical systems and emphasize on its study through several applications.
- The concepts of the periodic points, hyperbolicity and chaos explained through examples.
- Symbolic dynamics which help to represent and understand various dynamical systems.

Learning Outcomes: This course will enable the students to:

- Understand and demonstrate the basic concepts of dynamical systems and properties.
- Obtain fixed points and discuss the stability of the dynamical system.
- Understand Sharkovsky's theorem, Schwarzian derivative and Devaney chaos.
- Gain command in understanding subshifts of finite type and Markov chain which eventually leads to various areas of dynamical systems.

SYLLABUS OF DSE-5(ii)

UNIT – I: Orbits under Discrete Dynamical Systems (12 hours)

Dynamical systems: Discrete and continuous, Population Models, Newton's Method; Discrete dynamical system: Definition, examples and orbits, Periodic and eventually periodic points, Stable and unstable sets, Phase portrait, Graphical analysis of one-dimensional maps; Hyperbolicity, A glimpse of bifurcations, Analysis of families of logistic maps.

UNIT – II: Introduction to Chaos (15 hours)

Symbolic dynamics, Sequence space, Shift map, Itinerary map, Subshifts of finite type, Conjugacy and chaos, Sensitive dependence on initial conditions, Topological transitivity, Devaney chaos, Expansive homeomorphisms, Expansivity of interval and circle maps; Structural stability, Sharkovsky's theorem and examples, Schwarzian derivative; Period 3 case.

UNIT – III: More on Symbolic Dynamics (18 hours)

Full shifts, Shift spaces, Languages, Higher block shifts and higher power shifts, Sliding block codes; Finite type constraints, Graphs and their shifts, Graph representations of shifts of finite type, Markov chain; Shadowing property and subshifts of finite type.

Essential Readings

1. Aoki, N. and Hiraide, K. (1994). Topological Theory of Dynamical Systems: Recent Advances. Elsevier Science, North-Holland.
2. Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press, Taylor & Francis Group.
3. Lind, Douglas and Marcus, Brian (2021). An Introduction to Symbolic Dynamics and Coding (2nd ed.). Cambridge University Press.

Suggestive Readings

- Bruin, Henk (2022). Topological and Ergodic Theory of Symbolic Dynamics. Graduate Studies in Mathematics (228), American Mathematical Society.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.
- Robinson, Clark (1998). Dynamical Systems: Stability, Symbolic Dynamics, and Chaos (2nd ed.). CRC press.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(iii): FUNDAMENTALS OF TOPOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Fundamentals of Topology	4	3	1	0	Class XII pass with Mathematics	Metric Spaces

Learning Objectives: The main objective of this course is to:

- Having in depth understanding of metric spaces and realizing strength of notions like path connectedness, countability axioms and theorems due to Tietze and Baire.
- Create Topological spaces fundamentals, naturally abstracting out from metric spaces.
- Study powerful notions like connectedness, compactness, product topology leading to major results like Tychonoff Theorem.

Learning Outcomes: This course will enable the students to:

- Realize beautiful transitions of some of the major notions and results from metric spaces to topological spaces wherein we do not have facility of distance.
- Appreciate possibility of continuous deformation of several spaces into known spaces through notions developed during the course work.
- Enhance ability to create examples and counter examples classifying various notions.
- Have better understanding of Euclidean spaces and its subspaces, infinite dimensional spaces, and several non-Euclidean spaces.
- Acquire a detailed elucidation of connectedness and compactness of topological spaces.

SYLLABUS OF DSE-5(iii)

UNIT-I: Countability Axioms, Separability and Lindelöf Spaces (12 hours)

Review of the properties of metric spaces; Spaces of sequences of numbers, their convergence and completeness, Completion of a metric space; Local base and base, First and second axiom of countability, Separable and Lindelöf spaces.

UNIT-II: Baire Category Theorem and Localized Versions of Connectedness (12 hours)

Nowhere dense subsets, Category I and category II sets, Baire category theorem; Extension theorems; Tietze's Extension Theorem; Local connectedness, Arcwise connectedness; Totally bounded sets and its connection with completeness and compactness.

UNIT-III: Introduction to Topological Spaces (21 hours)

Topology; Basis and subbasis for a topology; Product and subspace topology; Closed sets, Closure, Interior and limit points of a set, Hausdorff spaces; Continuous functions, Homeomorphism; Product topology for an indexed family of spaces; Connectedness and Compactness.

Essential Readings

1. Munkres James R. (2002). Topology (2nd ed.). Prentice Hall of India Pvt. Ltd.
2. Shirali Satish and Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

Suggestive Readings

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. Delhi.
- Searcóid, Mícheál Ó (2007). Metric Spaces. Springer-Verlag.
- Simmons, G. F. (2017). Introduction to Topology and Modern Analysis. McGraw Hill Education. Delhi.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(iv): INFORMATION THEORY AND CODING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Information Theory and Coding	4	3	1	0	Class XII pass with Mathematics	Probability and Statistics, Linear Algebra

Learning Objectives: The main objective of this course is to:

- Define and comprehend the concepts of information and its relationship with uncertainty and entropy.
- Apply basic principles of probability theory to measure information content.
- Learn basic information inequalities and their applications.
- Introduce error-detecting and error-correcting codes.
- Learn various decoding techniques.
- Get exposure to linear codes and bounds on linear codes.

Learning Outcomes: This course will enable the students to:

- Understand information and entropy, and calculate various entropies.
- Apply mutual information, conditional entropy, and information-theoretic measures.
- Know about the detection and correction of errors while transmission.
- Understand and demonstrate encoding and decoding of linear codes, and gain knowledge about some bounds on linear codes.

SYLLABUS OF DSE-5(iv)

UNIT – I: Concepts of Information Theory **(15 hours)**

A measure of uncertainty, H function as a measure of uncertainty, Sources and binary sources, Measure of information for two-dimensional discrete finite probability schemes. Entropy, Joint entropy and conditional entropy, Relative entropy and mutual information, Chain rules for entropy, Conditional relative entropy and conditional mutual information, A measure of mutual information.

UNIT – II: Information Inequality and Coding Theory (15 hours)

Interpretation of Shannon's fundamental inequalities, Redundancy, Efficiency and channel capacity, Jensen's inequality and its characterizations, The log sum inequality and its applications. Introduction to error detecting and correcting codes, Maximum likelihood decoding, Hamming distance, Nearest neighbour/minimum distance decoding, Distance of a code, Main coding theory problems, Equivalence of codes, Sphere-packing bound, Perfect codes, Balanced block designs, Finite fields, The ISBN code.

UNIT – III: Linear Codes (15 hours)

Introduction to vector space over finite fields, Linear codes, Bases for linear codes, Encoding and decoding with a linear code, Dual code, Generator and parity check matrices, Nearest neighbour decoding for linear codes, Syndrome decoding. Binary Hamming codes, *q*-ary Hamming codes.

Essential Readings

1. Cover, Thomas M. and Thomas, Joy A. (2006). Elements of Information Theory (2nd ed.). Wiley India. Indian Reprint 2017.
2. Hill, Raymond. (1996). A First Course in Coding Theory. Oxford University Press.
3. Reza, Fazlollah M. (1961). An Introduction to Information Theory. Dover Publications Inc, New York. Reprint July 2022.

Suggestive Readings

- Bose, R. (2016). Information Theory, Coding and Cryptography (3rd ed.). McGraw-Hill.
- Hamming, R. W. (1980). Coding and Information Theory, Prentice Hall, Englewood.
- Ling, S. and Xing, C. (2004). Coding Theory: A First Course. Cambridge University Press.
- Pless, V. (1998). Introduction to the Theory of Error-Correcting Codes. John-Wiley.
- Sloane, N. J. A. and MacWilliams, F. J. (2007). Theory of Error Correcting Codes. North-Holland Mathematical Library 16, North-Holland.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(v): OPTIMIZATION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

Learning Objectives: The main objective of this course is to introduce:
- Nonlinear optimization problems.
- Convex and generalized convex functions and their properties.
- Optimality and duality in nonlinear optimization.
- Methods to solve unconstrained optimization problems, quadratic and fractional programming problems with linear constraints.

Learning Outcomes: This course will enable the students to:
- Learn about the optimal solutions of nonlinear optimization problems.
- Understand and apply Karush-Kuhn-Tucker (KKT) necessary and sufficient optimality conditions for nonlinear optimization problems.
- Demonstrate and apply Lagrangian duality results, and techniques to solve certain classes of nonlinear optimization problems.

SYLLABUS OF DSE-5(v)

UNIT – I: Nonlinear Optimization and Convex Functions (15 hours)
Problem statement of a nonlinear optimization problem, Example of production inventory, Location facilities, Stochastic resource allocation, Convex sets, Convex functions, Epigraph and hypograph of a function, Differentiable convex function, Twice differentiable convex function, Minima of convex function, Quasiconvex functions, Psuedoconvex functions.

UNIT – II: Optimality and Duality Theory in Nonlinear Optimization (15 hours)
Unconstrained problems: Necessary optimality conditions, Sufficient optimality conditions; Problems having inequality constraints: Fritz John optimality conditions, Karush-Kuhn-Tucker (KKT) necessary optimality conditions; Fritz John conditions, KKT necessary and sufficient optimality conditions for problems with inequality and equality constraints; Lagrangian dual problem, Weak duality theorem, Duality gap, Strong duality theorem.

UNIT – III: Numerical Methods to Solve Nonlinear Optimization Problems (15 hours)
Descent property, Order of convergence, Global convergence, Steepest descent method, Newton's method, Wolfe's method for quadratic programming problem; Linear fractional programming problem and simplex algorithm.

Essential Readings
1. Bazaraa, Mokhtar S., Sherali, Hanif D. & Shetty, C. M. (2006). Nonlinear Programming: Theory and Algorithms (3rd ed.). John Wiley & Sons. Wiley India (2017).
2. Chandra, Suresh, Jayadeva and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.

Suggestive Readings
- Durea, Marius and Strugariu, Radu. (2014). An Introduction to Nonlinear Optimization Theory. de Gruyter Open.
- Eiselt, H. A. and Sandblom, Carl-Louis. (2019). Nonlinear Optimization: Methods and Applications. Springer Nature Switzerland.
- Luenberger, David, G. and Ye, Yinyu. (2021). Linear and Nonlinear Programming (5th ed.). Springer Nature Switzerland.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(vi): RESEARCH METHODOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

Learning Outcomes: The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

SYLLABUS OF DSE - 5(vi)

UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics;

Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
3. Declaration on Research Assessment. https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment
4. Evaluating Journals using journal metrics; (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till,Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

Practical (30 hours): Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

Syllabi of Semester - VIII based on UGCF - 2022

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-VIII

DISCIPLINE SPECIFIC CORE COURSE (DSC)– 20: FIELD THEORY AND GALOIS EXTENSION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Field Theory and Galois Extension	4	3	1	0	Class XII pass with Mathematics	Group Theory, Ring Theory

Learning Objectives: The objective of the course is to introduce:
- Tools of field theory such as field extensions, splitting fields, normal extensions, separability, and separable extensions.
- Galois extensions and the Fundamental theorem of Galois theory.
- Link between group theory and the roots of polynomials, developed by Galois, to solve the problem of solvability of polynomial equations by radicals.
- Some applications, such as cyclotomic polynomials, finite fields, and simple extensions.

Learning Outcomes: This course will enable the students to:
- Identify and construct examples of fields, distinguish between algebraic and transcendental extensions, and characterize normal extensions in terms of splitting fields.
- Identify and characterize separable extensions, define Galois extensions, construct examples of automorphism groups of a field as well as prove the fundamental theorem of Galois theory.
- Use the Galois theory of equations to prove that a polynomial equation over a field is solvable by radicals if and only if its Galois group is solvable and hence deduce that a general quintic equation is not solvable by radicals.
- Define cyclotomic polynomials and find its Galois group using roots of unity, classify finite fields and prove that every finite separable extension is simple.

SYLLABUS OF DSC-20

UNIT – I: Field Extensions **(15 hours)**

Fields and prime subfields, Field extensions, Degree of field extensions, Tower theorem, Algebraic and transcendental elements, Algebraic and transcendental extensions, Monomorphism of field extensions, Ruler and compass constructions, Splitting fields, Extensions of monomorphisms, Uniqueness of splitting field.

UNIT – II: Galois Extensions and the Fundamental Theorem (15 hours)

Normal extensions, Separability and separable extensions, Monomorphisms and automorphisms of field extension, Galois extensions, Automorphism/Galois groups and fixed fields, Galois theory of polynomials, The fundamental theorem of Galois theory.

UNIT – III: Some Applications and Solvability by Radicals (15 hours)

The Discriminant, Cyclotomic polynomials, extensions and its Galois group, Solution by radicals, Existence and Uniqueness of finite fields, Simple extensions, and the primitive element theorem.

Essential Readings

1. Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.
2. Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.). Wiley.

Suggestive Readings

- Stewart, Ian (2022). Galois Theory (5th ed.). CRC Press. Chapman and Hall.
- Cox, David A. (2012). Galois Theory (2nd ed.). John Wiley & Sons.
- Cohn, P. M. (2003). Basic Algebra, Springer International Edition.

DSE Courses of B.Sc. (Hons) Mathematics, Semester -VIII

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(i): ADVANCED MECHANICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Mechanics	4	3	1	0	Class XII pass with Mathematics	Calculus, Differential Equations, Mechanics

Learning Objectives: The main objective of this course is to:

- Provide the students an enriching experience of the basic concepts of mechanics in space and its related concepts.
- Impart quality understanding to the students about Newtonian, Lagrangian and Hamiltonian mechanics along with practical applications of these concepts in real life.
- Understand the concept of fluid, their classifications, model, and approaches to study the fluid flow.

Learning Outcomes: This course will equip the students with the:

- Fundamental concepts of force systems, generalized coordinates, kinematics of a particle and a rigid body.
- Thorough and in depth understanding of the classification of dynamical systems, Lagrangian and Hamiltonian's equations.
- Formulation of mass and momentum conservation principle; solution for non-viscous flow, the motion of sphere, cylinder, and two-dimensional flow.
- Understanding of the concepts of stress and strain in viscous flow; derivation of Navier-Stokes equation of motion and related problems.

SYLLABUS OF DSE-6(i)

UNIT – I: Newtonian Mechanics (15 hours)

General force systems, Equilibrium of a system of particles, Reduction of a force systems, Equilibrium of a rigid body, Generalized coordinates and constraints, Work and potential energy, Kinematics of a particle and a rigid body. Moments and product of inertia. Kinetic energy and angular momentum, Motion of a particle and a system, Moving frame of reference, Motion of a rigid body.

UNIT – II: Lagrangian and Hamiltonian Mechanics (12 hours)

Lagrange's equations for a particle in plane, Classification of dynamical systems, Lagrange's equations for any simple dynamical system, general dynamical system and for impulsive motion; Applications of Lagrange's equations, Hamiltonian and the Canonical equations of motion, The passage from the Hamiltonian to the Lagrangian, Conservative systems.

UNIT – III: Fluid Mechanics (18 hours)

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Velocity of a fluid particle, Irrotational flow, Velocity potential, Equipotential surfaces, Streamlines and Pathlines, Mass flux density, Conservation of mass leading to equation of continuity, Boundary surface; Forces in fluid flows, Conservation of linear momentum and its mathematical formulation (Euler's equation of motion), Bernoulli's equation, Axi-symmetric flows and motion of sphere; Two-dimensional flows, Motion of cylinder, Stream function, Complex potential, Line sources and line sinks, Line doublet, Milne-Thomson circle theorem; Viscous flow, Stress components in a real fluid, Stress and strain analysis, Navier-Stokes equations of motion and its applications.

Essential Readings

1. Chorlton, F. (2005). Textbook of Fluid Dynamics. CBS Publishers, Delhi. Reprint 2018.
2. Synge, J. L. and Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.

Suggestive Readings

- Gantmacher, F. (1975). Lectures in Analytic Mechanics. MIR publisher, Moscow.
- Goldstein, H., Poole, C.P. and Safco, J.L. (2002). Classical Mechanics. (3rd ed.). Addison Wesley.
- Kundu, Piyush K. and Cohen, Ira M., Dowling, David R. (2016). Fluid Mechanics (6th ed.). Academic Press.
- Mitchell, John W. (2020). Fox and McDonald's Introduction to Fluid Mechanics. (10th ed.). John Wiley & Sons.
- Taylor, John R. (2005). Classical Mechanics. University Science Books.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(ii): CRYPTOGRAPHY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Cryptography	4	3	1	0	Class XII pass with Mathematics	Group Theory, Linear Algebra

Learning Objectives: Primary objective of this course is to:

- Learn challenges and types of attacks on the security of cryptographic protocols.
- Understand concept of confusion and diffusion, that is central to the security of symmetric key cryptography.
- Learn mathematical hard problems, which can be used to build various public key cryptosystems.
- Gain knowledge of post quantum cryptography that resist quantum attacks.

Learning Outcomes: This course will enable the students to:

- Learn classical cryptosystems Caesar cipher, Monoalphabetic cipher, Hill cipher, Vigenère cipher and their security analysis.
- Understand Feistel cipher structure to achieve confusion and diffusion in case of Data Encryption Standard (DES).
- Understand Advanced Encryption Standard (AES) structure and its operations along with key generation.
- Learn key sharing protocol – Diffie Hellman key exchange, Public-key cryptosystems – RSA, Elgamal, and Elliptic curve cryptography.
- Learn Lagrange interpolation secret sharing scheme.
- Learn hash functions and their applications, digital signatures scheme.
- Gain knowledge of code-based cryptography – McEliece cryptosystem.

SYLLABUS OF DSE-6(ii)

UNIT–I: Classical Cryptosystems and Review of Finite Fields (15 hours)

Overview of Cryptography, Symmetric key and Public-key cryptography, Security attacks, Relation between key length and security, Objectives and applications of cryptography primitives, Types of attacks from cryptanalyst view, Kerckhoff's principle; Substitution techniques - Caesar cipher, Monoalphabetic cipher, Hill cipher, Vigenère cipher, One-time pad; Euclidean Algorithm, Modular Arithmetic, Statement of Fermat's, Euler's and Chinese Remainder theorems, Discrete logarithm, Finite fields of the form GF(p) and GF(2^n), Binary and ASCII representation, Pseudo-random bit generation.

UNIT – II: Modern Block Ciphers (12 hours)

Introduction to stream and block ciphers, Diffusion and Confusion, The Feistel cipher Structure, Data Encryption Standard (DES); Advanced Encryption Standard (AES) Structure, AES transformation functions, Key expansion, AES Example.

UNIT – III: Public-key Cryptography, Hash Functions, Digital Signatures and Post Quantum Cryptography (18 hours)

Introduction to Public key cryptography, RSA cryptosystem, Diffie Hellman key exchange, Man in the middle attack, Elgamal cryptosystem, Elliptic curve arithmetic, Elliptic curve cryptography, Secret sharing; Hash functions, Applications of hash functions – MAC and digital signature, Simple Hash functions, Security requirements of Hash functions, Properties of SHA family of hash functions; Digital signatures, Elgamal and Schnorr digital signature scheme; Introduction to post quantum cryptography, Linear codes, Generating matrix, Parity check matrix, McEliece cryptosystem.

Essential Readings

1. Stallings, William (2023). Cryptography and Network Security, Principles and Practice (8th ed.). Pearson Education Limited. Global Edition.
2. Stinson, Douglas R. and Paterson, Maura, B. (2019). Cryptography: Theory and Practice (4th ed.). CRC Press.
3. Trappe, Wade and Washington, Lawrence C. (2020). Introduction to Cryptography with Coding Theory (3rd ed.). Pearson Education International.

Suggestive Readings

- Hoffstein, Jeffrey. Pipher, Jill & Silverman, Joseph H. (2014). An Introduction to Mathematical Cryptography (2nd ed.). Springer New York.
- Goldreich O. (2005). Foundations of Cryptography: Basic tools - Vol.1, Cambridge University Press.
- Goldreich O. (2009). Foundations of Cryptography: Vol.2, Basic applications, Cambridge University Press.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(iii): INDUSTRIAL MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Industrial Mathematics	4	3	0	1	Class XII pass with Mathematics	Calculus, Real Analysis, Linear Algebra, Ordinary and Partial Differential Equations

Learning Objectives: The main objective of this course is to:

- Orient the learners to understand nature and working of industrial systems and their models.
- Familiarize the learners with control and maneuvering of industrial processes through sample case-studies and encourage design-thinking and understanding.

Learning Outcomes: This course will enable the students to:

- Determine the controllability, stability, and observability of a system from the model description.
- Comprehend the signal processing landscape and analyse signals using real and spatial domain representations.
- Model/analyse an industrial system from its description and use mathematical formulations to investigate and manipulate the system for specific objectives.

SYLLABUS OF DSE-6(iii)

UNIT – I: Understanding Systems from their Mathematical Description (15 hours)

Continuous-time linear systems, Laplace transform, Transfer function and analogous systems, State-space models, Block-diagram algebra, Signal flow graph, Order of a system and reduced-order models; Discrete-time systems, Z-transform and its inverse, Feedback systems, Stability: Routh-Hurwitz criterion, Root locus method, Controllability and Observability.

UNIT – II: Mathematical Tools for Signals (15 hours)

Signal-to-noise ratio, Analog and digital messages, Channel bandwidth and rate of communication, Modulation, Randomness and redundancy; Signal energy and power, Period and aperiodic signals, Signal operations, Unit impulse function, Vector representation of signals, Orthogonality, Correlation of signals, Signal representation by orthogonal signal sets.

UNIT – III: Case Studies **(15 hours)**

Sample Cases: Continuous casting, Water filtration, Factory fires, Irrigation.

Essential Readings

1. Fulford, Glenn R., and Broadbridge, Philip (2002). Industrial Mathematics: Case Studies. Cambridge University Press.
2. Kheir, Naim A. (Ed.). (1996). Systems Modeling and Computer Simulation, CRC Press.
3. Lathi, B.P., and Ding, Zhi (2019). Modern Digital and Analog Communication Systems (5th ed.). Oxford University Press.

Suggestive Readings

- Friedman A., and Littman W. (1994). Industrial Mathematics: A Course in Solving Real-World Problems. SIAM (Society for Industrial and Applied Mathematics).
- Kreyszig, Erwin (2011). Advance Engineering Mathematics (10th ed.). John Wiley & Sons.
- MacClauer, Charles R. (2000). Industrial Mathematics: Modeling in Industry, Science, and Government. Prentice Hall, Inc.

Practical (30 hours)- Practical/Lab work using:

Mathematica/MATLAB/SciLab/C/C++/Python/R/FORTRAN or similar as per availability.

1. Use following methods to study, describe, and evaluate continuous/discrete systems:
   (a) Root locus method.
   (b) Routh-Horowitz criterion.
   (c) Transfer function using Laplace transform.
   (d) z-transform to convert continuous systems to equivalent discrete systems.
2. To apply controllability and observability analysis on a system description, using corresponding tools/libraries available.
3. To represent a signal/wave as vector data (sampling, choosing basis, and checking orthogonality).
4. To convolve and deconvolve signal/wave functions and represent the result as graphs.

Case Studies:

Besides reading the mentioned case-studies, ONE case may be chosen (in consultation with the instructor) as Semester Assignment for a brief similar study and analysis.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(iv): GEOMETRY OF CURVES AND SURFACES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Geometry of Curves and Surfaces	4	3	1	0	Class XII pass with Mathematics	Calculus

Learning Objectives: The main objective of this course is to:

- Introduces the concept of curves and surfaces in Euclidean spaces $\mathbb{R}^n$.
- Study of the curves and surfaces via the tools of calculus and introduction of concepts like first and second fundamental forms, curvatures, and differential forms.
- Complete the celebrated Gauss-Bonnet theorem that establishes a connection between curvature of a geometric object with its topology.

Learning Outcomes: This course will enable the students to:

- Understand the concept of curves and surfaces embedded in the Euclidean spaces $\mathbb{R}^n$.
- Compute the curvature and torsion for a curve in the space.
- Understand the concept of differential forms and their integration.
- Make sense of the infinitesimal distance element via the study of the Riemannian metric.
- Get prepared to venture into further study of modern differential geometry of manifolds.

SYLLABUS OF DSE-6(iv)

UNIT – I: Geometry of Curves **(15 hours)**

Concept of plane and space curves with examples, Parametrized plane and space-curves, Concepts of curvatures for curves, Frenet-Serret's formula for space curves, Global theorems for plane and space curves.

UNIT – II: Local Theory of Surfaces in the Space **(15 hours)**

Concept of surfaces in the space with examples, Fundamental forms and curvatures with examples, Orthonormal frames, Exterior differential forms in two variables and their uses.

UNIT – III: Geometry of Surfaces **(15 hours)**

Riemannian metric on a surface, Vector fields, Covariant derivatives, Concept of geodesic, Integration of exterior differential forms, Gauss-Bonnet theorem.

Essential Reading

1. Kobayashi, Shoshichi (2019). Differential Geometry of Curves and Surfaces. Springer Nature Singapore Pte Ltd. https://doi.org/10.1007/978-981-15-1739-6

Suggestive Readings

- Abbena Elsa, Salamon Simon, and Gray Alfred (2006). Modern Differential Geometry of Curves and Surfaces with Mathematica (3rd ed.). CRC Press.
- Carmo, Manfredo P. Do (2016). Differential Geometry of Curves and Surfaces (Revised and Updated Second Edition). Dover Publications.
- Pressley, Andrew (2010). Elementary Differential Geometry (2nd ed.). Springer-Verlag.
- Tapp, Kristopher (2016). Differential Geometry of Curves and Surfaces. Springer.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(v): INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Integral Equations and Calculus of Variations	4	3	1	0	Class XII pass with Mathematics	Ordinary, and Partial Differential Equations, Multivariate Calculus

Learning Objectives: The main objective of this course is to:

- Familiarize the learner with methods for solving Volterra and Fredholm integral equations.
- Know the determination of extremum of functional, necessary condition for an extremum, Euler's equation, and its generalization.

Learning Outcomes: This course will enable the students to:

- Compute the solutions to Volterra integral equations by method of resolvent kernel, method of successive approximations, method of Laplace transform, system of Volterra integral equations and integro-differential equation.
- Determine the solutions of Fredholm integral equations and derivation of Hilbert-Schmidt theorem.
- Understand the formulation of variational problems, the variation of a functional and its properties, extremum of functional, necessary condition for an extremum.

SYLLABUS OF DSE-6(v)

UNIT – I: Volterra Integral Equations (12 hours)

Integral equations, Introduction and relation with linear differential equations; Volterra integral equations and its solutions, Method of resolvent kernel, Method of successive approximations, Convolution type of equation, Method of Laplace transform, System of Volterra integral equations, Integro-differential equation, Abel's integral equation and its generalizations.

UNIT – II: Fredholm Integral Equations (18 hours)

Fredholm integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Integral equations with degenerate kernels, Eigenvalues and eigen functions and their properties, Hilbert-Schmidt theorem, Nonhomogeneous Fredholm integral equation with symmetric kernel, Fredholm alternative.

UNIT – III: Calculus of Variations (15 hours)

Variational problems, Variation of a functional and its properties, Extremum of functional, Necessary condition for an extremum, Euler's equation and its generalization, Variational derivative, General variation of a functional and variable end point problem, Sufficient conditions for the extremum of a functional.

Essential Readings

1. Gelfand, I. M. and Fomin, S.V. (2000). Calculus of Variations. Dover Publications, Inc.
2. Krasnov, M., Kiselev, A. and Makarenko, G. (1971). Problems and Exercises Integral Equations, Mir Publication Moscow.
3. Logan, J. David (1987). Applied Mathematics: A Contemporary Approach, John Wiley & Sons, Inc.

Suggestive Readings

- Hildebrand, F. B. (1992). Methods of Applied Mathematics (2nd ed.). Dover Publications.
- Zemyan, Stephen M. (2012). The Classical Theory of Integral Equations: A Concise Treatment. Birkhäuser.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(vi): MACHINE LEARNING: A MATHEMATICAL APPROACH

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Machine Learning: A Mathematical Approach	4	3	0	1	Class XII pass with Mathematics	Basic Knowledge of Python

Learning Objectives: The main objective of this course is to:

- Gain mathematical insights into the functioning of popular methods of Regression, Classification, Clustering and Dimension reduction.
- Understand the mathematical framework of learning and apply it to assess the performance of a number of regression, classification and density estimation algorithms
- Detect overfitting and employ regularization techniques to control it.

Learning Outcomes: This course will enable the students to:

- Learn how to build popular models of regression and classification including Linear regression, Polynomial regression, Logistic classifier, Support vector machine, Decision Tree, Random forests, Naïve Bayes classifier.
- Evaluate the performance of models on test data through analytical techniques (VC bounds and dimension) and Cross-validation to facilitate model selection and feature selection.
- Improve model performance by controlling overfitting through regularization techniques like Ridge and Lasso.

- Understand when to apply dimension reduction and combine it with other supervised learning methods.
- Understand and implement the key principles of Artificial Neural Networks in the context of regression and classification and employ them in function approximation.

SYLLABUS OF DSE-6(vi)

UNIT – I: Introduction to Machine Learning and its Applications (18 hours)

Overview of different tasks: Regression, Classification, and Clustering. Evaluation metrics– Mean Absolute error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE). Linear Regression, Cost function, Polynomial Regression, Gradient Descent Algorithm (GDA). Logistic Regression: Evaluation metrics - accuracy, precision, recall, confusion matrix, Receiver Operating Characteristic Curve (ROC curve) and Area Under ROC Curve (AUC), Vapnik-Chervonenkis (VC) dimension, VC bounds (only statement). *k*-fold validation, Concepts of training set, validation set and test set, Underfitting-Overfitting, Regularization techniques– Ridge, Lasso for Linear Regression and Logistic Regression, Bias-variance tradeoff.

UNIT – II: Popular Machine Learning Techniques (18 hours)

Cross-entropy and Gini Index, Decision Tree, Regression Tree, Random Forest and Bagging, Tree Pruning. Support Vector Machine (SVM), Kernel SVM (Gaussian) Similarity Criterion, *k*-Means clustering technique. Naive Bayes classifier- Linear Discriminant Analysis (LDA), Quadratic Discriminant Analysis (QDA). Dimensionality Reduction, Feature Selection, Principal Component Analysis (PCA).

UNIT – III: Introduction to Deep Learning (9 hours)

Artificial neural network (ANN), Activation functions – definition and examples (Sigmoid, ReLU, Tanh), neurons, layers, Cost function, Information passing, Back propagation algorithm, Optimizers, Learning rate, Statement of Universal Approximation Theorem for continuous functions, Regularization with ANN, Normalization.

Essential Readings

1. Abu-Mostafa, Y. S., Magdon-Ismail, M. & Lin, H.-T. (2012), Learning from Data, AML Book.
2. James, Gareth., Witten, D., Hastie, T., Tibshirani, R. and Taylor, J. (2023), An Introduction to Statistical Learning: with Applications in Python, Springer Nature Switzerland.
3. Ovidiu Calin, Springer. (2020). Deep Learning Architectures: A Mathematical Approach, Springer Nature Switzerland.

Suggestive Readings

- Deisenroth, M. P., Faisal, A. A., and Ong, C. S. (2020), Mathematics for Machine Learning, Cambridge University Press.
- Shalev-Shwartz, S., and Ben-David, S. (2014), Understanding Machine Learning - From Theory to Algorithms, Cambridge University Press.
- Phillips, Jeff. (2020), Mathematical Foundations for Data analysis, Springer.
- Goodfellow, I., Bengio, Y., and Courville, A. (2016), Deep Learning, MIT Press.
- Hastie, T.; Tibshirani, R., and Friedman, J. (2001), The Elements of Statistical Learning, Springer New York Inc.

Practical (30 hours)- Practical work to be performed in computer lab using Python.

Following lab exercises should be done for at least two classification problems or two regression problems or both, whenever applicable.

Following datasets can be used for classification problems

- https://scikit-learn.org/stable/datasets/toy_dataset.html
  (Toy datasets, Iris plants dataset, handwritten digits dataset, Wine recognition dataset, Breast cancer diagnostic dataset)
- https://pypi.org/project/ISLP/
  (Smarket dataset)

Following datasets can be used for regression problems

- https://scikit-learn.org/0.15/modules/classes.html#module-sklearn.datasets
  (Diabetes dataset, Boston house dataset, California housing dataset, Advertising dataset)

Following tasks needs to be performed for the below mentioned ML techniques in scikit learn (https://scikit-learn.org/stable/), whenever applicable:

- Split the dataset into two parts: training and test. Create and train model on training set and report model performance on test set.
- Test the model performance using k-fold cross validation (take k = 5 or 10) in terms of applicable metrics like – Accuracy, Precision, Recall, MAE, RMSE etc.
- Finding optimal parameters using Grid Search CV, whenever applicable; for example: in case of polynomial regression, employ Grid search CV to find the optimal value of the degree d for which the MSE is least.

<u>Practicals List</u>:

1. Create a Linear regression model. Use one variable at a time, all variable at a time, and statistically significant variables (using co-relation matrix) at a time, and observe the model performance. Preferably work with advertising dataset to predict sales in terms of the above features.
2. Fix a 10th order polynomial and sample 15 noisy data points (that is all 15 points do not lie on this polynomial). This is usually done by adding a white noise $\epsilon \sim N(0,1)$ to the polynomial $f(x)$. Using polynomial regression fit two models: one of order 10 and one of order 2. Compare the in-sample and out-sample errors for both models. Try to observe underfitting-overfitting, if any. In another scenario, take $f(x)$ to be a polynomial of order 50 and sample 15 noiseless data points (all lie on the graph of $f(x)$) and again fit a polynomial model of order 2 and 10. Compare the in-sample and out-sample errors. (refer to Exercise 4.2 and Problem 4.4 of [1]).
3. On the Smarket data, predict direction based on features Lag1 and Lag2. Split the Smarket dataset into training and testing parts in the ratio 80-20. Fit logistic regression on the training data and evaluate its accuracy on the test data via confusion matrix, ROC, and AUC. Plot decision boundary for the logistic regression in the 2D feature space spanned by Lag1 and Lag2 (you might need to rescale the variables).
4. Create decision tree models for classification and regression. Observe the effect of various parameters like - splitting criterion (Gini index, Cross entropy), max depth (for tree pruning). Examine overfitting-underfitting in the associated tree model. Display a decision tree.
5. Create Random Forest models for classification and regression. Observe the effect of number of estimators in the context of overfitting.

6. Create SVM models for classification and regression. Observe the effect of the parameter - kernel.
7. Create LDA and QDA models and assess them preferably on the digits dataset.
8. Create *k*-means cluster model for clustering. Observe the effect of parameter *k* (number of clusters). Plot *k* versus error to find out best *k* (Elbow criterion). Plot clusters in case of 2-dimensional data.
9. Demonstrate Principal Component Analysis (PCA) on a dataset with large number of features.
10. Create an ANN model for both classification and regression. Observe the effect of parameters- hidden layer sizes, activation functions (ReLU, Logistic/Sigmoid, Tanh), optimizers (Adam, Sgd), batch size, learning rate, early stopping, validation fraction, maximum number of iterations. Plot iteration number versus accuracy on training and validation dataset. The mnist dataset may be used to explore real strength of ANN. (https://www.kaggle.com/datasets/oddrationale/mnist-in-csv?resource=download in csv format).