Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part II Semester III
Math304(i): Coding Theory

Time: 3 hours Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Question No. **1** is compulsory. Attempt any **two** parts from questions 2-5 • All questions carry equal marks

(1) Define minimum weight and minimum distance of a linear code. Prove that the minimum weight of a linear code is same as the minimum distance of the code. Also, prove that a linear code that is the null space of a matrix H has minimum distance at least w if and only if every linear combination of $w-1$ or fewer columns of H is linearly independent.

(2) (a) Show or justify that it is not always possible to put the generator matrix of a linear code into echelon canonical form unless the code symbols are chosen from a field.
(b) Prove that step-by-step decoding always results in a code vector.
(c) Prove that the sum of the weights of all code words in an (n, k) linear code over $GF(q)$ is $n(q-1)q^{k-1}$.

(3) (a) If $n \geq (qd-1)/(q-1)$, prove that the number of parity-check digits required to achieve minimum weight d in an (n, k) linear code over $GF(q)$ is at least $(qd-1)/(q-1) - 1 - \log_q d$.
(b) Prove that it is always possible to construct an (n, k) linear code over $GF(q)$ with minimum distance at least d for which the following inequality holds:

$$q^{n-k} > \sum_{i=0}^{d-2} \binom{n-1}{i} (q-i)^i.$$

Also, derive asymptotic form of the bound in the binary case.
(c) Define a 'burst' of length b. Prove that for detecting all burst errors of length b or less with a linear code, b parity-check symbols are necessary and sufficient.

(4) (a) State and prove Reiger's bound for burst codes.
(b) Define a 'quasi-perfect' code. Let H be an $m \times n$ matrix, $2^{m-1} \leq n < 2^m - 1$, where columns are binary representation of the column number. Show that the code which is null space of H is a quasi-perfect code.
(c) Define a product code. Determine the product code of the code generated by

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

with itself. Write down the code words of the product code both in array form as well as in the vector form. Verify that the minimum distance of the product code is the product of minimum distances of the component codes.

(5) Write notes on the following:
(a) Reed-Muller codes
(b) Extended binary Hamming codes
(c) Codes derived from Hadamard matrices.

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