Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part I Semester I

Math-102: FUNCTIONAL ANALYSIS

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Attempt any **Five** questions • Each question carries 14 marks

---

(1) (a) Show that the space l^∞ is complete. [6 Marks]

(b) Prove that every finite dimensional subspace Y of a normed space X is complete. [4 Marks]

(c) In a finite dimensional normed space X, show that any subset $M \subset X$ is compact iff M is closed and bounded. [4 Marks]

(2) (a) Let $T : \mathcal{D}(T) \to Y$ be a linear operator, where $\mathcal{D}(T) \subset X$ and X, Y are normed spaces. Then prove that T is continuous if and only if T is bounded. [5 Marks]

(b) If X is a normed space and Y is a Banach space then show that $B(X, Y)$ is a Banach space. [6 Marks]

(c) Define bounded linear functional and give an example of it. [3 Marks]

(3) (a) If Y is a closed subspace of a Hilbert space H, then prove that $Y = Y^{\perp\perp}$. [7 Marks]

(b) State and prove the Riesz representation theorem for bounded sesquilinear form. [7 Marks]

(4) (a) Show that dual space of the space l^2 is l^2. [4 Marks]

(b) Let X be an inner product space and $T : X \to X$ an isometric linear operator. If dim $X < \infty$, show that T is unitary. [5 Marks]

(c) State and prove Hahn-Banach theorem for normed spaces. [5 Marks]

(5) (a) If the dual space X' of a normed space X is separable, then show that X itself separable. [6 Marks]

(b) If in a normed space X, the closed unit ball $M = \{x : ||x|| \leq 1\}$ is compact, then prove that X is finite dimensional. [6 Marks]

(c) State open mapping theorem. [2 Marks]

(6) (a) Let X and Y be Banach spaces and $T : \mathcal{D}(T) \subset X \to Y$ a closed linear operator. Then if $\mathcal{D}(T)$ is closed in X, show that the operator T is bounded. [7 Marks]

(b) If Y is a closed subspace of a Hilbert space H, then show that $H = Y \oplus Y^{\perp}$. [7 Marks]

✠