Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part II Semester III
MATH303(iv): Computational Fluid Dynamics

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Answer any **five** of the following questions • Each question carries 14 marks.• Symbols are having their usual meaning. • A scientific calculator is allowed for this examination.

(1) (a) Use general method to derive the central difference approximation with truncation error $O(\Delta x)^4$ for $\frac{\partial^2 u}{\partial x^2}$ at a point (i, j) using 5 grid points $u_{i+2,j}$, $u_{i+1,j}$, $u_{i,j}$, $u_{i-1,j}$, $u_{i-2,j}$. [8 marks]

(b) Derive the following expression and find its order of accuracy: [6 marks]

$$\left(\frac{\partial u}{\partial y}\right)_{i,j} = \frac{1}{6\Delta y}(-11u_{i,j} + 18u_{i,j+1} - 9u_{i,j+2} + 2u_{i,j+3}).$$

(2) Using Von-Neumann stability Analysis, discuss the stability of Dufort-Frankel scheme for 1-D heat conduction equation. Solve the following IBVP using the Dufort-Frankel method for two time steps described by the partial differential equation [14 marks]

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \quad 0 \leq x \leq 1,$$

subject to the initial and boundary conditions respectively given by

$$T(x, 0) = \cos(\frac{\pi x}{2}), \quad T(0, t) = 1, \quad T(1, t) = 0, \text{ for } t > 0$$

taking $\Delta x = 0.33$, $r = 0.33$.

(3) Solve the wave equation [14 marks]

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq x \leq 1,$$

subject to the initial conditions $u(x, 0) = x^2$, $\frac{\partial u(x,0)}{\partial t} = 1$ and the boundary conditions $u(0, t) = 0$ and $u(1, t) = 1 + t$, for $t > 0$ by taking $\Delta x = \Delta t = 0.2$ and compute the finite difference solution for $x = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and $t = 0.0, 0.2$.

(4) Derive standard and diagonal 5-point formula of a 2-D Laplace equation. Use them to compute approximate solutions at internal grid points of [14 marks]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

subject to $u(0,y) = 0$, $u(x,0) = 0$, $u(x,1) = 100x$, $u(1,y) = 100y$. Given that computational domain is a square bounded by $0 \leq x, y \leq 1$. Take $\Delta x = \Delta y = h = 1/4$. Compute $u_{1,1}$, $u_{1,2}$, $u_{1,3}$, $u_{2,1}$, $u_{2,2}$, $u_{2,3}$, $u_{3,1}$, $u_{3,2}$, $u_{3,3}$.

(5) With the aid of a neat diagram, elucidate the calculation procedure for evaluating pressure and velocity components for a two-dimensional flow using **Staggered grid**. What is **SIMPLE** algorithm? Give the full sequence of the operations in **SIMPLE** algorithm? How is it different from **SIMPLER** algorithm? Elucidate. [14 marks]

(6) Give a suitable diagram for the problem of heat conduction with sources govern by the equation [14 marks]

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + q = 0.$$

A large plate of thickness $L = 2\ cm$ with constant thermal conductivity k=0.5 W/m.K and uniform heat generation q=1000 kW/m^3. The two faces are at temperatures of 100^{o}C and 200^{o}C respectively. Assuming that the dimensions in the y- and z- directions are so large that temperature gradients are significant in the x-direction only. Divide the whole domain into three control volumes, Calculate the steady state temperature distribution.

(7) Define: (i) Explicit and Implicit finite difference scheme (ii) Consistency, stability and convergence of a finite difference scheme. Find the order of accuracy and discuss the stability of Schmidt and Crank-Nicolson methods with regard to 1-D heat conduction equation using Von-Neumann analysis. [14 marks]

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