Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, Nov.- Dec. 2016
Part II Semester III

MATH-302(iv): COMPUTATIONAL METHODS FOR ODE

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Answer any **5** questions choosing atleast two from each section. • Each question carries 14 marks.• All the symbols have their usual meaning.

Section A

(1) (a) Define the zero stability of a general linear multistep method. Discuss the convergence of two-step method $y_{n+2}-(1+\alpha)y_{n+1}+\alpha y_n=(1/2)h[f_{n+2}+(1-\alpha)f_{n+1}-\alpha f_n]$ for the solution of IVP $y'=-y, y(0)=1$. [7 marks]

(b) Define the consistency of a numerical method $\sum_{j=0}^{k}\alpha_j y_{n+j}=h\phi_f$ for the solution of the IVP $y'=f(x,y), y(a)=\eta$. Derive the conditions for the numerical method to be consistent. [7 marks]

(2) (a) Define the s- stage explicit RK method for the IVP $y'=f(x,y)$, $y(a)=\eta$. Derive the two-stage explicit RK method of order one and two. [7 marks]

(b) Use fourth order RK method to solve IVP $y'=-2xy^2, y(0)=1$ with $h=0.2$ on $[0,0.4]$ and compare it with exact solution. Tabulate the error up to six decimal places. [7 marks]

(3) (a) Define the absolute stability and the region of absolute stability of linear multistep method for the solution of IVP $y'=Ay, y(a)=\eta$ where A is $m\times m$ constant matrix with eigen values $\lambda_t, t=1,2,\cdots m$ with $\mathrm{Re}(\lambda_t)<0, t=1,2,\cdots m$. Find the region of absolute stability for the Euler s backward method and Trapezoidal rule. [7 marks]

(b) Derive the stability function for the absolute stability of general s-stage RK method for the solution of IVP $y'=\lambda y$, where $Re(\lambda)<0$. Also find the interval of absolute stability for two stage explicit RK method of order two. [7 marks]

(4) (a) Discuss the stiffness of the linear system $y'=Ay+\varphi(x)$ where $y,\varphi\in\mathbb{R}^m$ and the constant matrix A has distinct eigen values $\lambda_t\in\mathbb{C}, t=1,2,..m$ with $Re(\lambda_t)<0$. Also define the stiffness and stiffness ratio of a system of differential equation. [7 marks]

(b) Find the stiffness ratio for the system $u'=-10u+9v$, $v'=10u-11v$. What is the largest step length which can be used with four stage Runge-Kutta method of order four. [7 marks]

Section B

(5) (a) Define linear difference operator associated with general linear multistep method. Also define the order and error constant of a general linear multistep method. Prove that the order and error constant are independent of choice of origin in Taylor expansion. [7 marks]

(b) Define the convergence and consistency of the method $\sum_{j=1}^{k} \alpha_j y_{n+j} = h^2 \sum_{j=1}^{k} \beta_j f_{n+j}$ for solution of IVP $y'' = f(x, y)$, $y(a) = \eta_1, y'(a) = \eta_2$. Derive the method $y_{n+2} - 2y_{n+1} + y_n = h^2 f_{n+1}$ for the solution of the IVP and find the order of the method. [7 marks]

(6) (a) Define the local truncation error (LTE) and global truncation error (GTE) of the linear multistep method $\sum_{j=1}^{k} \alpha_j y_{n+j} = h \sum_{j=1}^{k} \beta_j f_{n+j}$. Prove that the LTE of an explicit LMM under the localizing assumption is difference between exact solution and numerical solution. [7 marks]

(b) Show that the method of finite difference reduces the Sturm-Liouville problem $(p(x)w'(x))' + q(x)w(x) = \lambda w(x); w(0) = 0$, $w(1) = 0$ into the system $[A - \lambda I]w = 0$ to find the eigen values and eigen vectors. [7 marks]

(7) (a) Describe the method of finite difference to find the solution of the Non-linear BVP $Y'' = f(t, Y, Y'), a < t < b, Y(a) = g_1$, $Y(b) = g_2$. Also discuss the error in the method. [7 marks]

(b) Reduce the BVP $-(d^2u/dx^2) - u + x^2 = 0, 0 < x < 1$ under the boundary conditions $u(0) = 0, u(1) = 0$ into the equivalent variational form and find the 2-parameter Rayleigh-Ritz approximate solution of the problem. [7 marks]

(8) (a) Reduce the BVP $-(d/dx)[a(x)du/dx] = q(x), 0 < x < L, u(0) = u_0, (adu/dx)_{x=L} = Q_0$ into a weak or variational form explaining the necessary steps involved. [7 marks]

(b) Define general linear multistep method for the solution of an IVP $y' = f(x, y), y(a) = \eta, a \leq x \leq b$. Derive the two step Adam-Bashforth method of order two for solution of the IVP. [7 marks]

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