Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part II Semester III
MATH303(ii): Advanced Group Theory

Time: 3 hours Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Answer **any five** questions • Each question carries 14 marks.

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(1) (a) Let a group G acts on itself via left translations. If $a \in G$, then find the orbit $\mathcal{O}(a)$, and the stabilizer G_a. [4 Marks]

(b) Prove that if a finite group G acts on a set X, then the number of elements in any orbit is a divisor of $|G|$. [6 Marks]

(c) Define a right regular representation of a group. If G is a finite group then show that the right regular representation is a permutation of G. [4 Marks]

(2) (a) Prove that every finite group has a composition series. Give an example of an infinite group which has a composition series. [5,2 Marks]

(b) Prove that the set of all permutation matrices over a field k is a group isomorphic to the permutation group S_n. Deduce that a finite group G of order n can be imbedded in the general linear group $GL(n, k)$ of G. [5,2 Marks]

(3) (a) State and prove Schreier Refinement Theorem. [1,8 Marks]

(b) Prove that if H is a normal subgroup of G and if both H and G/H are solvable, then so is G. [5 Marks]

(4) (a) Prove the following: (i) If H char K and K char G, then H char G. (ii) If H char K and K is a normal subgroup of G, the H is normal subgroup of G. [2,2 Marks]

(b) Prove that a group G is solvable if and only if $G^{(n)} = 1$. [4 Marks]

(c) Show that if G is a finite solvable group, then every minimal normal subgroup is elementary abelian. [6 Marks]

(5) (a) If the number of distinct cosets of the center of a group G is finite then show that the commutator subgroup of G is finite.. [8 Marks]

(b) State Three Subgroups Lemma. Prove that if H, K and L are normal subgroups of a group G then $[L, H, K] \leq [H, K, L][K, L, H]$. [1,5 Marks]

(6) (a) Define the Frattini subgroup of a group. Prove that the Frattini subgroup of a group is the set of all nongenerators. [1,4 Marks]

(b) Prove that the direct product of two nilpotent groups is nilpotent. [4 Marks]

(c) Define the Fitting subgroup of a group. Prove that in a finite group, the Fitting subgroup is characteristic. [1,4 Marks]

(7) (a) Prove that the additive groups $\mathbb{R}$ and $\mathbb{Q}$ are indecomposable. [4 Marks]

(b) Define ascending and descending chain conditions. If a group G has either of the two chain conditions, then show that G is a direct product of a finite number of indecomposable groups. [2,6 Marks]

(c) Give an example of a group which has neither chain conditions. [2 Marks]

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