Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, Nov.-Dec. 2016
Part -II Semester- III
MATH301(iii): GENERAL TOPOLOGY

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Attempt any FIVE questions.

1(a) Let X be a normal Space and A, B be disjoint closed sets in X, then prove that there exists a continuous map $f : X \to [\sqrt{2}, \sqrt{5}]$ such that $f(A) = \{\sqrt{2}\}$ and $f(B) = \{\sqrt{5}\}$. [9 Marks]

(b) Let X_α, $\alpha \in A$, be a family of regular spaces. Then prove that the product space $\prod X_\alpha$ is regular. [5 Marks]

2(a) Let X be a completely regular space, F be a closed subset and K be a compact subset of X with $K \cap F = \emptyset$. Show that there exists a continuous map $f : X \to [0, 1]$ such that $f(K) = \{0\}$ and $f(F) = \{1\}$. [4 Marks]

(b) Let X be a normal space and $A \subseteq X$ be closed, then prove that any continuous map $f : A \to [0, 1]$ can be extended to a continuous map $g : X \to [0, 1]$. [10 Marks]

3(a) Define Stone-Čech compactification $\beta(X)$ of a completely regular space X. Prove that any Hausdorff compactification $\tilde{X}$ of X to which every continuous function of X into a compact, Hausdorff space has an extension, is homeomorphic to $\beta(X)$. [4 Marks]

(b) Prove Urysohn metrization theorem. [10 Marks]

4(a) Prove that a T_3-space X is paracompact if and only if every open covering of X has a σ-locally finite open refinement. [7 Marks]

(b) Define a paracompact space and give an example. Prove that every paracompact, Hausdorff space is a normal space. [7 Marks]

5(a) If every open covering of a T_3 space X has locally finite closed refinement, then prove that X is a paracompact space. [8 Marks]

(b) Prove that every closed subspace of a paracompact space is paracompact. Can we say that every comapct space is a paracompact space? Justify your answer. [6 Marks]

6(a) Let X be a topological space and (Y, d) be a metric space then prove that the set of all continuous functions $\mathcal{C}(X, Y)$ and the set of all bounded functions $\mathcal{B}(X, Y)$ are closed subspaces of Y^X in the uniform topology. [8 Marks]

(b) Let X be a locally compact, Hausdorff space and $\mathcal{C}(X, Y)$ have compact open topology then prove that the map $e : X \times \mathcal{C}(X, Y) \to Y$ defined by $e(x, f) = f(x)$ is continuous. [6 Marks]

7(a) Let X be a topological space, (Y, d) be a metric space, $\mathcal{C}(X, Y)$ have the topology of compact convergence and $\mathcal{F} \subseteq \mathcal{C}(X, Y)$. If $\mathcal{F}$ is equicontinuous under d and the set $\mathcal{F}_a = \{f(a) : f \in \mathcal{F}\}$ has compact closure for each $a \in X$ then prove that $\mathcal{F}$ is contained in a compact subspace of $\mathcal{C}(X, Y)$. [9 Marks]

(b) Let X be a topological space, (Y, d) be a metric space. If $\mathcal{F} \subseteq \mathcal{C}(X, Y)$ is totally bounded under the uniform metric corresponding to d then prove that $\mathcal{F}$ is equicontinuous under d. [5 Marks]

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