Your Roll Number: ................

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, Nov.-Dec. 2016
Part -II Semester- III

**MATH14-303(C): TOPOLOGY II**

Time: 3 hours | Maximum Marks: 70

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**Instructions:** • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Question 1 is compulsory and answer any 5 questions from Question 2 to Question 8.

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1 State, giving reasons, whether following statements are TRUE or FALSE. (Try any FIVE).

(a) Every Hilbert Space is locally compact. [2 Marks]

(b) Every identification map is a closed map. [2 Marks]

(c) Composition of two proper maps is a proper map. [2 Marks]

(d) If for a completely regular space $X$, Stone- Čech compactification $\beta(X)$ is connected, then $X$ is connected. [2 Marks]

(e) If $X$ is a compact space, then $\{\infty\}$ is a component of the one point compactification $X^*$ of $X$. [2 Marks]

(f) Product of two paracompact spaces is paracompact. [2 Marks]

2 (a) If $X$ is a compact, Hausdorff space and $A \subseteq X$ is closed, then prove that the quotient space $X/A$ is compact and Hausdorff. [3 Marks]

(b) Let $f : X \to Y$ be an identification map and $A \subseteq X$ be an $f$-saturated set. If $f$ is an open map or $A$ is an open set, then prove that $g : A \to f(A)$, $a \mapsto f(a)$ is an identification map. [6 Marks]

(c) Define cone $CX$ and suspension $\Sigma X$ of a space $X$. Prove that $\Sigma S^n$ is homeomorphic to $S^{n+1}$. [3 Marks]

3 (a) Prove that a subspace of a locally compact, Hausdorff space is locally compact if and only if it is locally closed. [5 Marks]

(b) Prove that the one point compactification of a locally compact, second countable, Hausdorff space is second countable. [4 Marks]

(c) If $f : X \to Y$ is a proper map and $Y$ is compact, then prove that $X$ is compact. [3 Marks]

4 (a) Let $X$ be a normal Space and $A$, $B$ be disjoint closed sets in $X$, then prove that there exists a continuous map $f : X \to [\sqrt{2}, \sqrt{5}]$ such that $f(A) = \{\sqrt{2}\}$ and $f(B) = \{\sqrt{5}\}$. [9 Marks]

(b) Does there exist a countable, connected, regular space? Justify your answer. [3 Marks]

5 (a) Let $X$ be a completely regular space, $F$ be a closed subset and $K$ be a compact subset of $X$ with $K \cap F = \emptyset$. Show that there exists a continuous map $f : X \to [0, 1]$ such that $f(K) = \{0\}$ and $f(F) = \{1\}$. [4 Marks]

(b) Let $X$ be a normal space and $A \subseteq X$ be closed, then prove that any continuous map $f : A \to (-1, 1)$ can be extended to a continuous map $g : X \to (-1, 1)$. [6 Marks]

(c) Give an example of a space which is completely regular but not normal. [2 Marks]

6 (a) Define Stone- Čech compactification $\beta(X)$ of a completely regular space $X$. Prove that any Hausdorff compactification $\tilde{X}$ of $X$ to which every continuous function of $X$ into a compact, Hausdorff space has an extension, is homeomorphic to $\beta(X)$. [4 Marks]

(b) Prove Urysohn metrization theorem. [8 Marks]

7 (a) Prove that a $T_3$-space $X$ is paracompact if and only if every open covering of $X$ has a $\sigma$-locally finite open refinement. [5 Marks]

(b) Prove that every paracompact, Hausdorff space is a normal space. [4 Marks]

(c) If $f : X \to Y$ is a proper surjection and $X$ is Hausdorff, then prove that $Y$ is also Hausdorff. [3 Marks]

8 (a) Prove that the product of a paracompact space and a compact space is paracompact. [4 Marks]

(b) Define partition of unity on a space $X$. Prove that every open covering of a paracompact, Hausdorff space has a partition of unity subordinate to it. [8 Marks]

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