Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part II Semester III

MATH14-304(C): COMPUTATIONAL METHODS FOR ODES

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper. • Question 1 is compulsory. • Answer **four** questions from 2-6.

(1) State true or false and justify

(a) Simpson's rule has empty region of absolute stability. [3 Marks]

(b) The Euler equation corresponding to the functional $I = \frac{1}{2}\int[(u')^2 + xu^2]dx$ is $-\frac{d}{dx}u' - xu = 0$. [2 Marks]

(c) The range of α so that the roots of the characteristic equation of the difference equation [3 Marks]

$$(1-5\alpha)y_{n+2} - (1+8\alpha)y_{n+1} + \alpha y_n = 0$$

is less than 1 in magnitude is $(-1, 0)$.

(d) The order of the linear multistep method [3 Marks]

$$y_{j+1} + (a-1)y_j - ay_{j-1} = \frac{h}{4}\left[(a+3)y'_{j+1} + (3a+1)y'_{j-1}\right]$$

is 2 if $a = -1$.

(e) Consistency is the necessary and sufficient condition for a numerical method to converge. [3 Marks]

(2) (a) Use the implicit method $y_{j+1} = y_j + \frac{1}{4}(3k_1 + k_2)$, where [6 Marks]

$$k_1 = hf(t_j + h/3, y_j + k_1/3), \qquad k_2 = hf(t_j + h, y_j + k_1)$$

to find the solution of the problem

$$y' = -2ty^2, \quad y(0) = 1, \ 0 \le t \le 0.2, \ h = 0.2.$$

(b) Derive a fourth order method for the solution of the boundary value problem [8 Marks]

$$y'' - q(x)y = d(x), \quad a < x < b$$

with boundary conditions

$$a_0y(a) - a_1y'(a) = \gamma_1, \quad b_0y(b) + b_1y'(b) = \gamma_2.$$

(3) (a) Consider the following predictor corrector method [7 Marks]

$$\begin{aligned} P: &\quad y_{j+1} = y_j + hf_j \\ C: &\quad y_{j+1} = y_j + \tfrac{h}{2}(f_{j+1} + f_j). \end{aligned}$$

Show that the necessary condition for the above method when applied to the equation $y' = \lambda y, \ \lambda < 0$ to converge is $|\lambda h| < 2$. Also, find the expression of the truncation error.

(b) Application of the Ritz finite element method using linear shape functions to the boundary value problem [7 Marks]

$$-y'' = x, \quad y(0) = 0,\ y(1) = 0$$

leads to the system of equations $AY = b$. Determine the matrix A and the coloumn vector b for two elements of equal length.

(4) (a) Obtain the numerical solution of the non-linear boundary value problem [7 Marks]

$$y'' = \frac{1}{2}(1 + x + y)^3,\ y'(0) - y(0) = -1/2, \quad y'(1) + y(1) = 1$$

with $h = 1/2$ using a second order finite difference scheme. Use initial approximation as $y_0^{(0)} = 0.001,\ y_1^{(0)} = -0.1,\ y_2^{(0)} = 0.001$.

(b) Prove that the necessary and sufficient condition for the linear multistep method of the form [7 Marks]

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h\phi_f(y_{n+k}, y_{n+k-1}, \ldots, y_n, x_n, h\ )$$

to be consistent is

$$\rho(1) = 0,\ \phi_f(y(x_n), y(x_n), y(x_n), \ldots y(x_n), x_n, 0)/\rho'(1) = f(x_n, y(x_n)).$$

(5) (a) Find the range of α for which the method [8 Marks]

$$y_{n+2} - (1+\alpha)y_{n+1} + \alpha y_n = \frac{h}{2}\left[f(x_{n+2}, y_{n+2}) + (1-\alpha)f(x_{n+1}, y_{n+1}) - \alpha f(x_n, y_n)\right]$$

applied to the initial value problem $y' = -y,\ y(0) = 1$ is convergent.

(b) Find the interval of absolute stability of R-K second order method. [3 Marks]

(c) Define : **i)** zero stability, **ii)** order of a numerical method, **iii)** error constant. [3 Marks]

(6) (a) Given $\rho(\psi) = \psi^2(\psi - 1)$ find an explicit method. [4 Marks]

(b) Derive the expression for the truncation error and propogation error for the equation [3 Marks]

$$y' = \lambda y, \quad y(t_0) = \eta_0$$

where a numerical method is defined by $y_{j+1} = E(\lambda h)y_j$.

(c) Consider the system [7 Marks]

$$Y' = AY(t) + G(t), \quad Y(t_0) = Y_0$$

where $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}, Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix},\ G(t) = \begin{bmatrix} -2e^{-t} + 2 \\ -2e^{-t} + 1 \end{bmatrix}$,

$Y_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Use Taylor series second order method to find the approximate solution using step size $h = 0.5$ for $0 \le t \le 1$.

✠