Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part II Semester III

MATH14-304(B): COMPUTATIONAL FLUID DYNAMICS

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Answer any **five** of the following questions • Each question carries 14 marks.• Symbols are having their usual meaning. • A scientific calculator is allowed for this examination.

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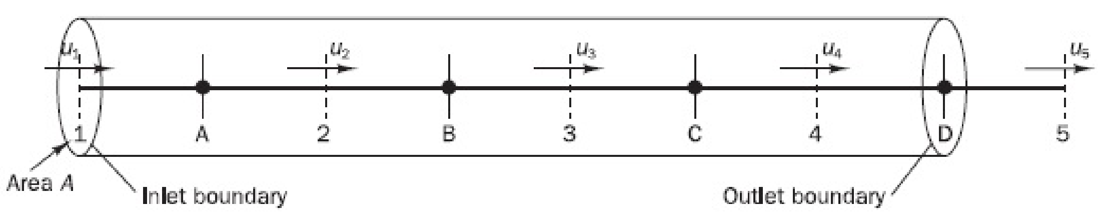
(1) Derive explicit finite volume discretized equation of one-dimensional unsteady heat conduction equation: [14 Marks]

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + S.$$

A thin plate is initially at a uniform temperature 400^0C. At a certain time $t = 0$, the temperature of the east side of the plate is suddenly reduced to 0^0C. The other surface is insulated. Use 3-node explicit finite volume method in conjunction with a suitable time step size to calculate the transient temperature distribution of the slab at time $t = 40s$. Recalculate the numerical solution using a time step size equal to the limit given by Δt in this method. The data are: plate thickness $L = 2cm$, thermal conductivity $k = 10W/m/k$, $\rho c = 10 \times 10^6 J/m^3/k$.

(2) (a) Elucidate **Staggered grid** with the aid of neat diagram. Use it to determine pressure and velocity components of the momentum equations on staggered grids centered around the cell faces. [7 Marks]

(b) Consider the steady, one-dimensional flow of a constant density fluid through a duct with constant cross-sectional area. Using staggered grid shown in the figure below, where the pressure p is evaluated at the main nodes $I = A, B, C$ and D, whilst velocity u is calculated at the backward staggered nodes i=1,2,3 and 4. [7 Marks]
Problem Data. The problem data are as follows: Density $\rho = 1.0kg/m^3$ is constant, Duct area A is constant, Multiplier d in $u' = d(p'_I + p'_{I+1})$ is assumed to be constant. Take d=1.0. Boundary conditions: $u_1 = 10m/s$, $P_D = 0Pa$. Initial guessed velocity field: say $u_2^* = 8.0m/s$, $u_3^* = 11.0m/s$, $u_4^* = 7.0m/s$.
Use the SIMPLE algorithm and above problem data to calculate pressure corrections at nodes I=A to D and obtain the corrected velocity fields at nodes i=2 to 4.



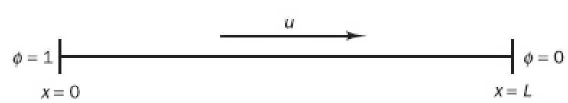
(3) Consider a suitable geometry of a cylindrical fin with uniform cross-sectional area A. The base is at a temperature of $100^0C(T_B)$and the end is insulated. The fin is exposed to an ambient temperature of 20^0C. One-dimensional heat transfer in this situation is governed by [14 Marks]

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0$$

where h is the conductive heat transfer coefficient, P the perimeter, k the thermal conductivity of the material and T_∞ the ambient temperature. Use a uniform grid and divide domain into five control volumes to calculate the temperature distribution along the fin. It is given that: $n^2 = hP/(kA)$, L is the length of the fin and x is the distance along the fin, $L = 1m$, $hP/(kA) = 25/m^2$ (note that kA is constant).

(4) What do you understand by Conservativeness, Boundedness and Transportiveness with regard to the finite volume discretization scheme? Give an assessment of the central difference, Upwind and Hybrid difference in schemes for 1-D Convection-Diffusion problem. What is power law scheme? How is it different from other schemes? [14 Marks]

(5) Derive by using QUICK scheme, the finite volume discretization for one-dimensional convection-diffusion problem. [14 Marks]
Using the QUICK scheme, solve the following problem for $u = 0.2m/s$ on a three point grid. A property Φ is transported by means of convection and diffusion through one-dimensional domain sketched as follows:



Use the required governing equations; the boundary conditions are $\Phi_0 = 1$ at $x = 0$ and $\Phi_L = 0$ at $x = L$. The data of this problem is $F = F_e = F_w = 0.2$, $D = D_e = D_w = 0.5$, $Pe_w = Pe_e = \rho u \delta x / \Gamma = 0.4$.

(6) Derive the Peaceman-Rachford ADI method for two-dimensional heat conduction equation $u_t = u_{xx} + u_{yy}$. Find the solution of the two dimensional heat conduction equations [14 Marks]

$$u_t = u_{xx} + u_{yy}$$

subject to the initial condition

$$u(x, y, 0) = sin(\pi x) sin(\pi y), \qquad 0 \leq x, y \leq 1$$

and the boundary condition

$$u = 0 \text{on the boundary } t \geq 0$$

using the Peaceman-Rachford ADI method with $h = \frac{1}{4}$ and $\lambda = \frac{1}{8}$ and integrate for one time step.

(7) Find the solutions of [14 Marks]

$$u_t + u_x = 0$$

subject to the initial conditions

$$u(x, 0) = \begin{cases} 0, & x < 0 \; and \; x > 4 \\ x/2, & 0 \leq x \leq 2 \\ 2 - \frac{x}{2}, & 2 \leq x \leq 4 \end{cases}$$

using the (i) Lax-Wendroff method and (ii) leap-frog scheme with h=$\frac{1}{2}$ and r=$\frac{1}{2}$. Integrate upto two time steps.