Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part II Semester III

MATH14-304(E): METHODS OF APPLIED MATHEMATICS

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper • Each question carries 14 marks.

Section A

(Answer any 3 questions)

(1) (a) Determine the Euler's equation for the functional depending on higher order derivatives. [6 Marks]

(b) Define the weak and strong extremum. If $\alpha(x)$ and $\beta(x)$ are continuous in $[a, b]$, and if [8 Marks]

$$\int_a^b [\alpha(x)h(x) + \beta(x)h'(x)]dx = 0$$

for every function $h(x) \in \mathcal{D}_1(a, b)$ such that $h(a) = h(b) = 0$, then $\beta(x)$ is differentiable, and $\beta'(x) = \alpha(x)$ for all x in [a,b].

(2) (a) Prove that a necessary condition for differentiable functional $\mathcal{J}[y]$ to have an extremum for $y = \hat{y}$ is that its variation vanishes. [6 Marks]

(b) State the Buckingham Π theorem. A physical phenomenon is described by a law $f(E, P, A) = 0$, where E, P, and A are energy, pressure, and area, respectively. Find the equivalent physical law in terms of dimensionless variables. [8 Marks]

(3) (a) Show that regular perturbation fails on the boundary value problem: $\epsilon y'' + (1+\epsilon)y' + y = 0$, $0 < t < 1$, $0 < \epsilon << 1$ with $y(0) = 0$ and $y(1) = 1$. Find the exact solution and hence, find the inner and outer approximations. [8 Marks]

(b) Show that a weak extremum may not be a strong extremum. [2 Marks]

(c) Find the resolvent kernel for Volterra integral equation ($\lambda = 1$) with kernel [4 Marks]

$$\kappa(x, t) = \frac{8(x-t)}{2x+1} - \frac{4x-2}{2x+1}.$$

(4) (a) State Hilbert Schimdt theorem. [2 Marks]

(b) Solve the following integro-differential equation: [5 Marks]

$$\phi''(x) + \int_0^x e^{2(x-t)}\phi'(t)dt = e^{2x}; \phi(0) = 0, \phi'(0) = 1.$$

(c) Convert the following Volterra integral equation of first kind to second kind and solve the resulting equation: [7 Marks]

$$\sinh x = \int_0^x e^{(x-t)}\phi(t)dt.$$

Section B

(Answer any 2 questions)

(5) (a) Find the general solution of the Euler equation corresponding to the functional: [7 Marks]

$$\mathcal{J}[y] = \int_a^b f(x)\sqrt{1+y'^2}dx.$$

(b) Define variation of a functional. Find the extremals of the following functional: [7 Marks]

$$\mathcal{J}[y] = \int_a^b (y^2 + y'^2 + 2ye^x)dx$$

(6) (a) Find the second iterated kernel $\kappa_2(x,t)$ of the kernel [7 Marks]

$$\kappa(x,t) = e^{|x|+t}; a = -1, b = 1.$$

(b) Define reciprocal kernels. Show that integral equation: [7 Marks]

$$\phi(x) - \lambda\int_0^1 (3x-2)t\phi(t)dt = 0$$

has no characteristic numbers and eigenfunctions.

(7) (a) Reduce the following BVP to an equivalent Fredholm integral equation by using Green's function: [7 Marks]

$$y'' + \lambda y(x) = x, \quad y(0) = \alpha,\ y'(1) = \beta.$$

(b) Investigate the solvability of the integral equation for different values of the parameter λ: [7 Marks]

$$\phi(x) - \lambda\int_0^{2\pi} |x-\pi|\phi(t)dt = x.$$

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