Your Roll Number: ....

Department of Mathematics, University of Delhi
M.A./M.Sc. Mathematics Examinations, November-December 2016
Part II Semester III
MATH14-304(A): Coding Theory

Time: 3 hours | Maximum Marks: 70

Instructions: • Question No. **1** is compulsory • Attempt any **two** parts from questions 2-5 • All questions carry equal marks

(1) For the binary linear code whose generator matrix is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) Find generator matrix G in reduced echelon form.
(b) Find the parity-check matrix H for the code in (a).
(c) Find the code word that has (110) as information symbols. Show that it is in the row-space of G and in the null-space of H.
(d) Form the standard array for this code. Find α_i , where α_i denotes the number of coset leaders of weight i.
(e) Find the modular representation vector.
(f) Find the weight vector.

(2) (a) Let H be a parity-check matrix for a linear code. Show that the coset whose syndrome is v contains a vector of weight w if and only if some linear combination of w columns of H equals v.
(b) If in each coset of the standard array of a linear code, the minimum weight vector that precedes all other minimum weight vectors is taken as coset leader, show that every descendant of a coset leader is a coset leader.
(c) State and prove Plotkin's bound. Also, prove that

$$B(n, d) \leq qB(n-1, d)$$

where $B(n, d)$ denotes the maximum number of code words possible in a linear code of length n over GF(q) with minimum weight at least d.

(3) (a) Show that if all the code words in an (n, k) linear code over $GF(q)$ are arranged as rows of a matrix, each field element appears q^{k-1} times in each column. Assume that no column consists of all zeros.
(b) State and prove Hamming's sphere-packing bound for linear codes. Discuss its geomerical interpretation as well.
(c) State and prove Fire's bound for a burst-error-correcting linear code.

(4) (a) Prove that there exists an (n, k) linear code over $GF(q)$ that corrects any single burst of length $b\,(<n/2)$ or less for which the following inequality is satisfied:

$$q^{n-k} > q^{2(b-1)}[(q-1)(n-2b+1)+1].$$

(b) How do we obtain extended binary Hamming codes from binary Hamming codes? Discuss their decoding as well.

(c) If H is an $n \times n$ Hadamard matrix, show that the matrix:

$$H' = \begin{bmatrix} H & H \\ H & -H \end{bmatrix}$$

is a $2n\times 2n$ Hadamard matrix. Further, prove that corresponding to an $n\times n$ Hadamard matrix, there exists a binary code of length n, $2n$ code vectors and minimum distance $n/2$.

(5) Write notes on the following:

(a) Systematic linear codes

(b) Nonbinary Hamming codes

(c) Product codes.