M.Phil./Ph.D. Coursework Syllabus

The course MATH17-01 Research Methodology is compulsory and the research scholar should study any three other courses. Each course has 4 credit. Final examination will be of 3 hours duration and for 70 marks. Internal assessment will be for 30 marks and will be based on tests, assignments, presentation. Qualifying marks: 55% in each paper (internal + final examination put together).

MATH17-R01 : Research Methodology

Scientific research and literature survey. History of mathematics, finding and solving research problems, role of a supervisor, survey of a research topic, publishing a paper, reviewing a paper, research grant proposal writing, copyright issues, ethics and plagiarism.

Research tools. Searching google (query modifiers), MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, h-index, Google Scholar, ORCID, JStor, Online and open access journals, Virtual library of various countries

Scientific writing and presentation. Writing a research paper, survey article, thesis writing; LaTeX, PSTricks, Beamer, HTML and MathJaX

Software for Mathematics. Mathematica/Matlab/Scilab/GAP

Reference:


MATH17-R02 Advanced Commutative Algebra

Localization of rings and its properties, Integral extensions, Discrete valuation rings, Dedekind domains, Graded rings and modules, Associated graded rings, l-adic completion, Krull’s intersection theorem, Hensel’s lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, Hom functor, Tensor functor, I-torsion functor, Flat modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and Ext functor.

References:

Uniform convergence and differentiation, The Stone-Weierstrass theorem, Contraction principle, Non-expansive maps and Browder fixed point theorem. Integration of vector functions—Bochner integrability.

Differential calculus in normed linear spaces, Gateaux and Frechet derivative of functions, mean value theorems, chain rule, higher order derivatives, Taylor's formula, local and global inverse function theorems, implicit function theorem, extremum problems and Lagrange multipliers.

Spherical distance in the extended complex plane, uniform convergence and local uniform convergence with respect to this metric for sequence of meromorphic functions, normality of families of meromorphic functions and various characterizations.


References:


Topological Vector Spaces. Types of topological vector spaces, separation properties, linear mappings, boundedness and continuity, quotient spaces, examples, Banach --Steinhaus theorem, open mapping theorem, closed graph theorem, Hahn Banach Theorem on topological vector spaces, Weal topologies, weak*-topology of a dual space, compact convex sets, extreme points, Milman's theorem, vector--valued integration, vector valued holomorphic functions.

Banach Algebras. Definition and examples of Banach algebras and *-Banach algebras, Complex homomorphisms, spectrum, Symbolic calculus, group of invertible elements, Ideals and quotient algebras, Gelfand transform, Applications to non-commutative Banach algebras, spectral theorem, symbolic calculus for normal operators, characterization of C*-algebras, unbounded operators

References


Topological transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for continuous interval maps, Characterizing topological mixing in terms of topological transitivity for
continuous interval maps, Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on intervals, Various other definitions of Chaos and their interrelationships.

**Topological Entropy:** Definition using open covers, Examples and properties, Bowen's definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov-Sinai Theorem, Topological entropy of an expansive homeomorphism, of the two sided shift, of the topological Markov chain, of any homeomorphism of the unit circle, of any homeomorphism of closed unit interval, an upper bound for the topological entropy of a diffeomorphism of a finite dimensional Riemannian manifold.

**References**


**MATH17-R06 Convex Analysis and Semidefinite Optimization**

Convex sets, Convexity-preserving operations for a set, Relative interior, Asymptotic cone, Extreme points, Face, Projection operator, Separation theorems, Bouligand tangent and normal cones.

Convex functions, Closedness, Affinity, Epigraphical hull and Lower-bound function of a set, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties.

Sublinear functions, Support function, Calculus of support functions, Norms and their duals, Polarity.

Subdifferential of convex functions, Geometric construction and interpretation, properties of subdifferential, Minimality conditions, Mean-value theorem.

Convex conjugate of a function, Biconjugate of a function, Coercivity, Subdifferentials of extended-valued functions, Calculus rule of conjugate functions.

Positive semidefinite cone, Primal and dual semidefinite program, Fenchel and Lagrangian duality in semidefinite optimization

**References**

MATH17-R07 Hyperbolic System of Conservation Laws and Boundary Layer Theory


*Boundary layer theory:* Laminar boundary layer, Turbulent flow, Turbulent boundary layer; Heat and Mass transfer, conduction, convection and radiation; Thermal boundary layer; Modeling and method of solution of the problems.

**References**


MATH17-R08 Partial Differential Equations: Theory and Numerics


Dispersion and Dissipation analysis of PDEs and its finite difference schemes, Discontinuous solutions. Finite difference schemes for systems of parabolic and hyperbolic PDEs. Analysis of well-posed initial value problem of parabolic and hyperbolic systems, Convergence estimates for parabolic and hyperbolic PDES. Finite difference schemes for curved boundaries of elliptic PDEs.

**References**