Discrete-time optimal control with infinite horizon with examples of applications to resource extraction problems

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After a short introduction to optimal control problems in discrete time and introducing the methodology based on the discrete time Bellman or HJB equation in general, I'm going to talk about sufficient conditions and their necessity. I will derive the weakest sufficient condition in the infinite horizon problems with unbounded payoffs guaranteeing that a solution of HJB solution is the value function. Moreover, it guarantees the existence of such solution and its uniqueness under very weak assumptions. Necessity of various parts of the condition will be also proved. I'll illustrate essentiality of all assumptions as well as essentiality of checking the terminal condition which is a part of the sufficient condition. The analysed class of problems is large enough to consider also both state constraints and state-dependent constraints on controls. Besides, I'll present examples showing emergence of problems, when using various heuristics instead of checking the terminal condition. I'll also mention continuous time analogue of some of the examples.

The weakest terminal condition was derived not in the chase for perfection. It was needed as a tool to make the results in a popular class of problems used in economics provable: e.g. Fish Wars and some economic growth models. The search for counterexamples was motivated by heuristic methods commonly used to derive the value function in papers on these models due to the fact that no standard terminal condition is fulfilled in them and lack of consciousness of its necessity.

I will also indicate the need for analogues working in continuous time.

The talk is partly based on [1], [2] and [3].

[1] A. Wiszniewska-Matyszkiel (2011), *On the terminal condition for the Bellman equation for dynamic optimization with an infinite horizon*, Applied Mathematics Letters 24, 943–949.

[2] A. Wiszniewska-Matyszkiel, R. Singh (2021), *Necessity of the terminal condition in the infinite horizon dynamic optimization problems with unbounded payoff*, Automatica 123, 109332.

[3] R. Singh R, AD. Dwivedi, G. Srivastava, A. Wiszniewska-Matyszkiel, X. Cheng (2020),*A game theoretic analysis of resource mining in blockchain*, Clust Comput 23(3), 2035–2046.

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