M.PHIL./PH.D. COURSEWORK SYLLABUS

The course MATH19-R01 Research Methodology is compulsory. Apart from this a research scholar is required to study any of the three courses from the nine courses MATH19-R02 to MATH19-R10. Total credits of the course work is 16 and credits for each course is 4.

Total Marks in Each Course: 100, Duration of Examination for Each Course: 3 Hrs.
Qualifying Marks for Each Course: 55 (Internal + Final taken together)

MATH19-R01: RESEARCH METHODOLOGY

Total Marks: 100 (Theory: 40, Practical: 30, Internal Assessment: 30)
Duration of Examination: 3 Hrs. (Theory: 2 Hrs., Practical: 1Hr.)
Lab/Theory: 4 Lectures per week

Scientific Research and Literature Survey: History of mathematics, finding and solving research problems, Role of a supervisor, Survey of a research topic, Publishing a paper, Reviewing a paper, Funding agencies, Research grant proposal writing, Copyright issues, Ethics and plagiarism.

Research Tools: MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, h-index, Google Scholar, ORCID, JStor, Online and open access journals.

Scientific Writing and Presentation: Writing a research paper, survey article, thesis writing; LaTeX, PSTricks, Beamer and HTML.

Software for Mathematics: Mathematica/Matlab/Scilab/GAP.

References

MATH19-R02: ADVANCED COMMUTATIVE ALGEBRA

Total Marks: 100 (Theory: 70, Internal Assessment: 30)
Duration of Examination: 3 Hrs. Theory: 4 Lectures per week

Localization of rings and its properties, Integral extensions, Discrete valuation rings, Dedekind domains, Graded rings and modules, Associated graded rings, $I$-adic completion, Krull’s intersection theorem, Hensel’s lemma, Hilbert function, Hilbert polynomial, Dimension theory of Noetherian local rings, Regular local rings, Hom functor, Tensor functor, $I$-torsion functor, Flat
modules, Projective and injective modules, Complexes, Projective and injective resolution, Derived functor, Tor and ext functor.

References

MATH19-R03: TOPICS IN ANALYSIS
Total Marks: 100 (Theory: 70, Internal Assessment: 30)
Duration of Examination: 3 Hrs. Theory: 4 Lectures per week

Uniform convergence and differentiation, Stone-Weierstrass theorem, Contraction principle, Non-expansive maps and Browder fixed point theorem; Integration of vector functions—Bochner integrability.

Differential calculus in normed linear spaces, Gâteaux and Fréchet derivative of functions, Mean value theorems, Chain rule, Higher order derivatives, Taylor's formula, Local and global inverse function theorems, Implicit function theorem, Extremum problems and Lagrange multipliers.

Spherical distance in the extended complex plane, Uniform convergence and local uniform convergence with respect to this metric for sequence of meromorphic functions, Normality of families of meromorphic functions and various characterizations.

Criterions for normality of families of holomorphic functions and their applications to Montel’s theorem, Miranda’s theorem and Bloch’s theorem; Criterions for normality of families of meromorphic functions and their applications to Montel’s theorem, Zalcman’s theorem and Gu’s theorem.

References
3. John B. Conway, Functions of One Complex Variable, Narosa, New Delhi, 2002

MATH19-R04: ADVANCED FUNCTIONAL ANALYSIS
Total Marks: 100 (Theory: 70, Internal Assessment: 30)
Duration of Examination: 3 Hrs. Theory: 4 Lectures per week

**Banach Algebras.** Definition and examples of Banach algebras and *-Banach algebras, Complex homomorphisms, Spectrum, Symbolic calculus, Group of invertible elements, Ideals and quotient algebras, Gelfand transform, Applications to non-commutative Banach algebras, Spectral theorem, Symbolic calculus for normal operators, Characterization of C*-algebras, Unbounded operators.

**References**

**MATH19-R05: TOPOLOGY AND MIXING**

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**  
Duration of Examination: **3 Hrs.** Theory: **4 Lectures per week**

**Topological Transitivity:** Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for continuous interval maps, Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on intervals, Various other definitions of chaos and their interrelationships.

**Topological Entropy:** Definition using open covers, Examples and properties, Bowen’s definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov-Sinai theorem, Topological entropy of an expansive homeomorphism, of the two sided shift, of the topological Markov chain, of any homeomorphism of the unit circle, of any homeomorphism of closed unit interval, an upper bound for the topological entropy of a diffeomorphism of a finite dimensional Riemannian manifold.

**References**
MATH19-R06: CONVEX AND NONSMOOTH ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs. Theory: 4 Lectures per week**

Convex sets, Convexity-preserving operations for a set, Relative interior, Asymptotic cone, Extreme points, Face, Projection operator, Separation theorems, Bouligand tangent and normal cones.

Convex functions, Closedness, Affinity, Epigraphical hull and lower-bound function of a set, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties; Sublinear functions, Support function, Calculus of support functions, Norms and their duals, Polarity.

Subdifferential of convex functions, Geometric construction, interpretation and properties of subdifferentials, Minimality conditions, Mean-value theorem; Calculus rule with subdifferentials.

References

MATH17-R07: HYPERBOLIC SYSTEM OF CONSERVATION LAWS AND BOUNDARY LAYER THEORY

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs. Theory: 4 Lectures per week**


Boundary layer theory: Laminar boundary layer, Turbulent flow, Turbulent boundary layer; Heat and Mass transfer, conduction, convection and radiation; Thermal boundary layer; Modeling and method of solution of the problems.

References


**MATH19-R08: PARTIAL DIFFERENTIAL EQUATIONS: THEORY AND NUMERICS**

Total Marks: **100** (Theory: 70, Internal Assessment: 30)

Duration of Examination: **3 Hrs. Theory: 4 Lectures per week**

Maximum principles for second order linear parabolic, elliptic and hyperbolic partial differential equations; Weak solutions for second order linear parabolic, elliptic and hyperbolic partial differential equations; Lax-Milgram theorem, Local existence, uniqueness and regularity results for second order linear parabolic, elliptic and hyperbolic partial differential equations.

Dispersion and dissipation analysis of PDEs and its finite difference schemes, Discontinuous solutions; Finite difference schemes for systems of parabolic and hyperbolic PDEs; Analysis of well-posed initial value problem of parabolic and hyperbolic systems, Convergence estimates for parabolic and hyperbolic PDEs, Finite difference schemes for curved boundaries of elliptic PDEs.

**References**


**MATH19-R09: OPERATORS AND FUNCTION THEORY ON THE UNIT CIRCLE**

Total Marks: **100** (Theory: 70, Internal Assessment: 30)

Duration of Examination: **3 Hrs. Theory: 4 Lectures per week**

Review of topics dealing with the basic properties of the Lebesgue and Hardy spaces on the unit circle and their inter-connections, Review of Beurling’s theorem with an elementary proof, Review of basic operator theory on Hilbert spaces, Review of definition and basic properties of Banach algebras.

Basic properties of algebras of functions on the unit circle that shall include the Disk Algebra, the algebra of bounded analytic functions, the algebra of essentially bounded functions and some of their important sub-algebras; Wermer’s Maximality theorem, Closed and maximal ideals of the algebras $C(T)$; Classifying closed ideals of the Disk Algebra via invariant subspaces, Properties of maximal ideals of the algebra of bounded analytic functions, Bohr’s inequality on the disk algebra and its application to the von Neumann inequality conjecture on non-unital Banach algebras, Uniform algebras: definition and simple examples, Bohr’s inequality on uniform algebras.

References

MATH19-R10: INTRODUCTION TO SEVERAL COMPLEX VARIABLES

Total Marks: 100 (Theory: 70, Internal Assessment: 30)
Duration of Examination: 3 Hrs. Theory: 4 Lectures per week

Definitions of holomorphic function and equivalence of these definitions, Cauchy formula, Cauchy’s inequality, identity theorem, open mapping theorem, Weierstrass’s theorem, Montel’s theorem, Holomorphic mapping.

Hartogs’s phenomenon, Weierstrass preparation theorem, division theorem and their consequences, Zero set of holomorphic functions, Analytic set, regular and singular points of an analytic set.

Automorphism group of bounded domains, Poincaré theorem: The ball and the polydisc are not biholomorphic.


References